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On gravity localization under Lorentz violation in warped scenario

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ABSTRACT

Recently Rizzo studied the Lorentz Invariance Violation (LIV) in a brane scenario with one extra dimension where he found a non-zero mass for the four-dimensional graviton. This leads to the conclusion that five-dimensional models with LIV are not phenomenologically viable. In this work we re-examine the issue of Lorentz Invariance Violation in the context of higher-dimensional theories. We show that a six-dimensional geometry describing a string-like defect with a bulk-dependent cosmological constant can yield a massless 4D graviton, if we allow the cosmological constant variation along the bulk, and thus can provides a phenomenologically viable solution for the gauge hierarchy problem.

1. Introduction

After a long time of experimental success showed by the data gathered until now, it has been suggested that the Special theory of Relativity and the General theory of Relativity are actually effective theories, which must be replaced by some other theory in specific scenarios. As an example, in the cosmological scale the energies of cosmic rays would exhibit a Greisen–Zatsepin–Kuz'min cut off below 100 EeV [1,2], predicted by General Relativity, whereas it had been detected cosmic rays above this threshold [3].

From the purely theoretical side, the incorporation of gravity in Quantum Field Theory naturally leads to a minimal measurable length in the ultraviolet regime. Some prominent approaches to Quantum Gravity, such as String Theory [4–6] and Loop Quantum Gravity [7], indicate the existence of a minimal length of the order of the Planck length $\ell_P \sim 10^{-35}$ m, yielding a minimal length measurable. This turns to be difficult to reconcile with the Lorentz Invariance, and then motivates investigation of Lorentz Invariance Violation (LIV) effects.

One route to achieve LIV is by spontaneous symmetry breaking, through the vacuum expectation value (v.e.v.) of a tensor, or in the more common cases, a four-vector which breaks the spacetime isotropy, giving a preferred frame of reference. In such context, it was proposed by Kostelecky and co-workers the so-called Standard Model Extension (SME), which furnishes a set of gauge-invariant LIV operators and then a framework to investigate LIV.

Within the Standard Model (SM) there is an interesting issue, which is the *gauge hierarchy problem*. The energy scale which characterizes the symmetry breaking for the SM is of the order of 10^{14} GeV (below of this scale the interactions are split into the electroweak and strong ones). However, the theory of electroweak interactions (the Glashow–Weinberg–Salam Model) itself predicts a symmetry breaking at the scale of 10^2 GeV; using the Higgs mechanism to implement such symmetry breaking it requires a parameter tuning up to 24 digits, which means perturbative corrections at $\mathcal{O}(10^{-24})$. This is taken as an imperfection of the SM, and has compelled the research of the Physics beyond it. Such a problem was addressed in the seminal work of Randall and Sundrum (RS) [8].

A framework with Lorentz Invariant Violation in an extra-dimensional scenario had already been considered. Namely, the five-dimensional case, with one extra *flat* dimension was studied using the Standard Model extension (SME) [9–11]. In Ref. [12], Rizzo showed that LIV in a Randall–Sundrum (RS) scenario induces a non-zero mass for the four-dimensional graviton, resulting both from the curvature of AdS_5 space and the loss of coordinate invariance. This leads to the conclusion that five-dimensional models with LIV are not phenomenologically viable.

However, this does not forbids LIV in higher-dimensional models. In this work we shall show that a six-dimensional geometry can yield a massless 4D graviton, if we allow the cosmological constant variation along the bulk.

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2. Randall–Sundrum gravity with LIV

In this section we review the work of Rizzo [12]. In particular, we restrict ourselves to the part concerning the Kaluza–Klein (KK) spectrum of the gravitational field.

The original setup of the RS model is based on the Einstein–Hilbert action

$$\mathscr{A} = -\frac{M_5^3}{2}\int d^4x\,dy\,\sqrt{-g}(R+2\Lambda), \tag{1}$$

where Λ is a bulk five-dimensional cosmological constant, M_5 is the five-dimensional reduced Planck scale, and y represents the extra dimension coordinate. The proposed extension for the action (1) which involves LIV follows Refs. [13,14]:

$$\mathscr{A} = -\frac{M_5^3}{2}\int d^4x\,dy\,\sqrt{-g}\big(R+2\Lambda-\lambda s^{ab}R_{ab}\big), \tag{2}$$

where λ is a dimensionless constant of the order of unity which measures the strength of the Lorentz invariance violation and $s^{ab}=u^a u^b$ is a constant tensor defined by the v.e.v. of the 5-vector $u^a=(0,0,0,0,1)$. The equations of motion following from the action (2) are

$$G_{ab} = \Lambda g_{ab} + \lambda \mathcal{F}_{ab}, \tag{3}$$

where G_{ab} is the Einstein tensor and

$$\mathcal{F}_{ab} = -\frac{1}{2}R_{cd}s^{cd}g_{ab} + 2g_{d(a}R_{b)c}s^{cd} + \frac{1}{2}\nabla_c\nabla_d s^{cd}g_{ab} + \frac{1}{2}\Delta s_{ab} - g_{d(a}\nabla_{|c|}\nabla_{|b)}s^{cd}$$

is a tensor arising from the LIV term in the (2). The parenthesis bracketing the indices denotes symmetrization,

$$T_{(ab)} = \frac{1}{2}(T_{ab}+T_{ba})$$

and Δ denotes the Laplace operator $\Delta = \nabla^a\nabla_a$.

In order to investigate the implication of such additional term in the four-dimensional effective field theory, we need first identify the massless gravitational fluctuations around the background solution. Since in this case the background solution is

$$ds^2 = e^{-2ky}\eta_{\mu\nu}\,dx^\mu\,dx^\nu - dy^2, \tag{4}$$

we take tensor fluctuations of the form

$$ds^2 = e^{-2ky}(\eta_{\mu\nu}+\gamma_{\mu\nu})\,dx^\mu\,dx^\nu - dy^2, \tag{5}$$

where $\gamma_{\mu\nu}$ represents the physical graviton in the four-dimensional theory. Therefore, the massless gravitational fluctuations are identified by the resulting linearized equations obtained from Eq. (3).

In the case of a *flat* five-dimensional background, the derivation was performed by Carroll and Tam [15], and it was found the equations of motion

$$\Box\gamma_{\mu\nu} = \lambda\partial_y^2\gamma_{\mu\nu}. \tag{6}$$

This gives the Kaluza–Klein masses $m_n^2 = n^2(1+\lambda)/R^2$, and wave-functions of trigonometric form. This result agrees with the results found for scalar and fermionic fields, already discussed in Ref. [12]. It is worthwhile to notice that the LIV parameter λ does not leads to a bulk mass term in the equation of motion, and thus it does not have any inconsistency with the 4D behavior for the graviton.

In curved spacetime however, the picture changes. The equation of motion for the yy component is non-dynamical and relates the cosmological constant to the LIV parameter λ:

$$\Lambda = -6k^2\left(1+\frac{\lambda}{3}\right).$$

The usual RS result is clearly obtained in the limit $\lambda \to 0$. The difference is that since the value of λ varies, its decreasing to large negative values causes a topology change $AdS_5 \to dS_5$. Then, for consistency with the RS model there must be imposed the condition $\lambda > -3$. Moreover, in this case there are no brane tensions to tune, and thus the other equations of motion are automatically satisfied.

The linearized equations of motion for the tensor fluctuations $\gamma_{\mu\nu}$ are given by

$$-\Box\gamma_{\mu\nu} + e^{-2ky}(1+\lambda)\partial_y^2\gamma_{\mu\nu} - 4ke^{-2ky}(1+\lambda)\partial_y\gamma_{\mu\nu} - 8k^2\lambda e^{-2ky}\gamma_{\mu\nu} = 0 \tag{7}$$

and under the Kaluza–Klein decomposition

$$\gamma_{\mu\nu}(x^\rho, y) = \sum_{n=0}^{\infty} M_{\mu\nu}^{(n)}(x^\rho)\phi_n(y), \tag{8}$$

where we require $\Box M_{\mu\nu}^{(n)} = -m_n^2 M_{\mu\nu}^{(n)}$, we can find the graviton equations of motion, namely

$$-\partial_y\big(e^{-4ky}\partial_y\phi_n\big) + m^2 e^{-4ky}\phi_n = \tilde{m}_n^2 e^{-2ky}\phi_n, \tag{9}$$

where $\tilde{m}_n^2 = m_n^2/(1+\lambda)$ and

$$m^2 = 8k^2\frac{\lambda}{1+\lambda}. \tag{10}$$

From Eq. (9) we can see that, unlikely the flat case, the curvature of the AdS_5 space (encoded in the constant k) together with the Lorentz violation led to a bulk mass term (10),

$$m^2 = 8k^2\frac{\lambda}{1+\lambda},$$

which is zero only when $\lambda = 0$, that is, when there is no Lorentz violation. Therefore the presence of LIV prevents the existence of a massless KK zero mode, leading to a phenomenological inconsistency since the zero mode is associated to the ordinary graviton in 4D. Since the bulk graviton mass depends on the nature of the background metric, in the next section we shall construct an example of a curved metric which yields a massless KK zero-mode.

3. Gravity on a string-like defect with LIV

Now we shall see how gravity behaves in a brane with specific structure. As our geometric background, we choose a local string defect, which was already studied in Ref. [16].

3.1. String-like defect: Einstein's equations and consistency relations

Let us recall the geometric setup of [16]. In six dimensions, we start from the field equations

$$G_{ab} = \Lambda g_{ab} + \frac{1}{M_6^4}T_{ab}, \tag{11}$$

where Λ is the bulk cosmological constant T_{ab} is the stress-energy tensor and M_6 is the bulk reduced mass scale. Also, we assume the existence of a six-dimensional geometry which respects the Poincaré invariance in the 4D brane, namely

$$ds^2 = A(\rho)\eta_{\mu\nu}\,dx^\mu\,dx^\nu - d\rho^2 - R_0^2 B(\rho)\,d\theta^2, \tag{12}$$

where the two extra dimensions are chosen to be polar coordinates $\rho \in [0,\infty)$ and $\theta \in [0, 2\pi)$.