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On principle of inertia in closed universe

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Abstract

If our universe is asymptotic to a de Sitter space, it should be closed with curvature in $O(\Lambda)$ in view of *dS* special relativity. Conversely, its evolution can fix on Beltrami systems of inertia in the *ds*-space without Einstein's 'argument in a circle'. Gravity should be local *ds*-invariant based on localization of the principle of inertia.

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1. Introduction

In classical physics, it is well known that for both Newton theory and Einstein's special relativity the principle of inertia (*PoI*) with Galilean symmetry and Poincaré symmetry, respectively, plays an extremely important role as the benchmark of physics for defining physical quantities and introducing physical laws. But, in Einstein's point of view, there is an 'argument in a circle' for the *PoI* as the benchmark.

Some eighty five years ago, Einstein claimed:

> The weakness of the *PoI* lies in this, that it involves an argument in a circle: a mass moves without acceleration if it is sufficiently far from other bodies; we know that it is sufficiently far from other bodies only by the fact that it moves without acceleration. Are there at all any inertial systems for very extended portions of the space–time continuum, or, indeed, for the whole universe? We may look upon the principle of inertia as established, to a high degree of approximation, for the space of our planetary system, provided that we neglect the perturbations due to the sun and planets. Stated more exactly, there are finite regions, where, with respect to a suitably chosen space of reference, material particles move freely without acceleration, and in which the laws of the special theory of relativity, ..., hold with remarkable accuracy. Such regions we shall call "Galilean regions" [1].

In fact, to avoid this 'weakness' is one of the main motivations for Einstein from special relativity to general relativity based on his equivalence principle and general principle of relativity as an extension of the special principle of relativity.

In general relativity, however, what is realized for the general principle of relativity is the principle of general covariance. Although it is always possible to analyze physics in terms of arbitrary (differentiable) coordinate systems at classical level, 'the principle of covariance has no forcible content' [2]. For the equivalence principle, it requires that physical quantities and laws are in 'their familiar special-relativistic forms' in local Lorentz frames [2]. The symmetry for physical quantities and laws, however, is local $GL(4, R)$ or its subgroup $SO(1, 3)$ without local translation in general. Thus, in 'Galilean regions', Poincaré symmetry of *PoI* as the benchmark in special relativity is partially lost. These seem away from Einstein's original intention and lead to the benchmark of physics with gravity is not completely in consistency with that in special relativity without gravity.

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Recent observations show that our universe is accelerated expanding [3,4]. It is certainly not asymptotic to a Minkowski (*Mink*)-space, rather quite possibly asymptotic to a de Sitter (*dS*)-space with a tiny cosmological constant Λ. These present great challenges to the foundation of physics on the cosmic scale (see, e.g., [5]). In fact, it is the core of challenges: What are the benchmarks of physics on the cosmic scale? Are they consistent?

In view of the *dS*-invariant special relativity [6–14], however, there is a *PoI* of *dS*-invariance on *dS*-space with Beltrami systems of inertia (denoted *BdS*-space). Here we show that if the universe is asymptotic to a *dS*-space, it should be closed with a tiny curvature in the order of Λ, $O(\Lambda)$. Conversely, the evolution of the universe can fix on the Beltrami systems. Thus, the universe acts as the origin of the *PoI* of *dS*-invariance without Einstein's 'argument in a circle' so that the benchmark of physics on the cosmic scale should still be the *PoI* of *dS*-invariance. Then, we explain that the benchmark of physics with gravity should be the localization of the *PoI* of the *dS* special relativity. Thus, the *PoI* of *dS*-invariance and its localization should play the role of the consistent benchmarks of physics on the cosmic scale in the universe.

Actually, based on the principle of relativity [6,7] and the postulate on invariant universal constants, the speed of light c and the curvature radius R [8,9], the *dS* special relativity can be set up on the *BdS*-space. While Einstein's special relativity is the limiting case of $R \to \infty$.

In the *dS* special relativity, Beltrami coordinate systems [15] with Beltrami time simultaneity are very similar to *Mink*-systems in Einstein's special relativity. Namely, in the *BdS*-space geodesics are all *straight world lines* so that there is a *PoI* with a *law of inertia* for free particles and light signals. All these issues are transformed symmetrically under the fractional linear transformations with a common denominator (*FLT*s) of *dS*-group $SO(1,4)$ in the Beltrami atlas chart by chart. It is significant that the Beltrami systems and their Robertson–Walker-like *dS*-counterpart with respect to proper-time simultaneity provide an important model. In this model, the *dS*-group as a maximum symmetry ensures that there are both the *PoI* and the cosmological principle on *dS*-space as two sides of a coin. On one side, there is the *BdS*-space with the *PoI*, while on the other there is a Robertson–Walker-like *dS*-space with the cosmological principle having an accelerated expanding closed 3d cosmos S^3 of curvature in the order of $O(R^{-2})$. Since the both can be transformed each other explicitly by changing the simultaneity just like flip a coin, the Robertson–Walker-like *dS*-space displays as an origin of the *PoI*, while the *PoI* provides a benchmark of physics on the *dS*-space.

If the universe is asymptotic to a *dS*-space with $R \simeq (3/\Lambda)^{1/2}$. In view of the *dS* special relativity, the universe should be asymptotic to the Robertson–Walker-like *dS*-space in the model so that it should be closed and the deviation from flatness is in the order of Λ, $O(\Lambda)$. This is an important prediction more or less consistent with recent data from WMAP [4] and can be further checked.

Conversely, the asymptotic behavior of the universe should naturally pick up a kind of the Robertson–Walker-like *dS*-systems with such a 'cosmic' time τ that its axis coincides with the revolution time arrow of the real cosmic time τ_u in the universe. Since the 'cosmic' time τ in the Robertson–Walker-like *dS*-space is explicitly related to the Beltrami time x^0, the universe should also fix on a kind of Beltrami systems with x^0 transformed from the 'cosmic' time τ. Therefore, via its evolution time arrow of τ_u picking up a 'cosmic' time τ on the Robertson–Walker-like *dS*-space, the universe should just act as an origin of such kind of Beltrami systems in which the *PoI* holds. Thus, there do exist the inertial systems in the universe and there is no Einstein's 'argument in a circle' for the *PoI*.

In general relativity, there is no special relativity in *dS*-space. In the *dS* special relativity, there is no gravity in *dS*-space. How to describe gravity?

In the light of Einstein's 'Galilean regions' [1], where his special relativity with full Poincaré symmetry should hold locally, the *PoI* should be localized. Therefore, in view of the *dS* special relativity, on spacetimes with gravity there should be local *dS*-frame anywhere and anytime so that the *PoI* of the *dS* special relativity should hold locally. If so, the localized *PoI* of the *dS* special relativity should be the benchmark of physics with gravity. This is in consistency with the role played by the *PoI* of the *dS* special relativity. We may further require that gravity have a gauge-like dynamics characterized by a dimensionless constant $g \simeq (\Lambda G\hbar/c^3)^{1/2} \sim 10^{-61}$ from the cosmological constant Λ and the Planck length. A simple model has implied this should be the case [23–25].

This Letter is arranged as follows. In Sections 2, we argue why there is a *PoI* on *dS*-space and very briefly introduce the *dS* special relativity. In Section 3, we introduce the relation between the *PoI* and the cosmological principle on *dS*-space as well as the cosmological meaning of *dS* special relativity. In Section 4, we explain why the universe can fix on the Beltrami systems of inertia without Einstein's 'argument in a circle'. In Section 5, we very briefly discuss that gravity should be based on localization of the *dS* special relativity with *PoI* and introduce the simple model for *dS*-gravity. Finally, we end with a few remarks.

2. On de Sitter special relativity

Is there special relativity with a *PoI* on *dS*-space?

Yes! Absolutely. This can be enlightened from two deferent but related angles [6–12].

Firstly, as is well known, weakening the Euclid fifth axiom leads to Riemann and Lobachevsky geometries on an almost equal footing with Euclid geometry. There is a physical analog via an inverse Wick rotation of 4d Euclid space, Riemann sphere and Lobachevsky hyperboloid $E^4/S^4/L^4$, respectively. Namely, there should be two other kinds of the dS/AdS-invariant special relativity on an almost equal footing with Einstein's special relativity [12]. In fact, there is a one-to-one correspondence between three kinds of geometries and their physical counterparts. We list the correspondence as follows: