Available online at www.sciencedirect.com
ScienceDirect
Physics Letters B 662 (2008) 405–412
PHYSICS LETTERS B
www.elsevier.com/locate/physletb

Hadron spectrum in a two-colour baryon-rich medium

Simon Hands [a,b,*], Peter Sitch [a], Jon-Ivar Skullerud [c,d]

[a] *Department of Physics, Swansea University, Singleton Park, Swansea SA2 8PP, UK* [1]
[b] *Isaac Newton Institute for Mathematical Sciences, 20 Clarkson Road, Cambridge CB3 0EH, UK*
[c] *School of Mathematics, Trinity College, Dublin 2, Ireland*
[d] *Department of Mathematical Physics, National University of Ireland, Maynooth, County Kildare, Ireland* [1]

Received 10 October 2007; received in revised form 5 December 2007; accepted 7 January 2008

Available online 19 March 2008

Editor: J.-P. Blaizot

Abstract

The hadron spectrum of SU(2) lattice gauge theory with two flavours of Wilson quark is studied on an $8^3 \times 16$ lattice using all-to-all propagators, with particular emphasis on the dependence on quark chemical potential μ. As μ is increased from zero the diquark states with non-zero baryon number B respond as expected, while states with $B = 0$ remain unaffected, until the onset of non-zero baryon density at $\mu = m_\pi/2$. Post onset the pi-meson mass increases in accordance with chiral perturbation theory while the rho becomes lighter. In the diquark sector a Goldstone state associated with a superfluid ground state can be identified. A further consequence of superfluidity is an approximate degeneracy between mesons and baryons with the same spacetime and isospin quantum numbers. Finally we find tentative evidence for the binding of states with kaon quantum numbers within the baryonic medium.

PACS: 11.15.Ha; 12.40.Yx; 21.65.+f; 24.85.+p

Keywords: Chemical potential; Hadron spectrum; Nuclear matter

1. Introduction

At large baryon chemical potential μ_B the properties of QCD are expected to change as the system moves from a confined nuclear matter phase to a deconfined quark matter phase where the relevant degrees of freedom are quarks and gluons. At low temperature T and high μ_B, the attraction between quarks is believed to be sufficient to promote diquark Cooper pairing leading to a colour superconducting ground state. Weak-coupling techniques can be used at asymptotic densities and have revealed a superconducting phase known as the colour-flavour locked phase. However as density is reduced towards phenomenologically reasonable values, the precise nature of the ground state appears very sensitive both to additional parameters such as isospin chemical potential and strange quark mass, and also to the nature of the non-perturbative assumptions made in the calculation. It seems natural to use Lattice QCD to investigate these issues, but unfortunately whilst the lattice has been used very successfully to investigate QCD with $T > 0$, the well-known "Sign Problem" has made progress for $\mu_B/T \gg 1$ impossible.

Orthodox simulation techniques can be applied, however, to the case of two colour QCD (QC$_2$D) with gauge group SU(2). Whilst this theory differs in important ways from QCD, for instance in having bosonic baryons in the spectrum, and in having a superfluid, rather than superconducting, ground state at large μ_B, it remains the simplest gauge theory in which a systematic non-perturbative treatment of a baryonic medium is possible. Moreover, recent simulations [1] have provided evidence that there exist two distinct forms of two colour matter: the dilute Bose gas formed from diquark bound states which forms at onset, i.e. for $\mu_B > \mu_{Bo} = M_\pi$, in which superfluidity arises via Bose–Einstein condensation of scalar diquarks, is

* Corresponding author at: Department of Physics, Swansea University, Singleton Park, Swansea SA2 8PP, UK.
E-mail address: s.hands@swan.ac.uk (S. Hands).
[1] Permanent address.

0370-2693/$ – see front matter © 2008 Elsevier B.V. All rights reserved.
doi:10.1016/j.physletb.2008.01.078

supplanted at larger densities by a deconfined "quark matter" phase in which a system of degenerate quarks is disrupted by BCS condensation at the Fermi surface. Studies in this regime may have qualitative or even quantitative relevance for QCD quark matter, particularly in the non-Goldstone sector. For instance, Schäfer [2] has stressed how the impact of instantons on the excitation spectrum at high baryon density could be elucidated by lattice simulations.

In this Letter we study the μ_B-dependence of the hadron spectrum in both meson and baryon sectors of QC$_2$D with $N_f = 2$ flavours of Wilson quark, which we find to vary dramatically as the onset from vacuum to a ground state with non-zero baryon density is traversed. We build on the pioneering work of the Hiroshima group [3], who found that beyond onset the pion mass did not change noticeably, but the rho meson became significantly lighter, so that the level-ordering is reversed. Two baryon channels were also studied but no significant μ_B-dependence found. This work was performed on small lattices, and only looked at a few states. One of our aims is to improve and update their results. We also study the nature of the Goldstone mode associated with superfluidity, as done for QC$_2$D with staggered lattice fermions in [4], and expose the specifically two colour phenomenon of "meson–baryon" mixing in the superfluid state, whereby since B is no longer conserved, states interpolated by mesonic operators $q\bar{q}$ and diquark operators qq have identical quantum numbers and hence exhibit an approximate degeneracy. Finally, we make the first measurements of strange meson masses in a baryonic medium, and find tentative evidence for bound states of kaons in nuclei.

2. Formulation

The gauge-invariant lattice action with $N_f = 2$ degenerate fermion flavours is [1]

$$S = \bar{\psi}_1 M(\mu)\psi_1 + \bar{\psi}_2 M(\mu)\psi_2 - \kappa j\left(\bar{\psi}_1 K \bar{\psi}_2^T - \psi_2^T K \psi_1\right), \tag{1}$$

with M the conventional Wilson fermion matrix (with lattice spacing $a = 1$)

$$M_{xy}(\mu) = \delta_{xy} - \kappa \sum_\nu \left[(1-\gamma_\nu)e^{\mu\delta_{\nu 0}} U_\nu(x)\delta_{y,x+\hat{\nu}} + (1+\gamma_\nu)e^{-\mu\delta_{\nu 0}} U_\nu^\dagger(y)\delta_{y,x-\hat{\nu}}\right], \tag{2}$$

κ the hopping parameter, μ the quark chemical potential, and j the strength of an $\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R$-invariant diquark source term needed to regularise IR fluctuations in the superfluid phase, which should be extrapolated to zero to reach the physical limit. The factor κ in the diquark source term matches a similar factor in the expression for the quark number density [1], so that j is conjugate to the density of diquark pairs. The subscript on the fermion fields is a flavour index. The anti-unitary operator $K = K^T \equiv C\gamma_5\tau_2$, where $C\gamma_\mu C^{-1} = -\gamma_\mu^T = -\gamma_\mu^*$ and the Pauli matrix τ_2 acts on colour indices. A useful relation is

$$M^T(\mu) = -K\gamma_5 M(-\mu) K\gamma_5. \tag{3}$$

The hadronic states examined in this Letter are $q\bar{q}$ mesons and qq, $\bar{q}\bar{q}$ diquark baryons and anti-baryons. In all cases we use local interpolating operators of the form $\bar{\psi}(x)\Gamma\psi(x)$, $\psi^T(x)K\Gamma\psi(x)$, $\bar{\psi}(x)K\bar{\Gamma}\bar{\psi}^T(x)$. The matrix $\Gamma = \gamma_0\bar{\Gamma}^\dagger\gamma_0$ determines the spacetime quantum numbers of the hadron, with inclusion of the K factor ensuring that mesons and baryons with the same Γ have the same J^P. In this Letter we will focus on states with $\Gamma \in \{\mathbb{1}, \gamma_5, \gamma_j, i\gamma_5\gamma_j\}$ with $j = 1, \ldots, 3$ corresponding to $J^P \in \{0^+, 0^-, 1^-, 1^+\}$.

2.1. Fermion propagators

The fermion action (1) can be written in the form $\bar{\Psi}\mathcal{M}(\mu, j)\Psi$ where $\bar{\Psi} \equiv (\bar{\psi}_1, \psi_2^T, \bar{\psi}_2, \psi_1^T)$ and $\Psi \equiv (\psi_1, \bar{\psi}_2^T, \psi_2, \bar{\psi}_1^T)^T$. It has the form:

$$\mathcal{M} = \begin{pmatrix} A & 0 \\ 0 & \bar{A} \end{pmatrix} \quad \text{with } A = \frac{1}{2}\begin{pmatrix} M & -\kappa j K \\ \kappa j K & -M^T \end{pmatrix}, \tag{4}$$

and $\bar{A}(j) = A(-j)$. Now consider the propagator

$$\left\langle \Psi(x)\bar{\Psi}(y)\right\rangle \equiv \begin{pmatrix} S_{11} & S_{12} & 0 & 0 \\ \bar{S}_{21} & \bar{S}_{22} & 0 & 0 \\ 0 & 0 & S_{22} & S_{21} \\ 0 & 0 & \bar{S}_{12} & \bar{S}_{11} \end{pmatrix} \tag{5}$$

where the zero entries arise from the assumption of isospin symmetry. This symmetry also implies that

$$S_{22} = S_{11} \equiv S_N; \qquad \bar{S}_{11} = \bar{S}_{22} \equiv \bar{S}_N;$$
$$S_{21} = -S_{12} \equiv S_A; \qquad \bar{S}_{12} = -\bar{S}_{21} \equiv -\bar{S}_A, \tag{6}$$

where the subscripts denote "normal" and "anomalous" propagation. Anomalous propagation arises from particle–hole mixing in a superfluid ground state; on a finite volume S_A vanishes in the limit $j \to 0$.

2.2. Mesons

The isovector ($I = 1$) meson operators M^1 are given by $\bar{\psi}_1\Gamma\psi_2$, $\bar{\psi}_2\Gamma\psi_1$ and $(\bar{\psi}_1\Gamma\psi_1 - \bar{\psi}_2\Gamma\psi_2)/\sqrt{2}$. The charged meson correlator is then

$$\begin{aligned}
&\left\langle M^1(x)M^{1\dagger}(y)\right\rangle \\
&\quad = \left\langle \bar{\psi}_1(x)\Gamma\psi_2(x)\bar{\psi}_2(y)\bar{\Gamma}\psi_1(y)\right\rangle \\
&\quad = -\operatorname{Tr}\left[S_N(y,x)\Gamma S_N(x,y)\bar{\Gamma}\right] \\
&\qquad + \operatorname{Tr}\left[\bar{S}_A(y,x)\Gamma S_A(x,y)\bar{\Gamma}^T\right].
\end{aligned} \tag{7}$$

For the neutral meson correlator, the "disconnected" parts made up from the product of two traces cancel because of isospin symmetry. The connected parts are:

$$\begin{aligned}
&\left\langle M^1(x)M^{1\dagger}(y)\right\rangle \\
&\quad = \left\langle \bar{\psi}_1(x)\Gamma\psi_1(x)\bar{\psi}_1(y)\bar{\Gamma}\psi_1(y)\right\rangle_c \\
&\qquad - \left\langle \bar{\psi}_1(x)\Gamma\psi_1(x)\bar{\psi}_2(y)\bar{\Gamma}\psi_2(y)\right\rangle_c \\
&\qquad - \left\langle \bar{\psi}_2(x)\Gamma\psi_2(x)\bar{\psi}_1(y)\bar{\Gamma}\psi_1(y)\right\rangle_c \\
&\qquad + \left\langle \bar{\psi}_2(x)\Gamma\psi_2(x)\bar{\psi}_2(y)\bar{\Gamma}\psi_2(y)\right\rangle_c \\
&\quad = -\operatorname{Tr}\left[S_N(y,x)\Gamma S_N(x,y)\bar{\Gamma}\right] \\
&\qquad + \operatorname{Tr}\left[\bar{S}_A(y,x)\Gamma S_A(x,y)\bar{\Gamma}^T\right]
\end{aligned} \tag{8}$$