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Target mass corrections and twist-3 in the nucleon spin structure functions

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Abstract

The Nachtmann moment is employed to study the contribution of twist-3 operator to the nucleon spin structure functions. Target mass corrections to the Cornwall–Norton moments of the spin structure functions $g_{1,2}$ are discussed. It is found that the corrections play a sizeable role to the contribution of the twist-3 $\tilde{d}_2$ extracted from the Cornwall–Norton moments.

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1. Introduction

We know that the study of Bloom–Gilman quark–hadron duality is essential to understand the physics behind the connection between perturbative QCD (pQCD) and non-perturbative QCD [1]. In 2000, the new evidence of valence-like quark–hadron duality in the nucleon unpolarized structure functions $F_2^{p,d}$ was reported by Jefferson Lab. [2]. The well-known Bloom–Gilman quark–hadron duality [3] tells that prominent resonances do not disappear relatively to the background even at a large Q^2. It also means that the average of the oscillate resonance peaks in the resonance region is the same as the scaling structure function at a large Q^2 value. The origin of the Bloom–Gilman quark–hadron duality has been discussed by Rujula et al. [4] with a QCD explanation. According to operator production expansion (OPE), it is argued that higher-twist effects turn to be small in the integral of the structure functions, and therefore, the leading-twist plays a dominate role to the moments of the nucleon structure functions [4]. So far, the nucleon structure functions and the higher-twist effects have already been carefully and systemically studied [5]. Some detailed calculations for the higher-twist effects were carried out based on various theoretical approaches, like bag model [6], QCD sum rule [7,8], constituent quark model [9], Lattice QCD [10], and chiral soliton model [11]. Moreover, there are also several empirical analyses of the spin structure functions of g_1 and g_2 at low Q^2. The higher-twist effects, like the ones of the twist-3 and twist-4 terms, have been extracted from the data [12–16]. Those analyses can be more and more accurate because more and more precisely new measurements of the nucleon spin structure functions of g_1 [17–19], and particularly of g_2 [15,16,20], are available.

Usually, the contribution of the twist-3 $\tilde{d}_2$ is extracted from the measured $g_{1,2}(x, Q^2)$ by calculating the moment of

$$I(Q^2) = \int_0^1 dx\, x^2\big(2g_1(x, Q^2) + 3g_2(x, Q^2)\big) \to \tilde{d}_2(Q^2). \quad (1)$$

We know that the first moment of g_1 can be generally expanded in inverse powers of Q^2 in OPE [5]. It is

$$g_1^{(1)} = \int_0^1 dx\, g_1(x, Q^2) = \sum_{\tau=2,\text{even}}^{\infty} \frac{\mu_\tau(Q^2)}{Q^{\tau-2}}, \quad (2)$$

with the coefficients μ_τ related to the nucleon matrix elements of the operators of twist $\leqslant \tau$. In Eq. (2), the leading-twist (twist-2) component μ_2 is determined by the matrix elements of the axial vector operator $\bar{\psi}\gamma_\mu\gamma_5\psi$, summed over various quark

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flavors. The coefficient of $1/Q^2$ term contains the contributions from the twist-2 $\tilde{a}_2$, twist-3 $\tilde{d}_2$, and twist-4 $\tilde{f}_2$, respectively. Thus [5],

$$\mu_4 = \frac{1}{9}M^2(\tilde{a}_2 + 4\tilde{d}_2 + 4\tilde{f}_2), \tag{3}$$

where M is the nucleon mass. In Eq. (3) $\tilde{a}_2$ arises from the target mass corrections, and it is of purely kinematical origin. It relates to the third moment of the twist-2 part of $g_1(x, Q^2; M = 0)$ $(2\tilde{a}_2 = \int_0^1 x^2\, dx\, g_1(x, Q^2; M = 0))$. The other higher-twist terms, like $\tilde{d}_2$ and $\tilde{f}_2$, result from the reduced matrix elements, which are of the dynamical origin since they show the correlations among the partons [7,21]. If the contribution of the twist-3 term is well defined by Eq. (1), then the contribution of the twist-4 term, like $\tilde{f}_2$, can be extracted according to Eq. (3).

It should be mentioned that the method of Eq. (1) to extract the twist-3 contribution $\tilde{d}_2$ ignores the target mass corrections to $g_{1,2}$, since the relations

$$\int_0^1 x^2 g_1(x, Q^2) = \frac{1}{2}\tilde{a}_2, \qquad \int_0^1 x^2 g_2(x, Q^2) = \frac{1}{3}(\tilde{d}_2 - \tilde{a}_2)$$

are used. In general, the nucleon target mass corrections should be considered completely in the studies of the nucleon structure functions [22], of the Bloom–Gilman quark–hadron duality [23–25], and of the Bjorken sum rule [26] at a moderate low Q^2 (∼1–5 GeV^2). We know that the target mass corrections to the nucleon structure functions are of the pure kinematical origin. They are different from the other higher-twist effects from dynamical multi-gluon exchanges or parton correlations. Before one can extract the higher-twist effects, it is important to remove the target mass corrections from the data [24]. There were several works about the target mass corrections to $F_{1,2}(x, Q^2)$ and $g_{1,2}(x, Q^2)$ in the literature [27–29]. Recently, the expressions of all the electromagnetic and electroweak nucleon spin structure functions with the target mass corrections have been explicitly given in Refs. [30,31].

In this work, in order to precisely extract the contribution of the twist-3 operators, the target mass corrections to Eq. (1) will be discussed. In Section 2, we explicitly give the target mass corrections to the integral $I(Q^2)$. Moreover, the advantage of the Nachtmann moments is stressed. In Section 3, the numerical estimate of the target mass corrections to $I(Q^2)$ is given comparing to the result of the Nachtmann moment. Section 4 is devoted for conclusions.

2. Twist-3 matrix elements and the target mass corrections

Here, we use the notations of Piccione and Ridolfi [30] for the spin structure functions and for their moments. We know that the well-known Cornwall–Norton (CN) moments are

$$g_{1,2}^{(n)}(Q^2) = \int_0^1 dx\, x^{n-1} g_{1,2}(x, Q^2). \tag{4}$$

In Refs. [30,31], the target mass corrections to the nucleon spin structure functions g_1 and g_2 are explicitly given in terms of the CN moments of the matrix elements of the twist-2 (leading-twist) operator and twist-3 one. Up to twist-3, the results are

$$\begin{aligned} g_1^{(n)}(Q^2) = {} & a_n + y^2\frac{n(n+1)}{(n+2)^2}(na_{n+2} + 4d_{n+2}) \\ & + y^4\frac{n(n+1)(n+2)}{2(n+4)^2}(na_{n+4} + 8d_{n+4}) \\ & + y^6\frac{n(n+1)(n+2)(n+3)}{6(n+6)^2}(na_{n+6} + 12d_{n+6}) \\ & + \mathcal{O}(y^8), \end{aligned} \tag{5}$$

with $y^2 = M^2/Q^2$, and

$$\begin{aligned} g_2^{(n)}(Q^2) = {} & \frac{n-1}{n}(d_n - a_n) \\ & + y^2\frac{n(n-1)}{(n+2)^2}\big(nd_{n+2} - (n+1)a_{n+2}\big) \\ & + y^4\frac{n(n-1)(n+1)}{2(n+4)^2}\big(nd_{n+4} - (n+2)a_{n+4}\big) \\ & + y^6\frac{n(n-1)(n+1)(n+2)}{6(n+6)^2} \\ & \times \big(nd_{n+6} - (n+3)a_{n+6}\big) + \mathcal{O}(y^8). \end{aligned} \tag{6}$$

In Eqs. (5) and (6), a_n and d_n are the reduced hadron matrix elements of the irreducible Lorentz operators [29,30]: $R_1^{\sigma\mu_1\cdots\mu_{n-1}}$ and $R_2^{\lambda\sigma\mu_1\cdots\mu_{n-2}}$. The matrix elements of the operators: $R_1^{\sigma\mu_1\cdots\mu_{n-1}}$ (twist-2) and $R_2^{\lambda\sigma\mu_1\cdots\mu_{n-2}}$ (twist-3) can be written as [29,30]

$$\langle p, s|R_1^{\sigma\mu_1\cdots\mu_{n-1}}|p, s\rangle = -2Ma_n M_1^{\sigma\mu_1\cdots\mu_{n-1}}, \tag{7}$$

and

$$\langle p, s|R_2^{\lambda\sigma\mu_1\cdots\mu_{n-2}}|p, s\rangle = Md_n M_2^{\lambda\sigma\mu_1\cdots\mu_{n-2}}, \tag{8}$$

where $M_1^{\sigma\mu_1\cdots\mu_{n-1}}$ is the general rank-n symmetric tensor which can be formed with one spin four-vector s and $n-1$ momentum four-vectors p, and $M_2^{\lambda\sigma\mu_1\cdots\mu_{n-2}}$ is antisymmetric in (λ, σ), symmetric in all other indices. The two tensors must be traceless. A typical example for a twist-3 operator is

$$d_2: \quad \bar{\psi}\gamma_{\{\alpha}\tilde{F}_{\beta\}\gamma}\psi \quad \text{twist-3}, \tag{9}$$

where $\{\cdots\}$ denotes symmetrizing the indices and subtracting the trace, and $\tilde{F}_{\alpha\beta} = \frac{1}{2}\epsilon_{\alpha\beta\gamma\delta}F_{\gamma\delta}$ is the dual gluon field strength [6,7]. We notice that the contribution of the leading-twist terms has a relation to the quark distributions functions. However, the contributions of the higher-twist operators (like twist-3) have no partonic interpretation [31]. In Eq. (5) if we fix $n = 1$, then,

$$g_1^{(1)} = a_1 + y^2\frac{2}{9}(a_3 + 4d_3) + \cdots. \tag{10}$$

Comparing to Eqs. (2), (3), we see that $\tilde{a}_2 = 2a_3$ and $\tilde{d}_2 = 2d_3$. In Eqs. (5) and (6) only the contributions of the leading-twist and twist-3 operators are considered.

From Eqs. (1) and (4)–(6), we see that