Physics Letters B 663 (2008) 317–321

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Ground-state triply and doubly heavy baryons in a relativistic three-quark model

A.P. Martynenko

Samara State University, Pavlov Street 1, Samara 443011, Russia

ARTICLE INFO

Article history:
Received 6 September 2007
Received in revised form 19 March 2008
Accepted 15 April 2008
Available online 18 April 2008
Editor: W. Haxton

PACS:
12.40.Yx
12.39.Ki
13.40.Hq

Keywords:
Triply and doubly heavy baryons
Relativistic quark model

ABSTRACT

Mass spectra of the ground-state baryons consisting of three or two heavy (b or c) and one light (u, d, s) quarks are calculated in the framework of the relativistic quark model and the hyperspherical expansion. The predictions of masses of the triply and doubly heavy baryons are obtained by employing the perturbation theory for the spin-independent and spin-dependent parts of the three-quark Hamiltonian.

The transition from two-quark bound states to three-quark bound states opens new problems which refer both to the form of the quark interaction in the baryon and the structure of three-quark relativistic wave equation describing this system. In general form they were studied and solved already by many authors [1–8] (see other references in Ref. [6]). In practice it is important to have an approach which can allow to obtain simple and reliable estimates for the different experimental quantities regarding to the baryon spectroscopy, the production and decay rates. Whereas heavy baryons with one or two heavy quarks were investigated both theoretically and experimentally, the triply heavy baryons $\Omega_{Q_1Q_2Q_3}$ containing b- and c-quarks have not been studied so much. The estimate of masses of the lowest-lying (ccc), (ccb), (bbc) and (bbb) states is presented in Refs. [9–11]. Their production in a c- or b-quark fragmentation is calculated in Refs. [12–14]. The doubly heavy baryons (Q_1Q_2q) represent a unique part of three-quark systems. Two heavy quarks compose a localized quark nucleus while the light quark moves around this color source at a distance of order ($1/m_q$). This picture leads to the quark–diquark model for doubly heavy baryons which was used in Refs. [15–17] for the description of the mass spectrum and decay widths. Moreover, relativistic and bound state corrections to the mass spectra of mesons and baryons (in the quark–diquark approximation) and their different decay rates were also considered in the relativistic quark model [15–19]. Estimates for the masses of baryons containing two heavy quarks have been presented by many authors [4,20–24] using different QCD inspired models for the quark interactions. The aim of the present paper is a twofold one. First, we go beyond the scope of the quark–diquark approximation in Refs. [15,16] treating the total baryon Hamiltonian as a sum of two-quark interactions and using the hyper-radial approximation for the ground state triply and doubly heavy baryons. Secondly, we take into account relativistic and bound state corrections of order v^2/c^2 by the perturbation theory. So, the purpose of our new investigation consists in the elaboration of an alternative calculational scheme of the baryon mass spectrum as compared with the earlier performed investigations in Refs. [15,16] through the use of the three-quark approach to the baryon problem formulated in Refs. [20–22] with the Hamiltonian containing the spin-independent and spin-dependent corrections of order v^2/c^2.

It should be noted, that the theoretical results on the mass spectrum of triply and doubly heavy baryons [5–7] remain untapped. In the past several years, the SELEX Collaboration has reported the first observation of doubly charmed baryons [25,26]. But most recently, BaBar Collaboration has reported that they have not found any evidence of doubly charmed baryons in e^+e^- annihilation [27]. Nevertheless, we can expect that the mass spectra and decay rates of triply and doubly heavy baryons will be measured before long. This gives additional grounds for new theoretical investigations of triply and doubly heavy baryon properties.

In order to describe the mass spectra of baryons the different three-quark Hamiltonians are used [7,15,20,28]. The effective Hamiltonian of Refs. [20–22] devoted to the calculation of the mass

E-mail address: mart@ssu.samara.ru.

0370-2693/$ – see front matter
doi:10.1016/j.physletb.2008.04.030

spectra of doubly heavy baryons in the three-body approach is a sum of the string potential $V^{\rm conf}$ and the Coulomb interaction potential V^C. A consistent derivation of the three-quark potential was done in the Wilson-loop approach in Refs. [28,29] which accounts for corrections of order $1/m^2$. A construction of the vacuum polarization contributions to the quark–antiquark interaction operator is performed by many authors (see Refs. [30–32]). Taking into account the growth of the strong coupling constant for the interaction of a light quark with the heavy quark and the color assumption $V_{QQ}(\bar 3)=\frac{1}{2}V_{Q\bar Q}(1)$, we have included in the pure non-relativistic three-quark Hamiltonian the vacuum polarization corrections of order α_s^2 in the form [19,30–32]:

$$H_0=\frac{\mathbf{p}_1^2}{2m_1}+\frac{\mathbf{p}_2^2}{2m_2}+\frac{\mathbf{p}_3^2}{2m_3}-\frac{2}{3}\sum_{i<j}\frac{\alpha_s^{ij}}{|\mathbf{r}_{ij}|}\left[1+\frac{\alpha_s^{ij}}{4\pi}\left(\tilde a_1+8\gamma_E\beta_0+8\beta_0\ln\left(\tilde\mu_{ij}|\mathbf{r}_{ij}|\right)\right)\right]+\sum_{i=1}^{3}\frac{1}{2}A(r_i+B). \tag{1}$$

We choose the Hamiltonian (1) as the initial approximation in our study of the baryon mass spectrum. The operator of quark momenta $\mathbf{p}_i=-i\frac{\partial}{\partial\mathbf{r}_i}$, $\mathbf{r}_i$ is the position of quark i with respect to a common string-junction point which coincides approximately with the center-of-mass point (for a more detailed discussion see Refs. [20–22]) and $\mathbf{r}_{ij}=\mathbf{r}_i-\mathbf{r}_j$ (compare with Fig. 1). The parameters of the confinement part of the potential are the following [15, 18,19]: $A=0.18$ GeV^2, $B=-0.16$ GeV. We take the quark masses $m_b=4.88$ GeV, $m_c=1.55$ GeV, $m_{u,d}=0.33$ GeV, $m_s=0.5$ GeV as in our previous [16,17,19] calculations of different hadron properties on the basis of the relativistic quark model. For the dependence of the QCD coupling constant $\alpha_s^{ij}=\alpha_s(\tilde\mu_{ij}^2)$ on the renormalization point $\tilde\mu_{ij}^2$ we use the two-loop result [33]:

$$\alpha_s(\tilde\mu_{ij}^2)=\pi\left[\frac{1}{\beta_0 L}-\frac{\beta_1\ln L}{\beta_0(\beta_0 L)^2}\right],\qquad \beta_0=\frac{1}{4}\left(11-\frac{2}{3}n_f\right),$$
$$\beta_1=\frac{1}{16}\left(102-\frac{38}{3}n_f\right),\qquad L=\ln(\tilde\mu_{ij}^2/\Lambda^2), \tag{2}$$

where $\Lambda=0.168$ GeV. The typical momentum transfer scale in a quarkonium is of order of the quark mass, so we choose the renormalization scale $\tilde\mu_{ij}=2m_im_j/(m_i+m_j)$, $\tilde a_1=(31-10n_f/3)/3$ and

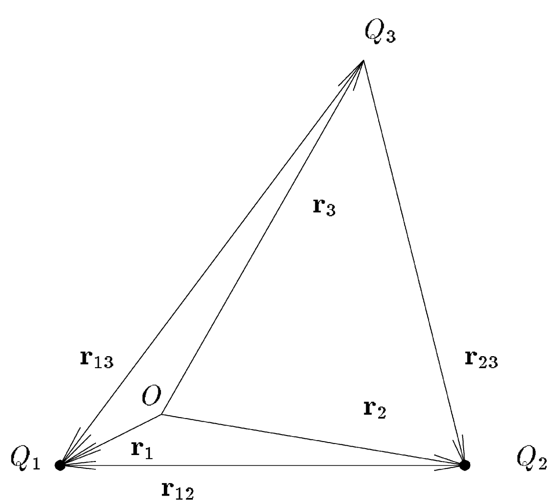

Fig. 1. The configuration of the three-quark system $(Q_1Q_2Q_3)$. O is the string-junction point. Q_1, Q_2 are the heavy quarks b or c. The quark Q_3 is treated as a heavy quark b,c in the triply heavy baryon or a light quark q in the doubly heavy baryon.

n_f is the number of flavors. The three-body Hamiltonian (1) and the Schrödinger equation can be reduced to the two-body form. For this aim, the three-body Jacobi coordinates become useful [20, 34,35]:

$$\boldsymbol{\rho}_{ij}=\alpha_{ij}(\mathbf{r}_i-\mathbf{r}_j),\qquad \boldsymbol{\lambda}_{ij}=\beta_{ij}\left(\frac{m_i\mathbf{r}_i+m_j\mathbf{r}_j}{m_i+m_j}-\mathbf{r}_k\right), \tag{3}$$

where the coefficients α_{ij}, β_{ij} are expressed in terms of the appropriate reduced masses:

$$\alpha_{ij}=\sqrt{\frac{\mu_{ij}}{\mu}},\qquad \beta_{ij}=\sqrt{\frac{\mu_{ij,k}}{\mu}},$$
$$\mu_{ij}=\frac{m_im_j}{(m_i+m_j)},\qquad \mu_{ij,k}=\frac{(m_i+m_j)m_k}{(m_i+m_j+m_k)}, \tag{4}$$

μ is an arbitrary mass parameter which disappears in final expressions. Evidently, the coordinate $\boldsymbol{\rho}_{ij}$ is proportional to the distance between quarks i and j, and the coordinate $\boldsymbol{\lambda}_{ij}$ is proportional to the distance between the quark k and the center-of-mass of quarks i,j. Together with the center-of-mass coordinate $\mathbf{R}_{c.m.}$ the Jacobi coordinates determine completely the position of the system.

In the center-of-mass frame ($\mathbf{R}_{c.m.}=0$) the operator of the kinetic energy can be written in the Jacobi coordinates $\boldsymbol{\rho},\boldsymbol{\lambda}$ as follows:

$$T_0=-\frac{1}{2\mu}\left(\frac{\partial^2}{\partial\boldsymbol{\rho}^2}+\frac{\partial^2}{\partial\boldsymbol{\lambda}^2}\right)=-\frac{1}{2\mu}\left(\frac{\partial^2}{\partial R^2}+\frac{5}{R}\frac{\partial}{\partial R}+\frac{K^2(\Omega)}{R^2}\right), \tag{5}$$

where $K^2(\Omega)$ is the angular momentum operator, whose eigenfunctions (hyperspherical harmonics) are determined by the equation [36]:

$$K^2(\Omega)Y_K(\Omega)=-K(K+4)Y_K(\Omega). \tag{6}$$

Here Ω designates five angular coordinates and R is the six-dimensional hyper-radius

$$R=\sqrt{\boldsymbol{\rho}_{ij}^2+\boldsymbol{\lambda}_{ij}^2},\qquad \rho=R\cos\theta,\qquad \lambda=R\sin\theta. \tag{7}$$

The baryon wave function $\Psi(\boldsymbol{\rho},\boldsymbol{\lambda})$ can be presented as an expansion over functions $Y_K(\Omega)$:

$$\Psi(\boldsymbol{\rho},\boldsymbol{\lambda})=\sum_K\Psi_K(R)Y_K(\Omega). \tag{8}$$

The radial wave functions $\Psi_K(R)$ appearing in Eq. (8) satisfy the system of differential equations [34,35]. However, in the further study of the ground state triply and doubly heavy baryons we use the hyper-radial approximation in which $K=0$ and the bound state wave function $\Psi=\Psi(R)$ does not depend on the angular variables in the six-dimensional space. Using the Jacobi coordinates the Schrödinger equation for the three-quark system can now be transformed into the following form:

$$\left[-\frac{1}{2\mu}\left(\frac{d^2}{dR^2}+\frac{5}{R}\frac{d}{dR}\right)-\frac{2}{3}\sum_{i<j}\frac{\alpha_s^{ij}\alpha_{ij}}{|\boldsymbol{\rho}_{ij}|}+\sum_{i<j}\frac{1}{2}\left(A\gamma_{ij}|\boldsymbol{\lambda}_{ij}|+B\right)-\frac{1}{6\pi}\sum_{i<j}\frac{\alpha_s^{ij^2}\alpha_{ij}}{|\boldsymbol{\rho}_{ij}|}\left(\tilde a_1+8\gamma_E\beta_0+8\beta_0\ln\frac{(\tilde\mu_{ij}|\boldsymbol{\rho}_{ij}|)}{\alpha_{ij}}\right)\right]\Psi(R)=E\Psi(R), \tag{9}$$

where coefficients γ_{ij} are given by

$$\gamma_{ij}=\sqrt{\frac{\mu(m_i+m_j)}{m_k(m_1+m_2+m_3)}}. \tag{10}$$