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CMB fluctuations and string compactification scales

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ABSTRACT

We propose a mechanism for the generation of temperature fluctuations of cosmic microwave background. We consider a large number of fields, such as Kaluza–Klein modes and string excitations. Each field contributes to the gravitational potential by a small amount, but an observable level of temperature fluctuations is achieved by summing up the contribution of typically of order 10^{14} fields. Tensor fluctuations are hardly affected by these fields. Our mechanism is based on purely quantum effects of the fields which are classically at rest, and is different from the one in slow-roll inflation. Using the observed data, we find constraints on the parameters of this model, such as the size of the extra dimensions and the string scale. Our model predicts a particular pattern of non-gaussianity with a small magnitude.

1. Introduction

Observation of cosmic microwave background (CMB) [1] provides a great opportunity for testing fundamental theories. It is generally believed that the fluctuations of CMB originate from quantum fluctuations generated at energy much higher than present accelerators can achieve.

It is hoped that string theory gives predictions for observational cosmology. But at the current stage of its development, string theory is defined only in limited classes of background spacetimes. The analysis of cosmology has been limited to those based on low energy effective field theory, and it has been difficult to make concrete predictions. (For attempts to formulate cosmology beyond the level of effective field theory, see e.g. [2] and [3].)

In this Letter we will focus on an aspect of fundamental theories, namely the presence of a large number of fields. These could be Kaluza–Klein (KK) modes from the compactification of extra dimensions or excited states of strings. Our analysis is mostly based on effective field theory, but we will also use a general property of perturbative string theory as an input.

It would be possible that the size of extra dimensions L is large in unit of Hubble scale of inflation H^{-1}. We already know a hierarchy of scales between the Planck scale m_{pl} and H: The fact that tensor perturbation (B-mode polarization of CMB) has not been observed implies H is at least 4 orders of magnitude smaller than m_{pl} [1]. The question is whether string scale m_s is close to m_{pl} or H. We will see below that with m_s slightly smaller than $H \sim 10^{-4} m_{pl}$, L can be as large as $L \sim 10^8 H^{-1}$. In such a case, there will be many KK modes with $m \lesssim H$. We will show that the quantum effects of these fields lead to interesting observable effects.

For concreteness, we shall consider a collection of N free fields ϕ_A $(A = 1, \dots, N)$ with mass m_A, which are classically at the bottom of the potential $\phi_A = 0$. We will not consider inflaton field, and take the background to be pure de Sitter, and find the temperature fluctuations generated during inflation. At the end of this Letter, we make comments on the effect of time dependence of Hubble, e.g. near the end of inflation.

In this Letter we point out that temperature fluctuations $\delta T/T$ can be generated by a mechanism different from the standard one based on slow-roll inflation. In the latter case, the classical value of a scalar field (inflaton) provides a preferred time slicing, and its fluctuation is interpreted as the difference of duration of inflation at different points in space, which results in the density (or temperature) anisotropy. In our case, the fields ϕ_A are classically at rest, and the above argument does not apply. It has been assumed that these fields do not contribute to $\delta T/T$. We will show that by keeping quadratic terms in ϕ_A, the gravitational potential Φ is generated through quantum effects of these fields. Each field gives small contribution, but when $N \gg 1$, this effect becomes

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important. The large ratio between scalar and tensor contributions to $\delta T/T$ is usually assumed to be the consequence of the smallness of the slope of the inflaton potential. In our approach, it is the consequence of a large number of fields.

In the following, we first review quantization in de Sitter space, and recall that fields with $m < \frac{3}{2}H$ do not oscillate at super-horizon scales, because the "friction" due to the cosmic expansion overdumps the oscillation. These are the fields that we will focus on. We then study Einstein equations and solve for the gravitational potential Φ in terms of the matter fields. Using this, we find the contribution to CMB fluctuations from KK modes. We will make an argument based on string perturbation theory that if string scale m_s is lower than H, the summation over the mass is effectively cut off at m_s. We then use the observed data on the amplitude of $\delta T/T$ and the tensor-to-scalar ratio $r_{t/s}$ to constrain the parameters, such as m_s and L.

There have been inflationary models involving a large number of fields. In "N-flation" [4] (or "assisted inflation" [5]), many fields (such as axions) classically roll down the potential, collectively producing an effect similar to chaotic inflation. Our mechanism is different from this, since we are not assuming classical motion. Also note that our mechanism is different from the curvatons scenario [6]. We demonstrate that curvature fluctuations are generated during inflations without such a late time mechanism.

2. Quantization in de Sitter background

We consider background de Sitter space

$$ds^2 = dt^2 - a^2(t)\,d\vec{x}^2, \quad a(t) = H^{-1}e^{Ht}, \tag{1}$$

and a collection of N free massive scalar fields,

$$S = \sum_{A=1}^{N} \int d^4x \sqrt{-g}\left\{\partial_\mu\phi_A\partial^\mu\phi_A - m_A^2\phi_A^2\right\}. \tag{2}$$

In the following we will often use the conformal time, $\tau = \int dt/a(t) = -e^{-Ht}$ $(-\infty \leqslant \tau \leqslant 0)$, and the rescaled field $\chi_A = a\phi_A$ which has the standard kinetic term. We will suppress the label A hereafter for brevity.

Fourier component of χ satisfies the equation of motion (prime denotes ∂_τ),

$$\chi_{\vec{k}}''(\tau) + \left\{|\vec{k}|^2 + \left(H^{-2}m^2 - 2\right)\frac{1}{\tau^2}\right\}\chi_{\vec{k}}(\tau) = 0. \tag{3}$$

Quantization is done by setting

$$\chi(\tau,\vec{x}) = \int \frac{d^3k}{(2\pi)^3}\frac{1}{\sqrt{2|\vec{k}|}}\left[u_{\vec{k}}(\tau)a_{\vec{k}}e^{i\vec{k}\cdot\vec{x}} + u_{\vec{k}}^*(\tau)a_{\vec{k}}^\dagger e^{-i\vec{k}\cdot\vec{x}}\right] \tag{4}$$

where $u_{\vec{k}}(\tau)$ is the solution of (3) which approaches $u_{\vec{k}}(\tau) \to e^{-i|\vec{k}|\tau}$ at early time $\tau \to -\infty$. This condition ensures that the flat space result is recovered in the short distance limit. The solution is given by $u_{\vec{k}}(\tau) = \sqrt{\frac{\pi}{2}}e^{i\frac{\pi}{2}(\nu+\frac{1}{2})}\sqrt{-|\vec{k}|\tau}\,H_\nu^{(1)}(-|\vec{k}|\tau)$ with $\nu = \sqrt{\frac{9}{4} - m^2H^{-2}}$. Asymptotic behavior at the late times (in the super-horizon $|\vec{k}|/a \ll H$ limit) is $u_{\vec{k}} \sim (-|\vec{k}|\tau)^{-\nu+\frac{1}{2}}$. Time dependence factorizes from space dependence, as is clear from the fact that $|\vec{k}|$ dependence drops out from (3) in this limit. Fields with small mass, $mH^{-1} < \frac{3}{2}$, do not oscillate in time. We will take the $mH^{-1} \ll 1$ limit in the following formulas, since this is the case of importance for our applications, as explained below.

The equal time two-point function of ϕ at late times is

$$\begin{aligned}\left\langle\phi(\tau,\vec{x})\phi(\tau,\vec{x}')\right\rangle &= \frac{1}{a^2(\tau)}\int\frac{d^3k}{(2\pi)^3}\frac{1}{2|k|}\left|u_k(\tau)\right|^2 e^{i\vec{k}(\vec{x}-\vec{x}')} \\ &\sim \frac{H^2}{4\pi^2\beta}\left(Ha|\vec{x}-\vec{x}'|\right)^{-\beta},\end{aligned} \tag{5}$$

where we have taken the $m^2H^{-2} \ll 1$ limit, and defined

$$\beta = \frac{2}{3}m^2H^{-2}. \tag{6}$$

3. Einstein equations

Quantum fluctuations of the fields ϕ induce gravitational potential. Let us look at the $(0,i)$ and (i,j) components of the Einstein equation,

$$\left(\Psi' + \mathcal{H}\Phi\right)_{,i} = 4\pi G\,\delta T_{0i}^{(S)}, \tag{7}$$

$$\begin{aligned}&\left[\Psi'' + \mathcal{H}(2\Psi + \Phi)' + \left(2\mathcal{H}' + \mathcal{H}^2\right)\Phi + \frac{\Delta}{2}(\Phi - \Psi)\right]\delta_{ij} \\ &\quad - \frac{1}{2}(\Phi - \Psi)_{,ij} = 4\pi G\,\delta T_{ij}^{(S)}\end{aligned} \tag{8}$$

where $\mathcal{H} = \frac{a'}{a} = -\frac{1}{\tau}$. The l.h.s. is the Einstein tensor expanded to the first order in metric fluctuations; Φ and Ψ are the two gauge invariant combinations constructed from the scalar modes (see e.g. [8]). On the r.h.s. we have energy–momentum tensor which is quadratic in ϕ,

$$\delta T_{\mu\nu} = \sum\left\{\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}\left(\partial^\rho\phi\partial_\rho\phi - m^2\phi^2\right)\right\}. \tag{9}$$

The sum is taken over the species of the fields. The superscript (S) denotes the scalar part (the part which can be written as derivatives of a scalar), e.g.,

$$\delta T_{0i}^{(S)} = \partial_i\left(\frac{1}{\Delta}\partial_k\delta T_{0k}\right) = \frac{1}{\Delta}\partial_i\partial_k\left(\phi'\partial_k\phi\right). \tag{10}$$

We shall use the Einstein equation to express Φ and Ψ in terms of the matter fields ϕ. We first find the difference $\Phi - \Psi$ from the $(i \neq j)$ component of (8),

$$\Phi - \Psi = -8\pi G\sum s, \quad s \equiv \frac{3}{2\Delta^2}\partial_i\partial_j\left(\partial_i\phi\partial_j\phi - \frac{\delta_{ij}}{3}\partial_k\phi\partial_k\phi\right)$$

and substitute it into (7) to get

$$\Phi' + \mathcal{H}\Phi = 4\pi G\sum\left\{-2s' + \frac{1}{\Delta}\partial_i\left(\phi'\partial_i\phi\right)\right\}. \tag{11}$$

At late times, the r.h.s. goes like $(-\tau)^{\beta-1}$, which implies $\Phi \sim (-\tau)^\beta$. The part containing s can be dropped when $\beta \ll 1$, since this gives smaller contribution in correlation functions than the second term.[1] Using the fact that time dependence $(\phi \sim (-\tau)^{\beta/2})$ enters as a multiplicative factor, and $\partial_i(\phi\partial_i\phi) = \frac{1}{2}\Delta\phi^2$, we find

$$\Phi = -\sum\pi G\beta\phi^2 \tag{12}$$

[1] In correlation functions, $\frac{\partial}{\partial x'}\langle\phi(x)\phi(x')\rangle$ gives a factor of order β (and further differentiation only gives order 1 factors like $-(\beta+1)$). Correlators involving the first term of (11) necessarily contain more of these factors than the ones involving only the second term of (11).