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Accurate calibration of the velocity-dependent one-scale model for domain walls

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ABSTRACT

We study the asymptotic scaling properties of standard domain wall networks in several cosmological epochs. We carry out the largest field theory simulations achieved to date, with simulation boxes of size 2048^3, and confirm that a scale-invariant evolution of the network is indeed the attractor solution. The simulations are also used to obtain an accurate calibration for the velocity-dependent one-scale model for domain walls: we numerically determine the two free model parameters to have the values $c_w = 0.34 \pm 0.16$ and $k_w = 0.98 \pm 0.07$, which are of higher precision than (but in agreement with) earlier estimates.

1. Introduction

A key consequence of cosmological phase transitions is the formation of topological defects [1,2]. While cosmic strings have attracted most of the community's attention, domain walls are useful as a testbed case with which one can gather information relevant for other more complex defects (despite being tightly constrained by observations [3,4]). Here we take advantage of ever-improving computing resources to carry out a large set of 2048^3 high-resolution simulations of domain walls, using the standard Press–Ryden–Spergel (PRS) algorithm [5]. This is a follow-up on [6], where the results of simulations of size up to 1024^3 were presented, and we confirm and expand their results.

Early generations of domain wall simulations [5,7–13] found some hints for late-time deviations from the scale-invariant evolution, which would be the expected behavior [14,12]. Our previous work [6] found no such deviations, which provided support for the hypothesis that the earlier results were simply a consequence of the limited dynamical range of numerical simulations. We believe that the present work clearly confirms this.

Macroscopic properties of defect networks can be accurately described by an analytic velocity-dependent model, first derived for cosmic strings [15–17]. The large-scale features of the network are described by a characteristic scale L (which one can interchangeably think of as a typical defect separation or correlation length) and a microscopically averaged (root-mean-squared) velocity v. This has the advantages of tractability and conceptual simplicity but must include phenomenological parameters which parametrize our ignorance about certain dynamical mechanisms. The only way to accurately determine the correct values of these parameters is by employing large-scale numerical simulations to calibrate them. The main goal of the current work is precisely to improve this calibration.

2. Numerical simulations

We will study simple (single-field) domain wall networks in flat homogeneous and isotropic Friedmann–Robertson–Walker (FRW) universes. (Throughout the Letter we shall use fundamental units, in which $c = \hbar = 1$.) A scalar field ϕ with Lagrangian density

$$\mathcal{L} = \frac{1}{2}\phi_{,\alpha}\phi^{,\alpha} - V_0\left(\frac{\phi^2}{\phi_0^2} - 1\right)^2, \tag{1}$$

provides the simplest case. By standard variational methods we obtain the field equation of motion (written in terms of physical time t)

$$\frac{\partial^2\phi}{\partial t^2} + 3H\frac{\partial\phi}{\partial t} - \nabla^2\phi = -\frac{\partial V}{\partial\phi}. \tag{2}$$

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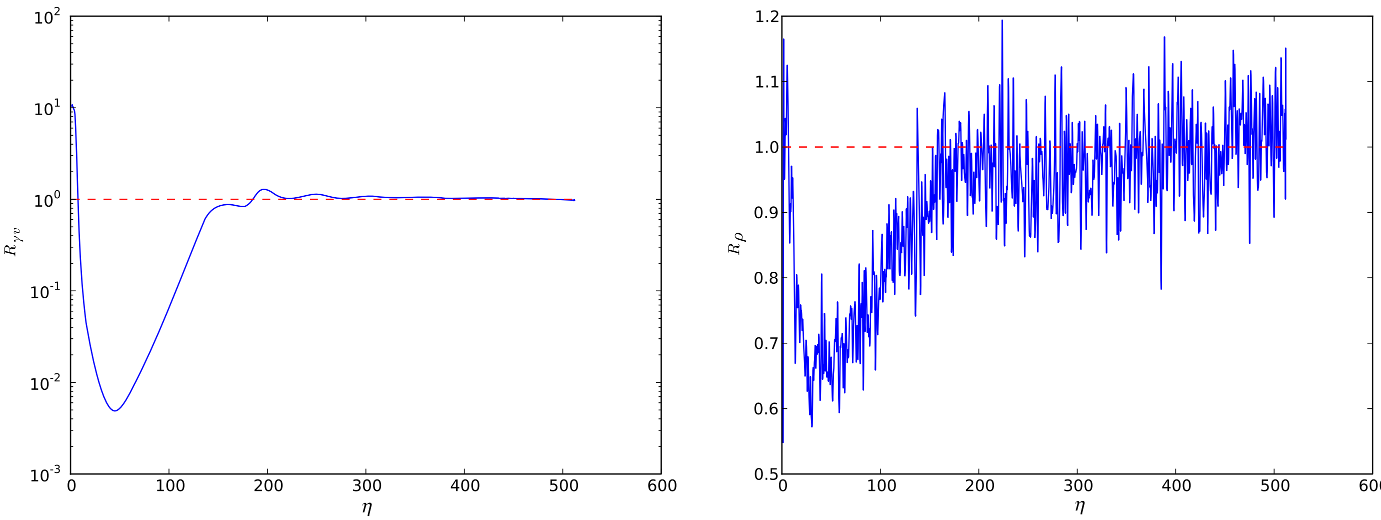

Fig. 1. The evolution of the velocity and density ratios ($R_{\gamma v}$ and R_ρ, defined in the text) for the average of two sets of ten 1024^3 matter era simulations with $W_0 = 100$ and $W_0 = 10$. As expected, both ratios become unity after a transient period.

where ∇ is the Laplacian in physical coordinates, $H = a^{-1}(da/dt)$ is the Hubble parameter and a is the scale factor, which we generically assume to vary as $a \propto t^\lambda$. In what follows we will study the network's evolution in several such cosmological epochs.

We follow the procedure of Press, Ryden and Spergel [5], modifying the equations of motion in such a way that the thickness of the domain walls is fixed in co-moving coordinates. The reliability of this method has been numerically tested in previous work [5,11,18]. In the PRS method, Eq. (2) becomes:

$$\frac{\partial^2\phi}{\partial\eta^2} + \alpha\left(\frac{d\ln a}{d\ln\eta}\right)\frac{\partial\phi}{\partial\eta} - \nabla^2\phi = -a^\beta\frac{\partial V}{\partial\phi}, \tag{3}$$

where η is the conformal time and α and β are constants: $\beta = 0$ is used in order to have constant co-moving thickness and $\alpha = 3$ is chosen to require that the momentum conservation law of the wall evolution in an expanding universe is maintained [5]. The specific parameters used in the simulations are $\phi_0 = 1$, $V_0 = \pi^2/2W_0^2$, where $W_0 = 10$ is the wall thickness; these choices are also justified by previous work on this algorithm [5,11,18].

Despite these previous results one may wonder whether the chosen wall thickness is sufficient to accurately calibrate the model. Fig. 1 compares two series of ten 1024^3 matter era runs with $W_0 = 10$ and $W_0 = 100$. Plotted are the ratios of the network velocities ($R_{\gamma v} = (\gamma v)_{W_0=100}/(\gamma v)_{W_0=10}$) and densities ($R_\rho = (\rho)_{W_0=100}/(\rho)_{W_0=10}$). After transients due to the choices of initial conditions, both of these ratios converge to unity, with statistical uncertainties well below 10%. This convergence is expected to be stronger with larger ensembles and/or larger boxes.

Eq. (3) is integrated using a standard finite-difference scheme. We assume the initial value of ϕ to be a random variable between $-\phi_0$ and $+\phi_0$ and the initial value of $\partial\phi/\partial\eta$ to be zero. This will lead to large energy gradients in the early timesteps of the simulation, and the network needs some time (which is proportional to the wall thickness) to wash away these initial conditions. Our simulations start at a conformal time $\eta_0 = 1$ and evolve in timesteps $\Delta\eta = 0.25\eta_0$ until a conformal time equal to half the box size (that is, $\eta = 1024$).

The conformal time evolution of the co-moving correlation length of the network ξ_c (specifically $A/V \propto \xi_c^{-1}$, A being the co-moving area of the walls) and the wall velocities (specifically γv, where γ is the Lorentz factor) are directly measured from the simulations, using techniques previously described in [12]. However here we use a newly parallelized version of the code, optimized for the Altix UV1000 architecture of the COSMOS Consortium's supercomputer.

3. Analytic model

In order to model a defect network one starts from the microscopic equations of motion (the Nambu–Goto equations, in the case of strings) and, through a suitable averaging, arrives at 'thermodynamic' evolution equations. The non-trivial part of this procedure is the inclusion of terms to account for defect interactions and energy losses. Such terms must be added in a phenomenological way, and for their calibration one must resort to numerical simulations.

For cosmic strings, this procedure leads to the velocity-dependent one-scale (VOS) model [15–17], which has been thoroughly tested against simulations. One can follow an analogous procedure both for the case of monopoles [19] and for domain walls. This latter case was first studied in [12], and more recently [6] provided a preliminary calibration; here we will provide a more quantitative one.

The evolution equation for the characteristic wall length scale L (which is related to the wall density ρ_w via $L = \sigma/\rho_w$, where σ is the domain wall energy per unit area) and their RMS velocity v, are as follows

$$\frac{dL}{dt} = \left(1 + 3v^2\right)HL + c_w v, \tag{4}$$

$$\frac{dv}{dt} = \left(1 - v^2\right)\left(\frac{k_w}{L} - 3Hv\right). \tag{5}$$

Here c_w and k_w are the free parameters: the former quantifies energy losses, while the latter quantifies the (curvature-related) forces acting on the walls. To a first approximation, these are expected to be constant. Note that in the context of the VOS model the characteristic length scale L can further be identified with the physical correlation length ξ_{phys}. The co-moving version of this was defined in the previous section, and the two are related via

$$\xi_{phys} = a\xi_c, \tag{6}$$

and we are therefore assuming that $\xi_{phys} \equiv L$. Note that if

$$\xi_c \propto \eta^{1-\delta}, \tag{7}$$

then, for an expansion rate λ defined as before,

$$\xi_{phys} \propto t^{1-\delta(1-\lambda)}. \tag{8}$$