Physics Letters B 702 (2011) 422–428

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

New eigen-mode of spin oscillations in the triplet superfluid condensate in neutron stars

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ARTICLE INFO

Article history:
Received 11 May 2011
Received in revised form 13 June 2011
Accepted 10 July 2011
Available online 18 July 2011
Editor: J.-P. Blaizot

Keywords:
Neutron star
Superfluidity
Spin waves

ABSTRACT

The eigen-mode of spin oscillations with $\omega \simeq \sqrt{58/35}\Delta$ is predicted to exist besides already known spin waves with $\omega \simeq \Delta/\sqrt{5}$ in the triplet superfluid neutron condensate in the inner core of neutron stars. The new mode is kinematically able to decay into neutrino pairs through neutral weak currents. The problem is considered in BCS approximation for the case of ${}^3P_2$–${}^3F_2$ pairing with a projection of the total angular momentum $m_j = 0$ which is conventionally considered as preferable one at supernuclear densities.

A superfluidity of the inner core of neutron stars plays a crucial role in theirs cooling scenario. The energy gap Δ arising in the quasi-particle spectrum below the critical condensation temperature T_c suppresses the most of neutrino emission mechanisms [1]. According to the minimal cooling paradigm [2–5], under these conditions, the most efficient energy losses from the star volume can take place at a recombination of thermal excitations in the form of broken Cooper pairs. The neutrino emission at the pair-recombination processes occurs through neutral weak currents in the axial channel of weak interactions[1] and can be very efficient, in the triplet superfluid neutron liquid, a few below the critical temperature [6]. However, the corresponding neutrino emissivity falls rapidly with lowering of the temperature because the number of broken pairs, having the excitation energy larger than 2Δ, decreases exponentially. In this case the *collective* excitations of the condensate can dominate in the neutrino production.

Since we assume that the condensate consist of neutron pairs in the triplet state it is natural to expect the collective modes associated with spin fluctuations of the condensate.[2] Such collective excitations with the energy lower than 2Δ might undergo the weak decay into neutrino pairs. Recently spin waves with the excitation energy $\omega = \Delta/\sqrt{5}$ was predicted to exist in the superfluid spin-triplet condensate of neutrons [8–10]. Because of a rather small excitation energy, the weak decay of such waves leads to a substantial neutrino emission at the lowest temperatures $T \ll T_c$, when all other mechanisms of the neutrino energy losses are killed by the superfluidity.

In Refs. [8–10], the eigen-mode of spin oscillations in the ${}^3P_2$ superfluid neutron liquid was studied in a simple model restricted to excitations of the condensate with $l = 1$. In this Letter we demonstrate that extending of the decomposition up to $l = 1, 3$ leads to a very small frequency shift of the known mode, $\omega = \Delta/\sqrt{5}$, but opens the new additional mode of spin oscillations with the finite energy gap $\omega(\mathbf{q} = 0) < 2\Delta$. The problem is considered for the case of ${}^3P_2$–${}^3F_2$ pairing with a projection of the total angular momentum $m_j = 0$ which is conventionally considered as preferable one at supernuclear densities.

We will examine the spin modes within the BCS approximation.[3] Let us remind briefly the theory of spin density excitations in the condensate. The order parameter, $\hat{D} \equiv D_{\alpha\beta}$, arising due to triplet pairing of quasiparticles, represents a 2×2 symmetric matrix in spin space, $(\alpha, \beta = \uparrow, \downarrow)$. The spin–orbit interaction among quasiparticles is known to dominate in the nucleon matter of a high density.

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[1] The vector channel of the neutrino radiation through neutral weak currents is strongly suppressed in the non-relativistic case [7,8].

[2] Previously spin modes have been thoroughly studied in the p-wave superfluid liquid ^{3}He with a central interaction between quasiparticles [11–15]. These results cannot be applied directly to the triplet-spin neutron superfluid condensate, where the pairing is caused mostly by the spin–orbit interaction between quasiparticles (see details in Ref. [9]).

[3] Throughout this Letter, we use the system of units $\hbar = c = 1$ and the Boltzmann constant $k_B = 1$.

0370-2693/$ – see front matter
doi:10.1016/j.physletb.2011.07.025

Therefore it is conventional to represent the triplet order parameter of the system as a superposition of standard spin-angle functions of the total angular momentum (j, m_j),

$$\Phi^{(j,l,m_j)}_{\alpha\beta}(\mathbf{n}) \equiv \sum_{m_s+m_l=m_j}\left(\frac{1}{2}\frac{1}{2}\alpha\beta|sm_s\right)(slm_sm_l|jm_j)Y_{l,m_l}(\mathbf{n}). \tag{1}$$

Assuming that the pair condensation occurs into the state with a total angular momentum $j = 2$ we use the vector notation which involves a set of mutually orthogonal complex vectors $\mathbf{b}_{l,m_j}(\mathbf{n})$ defined as

$$\mathbf{b}_{l,m_j}(\mathbf{n}) = -\frac{1}{2}\operatorname{Tr}\left(\hat{g}\hat{\boldsymbol{\sigma}}\hat{\Phi}^{2,l,m_j}\right), \qquad \mathbf{b}_{l,-m_j} = (-)^{m_j}\mathbf{b}^*_{l,m_j}, \tag{2}$$

where $\hat{\boldsymbol{\sigma}} = (\hat{\sigma}_1, \hat{\sigma}_2, \hat{\sigma}_3)$ are Pauli spin matrices, $\hat{g} = i\hat{\sigma}_2$, and the angular dependence of the order parameter is represented by the unit vector $\mathbf{n} = \mathbf{p}/p$ which defines the polar angles (θ, φ) on the Fermi surface. The vectors $\mathbf{b}_{l,m_j}$ are mutually orthogonal and are normalized by the condition

$$\left\langle \mathbf{b}^*_{l',m'_j}\mathbf{b}_{l,m_j}\right\rangle = \delta_{ll'}\delta_{m_jm'_j}. \tag{3}$$

Hereafter the angle brackets denote angle averages, $\langle\cdots\rangle \equiv (4\pi)^{-1}\int d\mathbf{n}\cdots$.

The block of interaction diagrams irreducible in the channel of two quasiparticles, $\Gamma_{\alpha\beta,\gamma\delta}$, is usually generated by expansion over spin-angle functions. The spin–orbit interaction among quasiparticles is known to dominate at high densities. This implies that the spin $\mathbf{s}$ and orbital momentum $\mathbf{l}$ of the pair cease to be conserved separately, and the complete list of channels includes the pair states with $j = 0, 1, 2$, and $|m_j| \leqslant j$. These nine complex states exhaust the number of independent components in the matrix order parameter arising at the P-wave pairing caused by the strong spin–orbit forces. The pairing in the $j = 2$ channel dominates, and due to relatively small tensor components of the neutron–neutron interaction the condensation of pairs occurs in the ${}^3P_2 + {}^3F_2$ state. In this pairing model, contributions from ${}^3P_2 \to {}^3P_0$ or ${}^3P_2 \to {}^3P_1$ transitions are deemed to be unimportant. Such assumption is somewhat vulnerable especially when considering excited state of the condensate. Unfortunately the detailed information on the in-medium effective interaction between neutrons in the channels $j = 0, 1$ is currently unavailable and requires a special investigation. Hence we take the approximation to neglect the $j = 0, 1$ coupling throughout this Letter. From now on we omit the suffix j everywhere by assuming that the interaction occurs in the state with $j = 2$. Thus we assume $l = j \pm 1$, and

$$\varrho\Gamma_{\alpha\beta,\gamma\delta}\left(\mathbf{p}, \mathbf{p}'\right) = \sum_{l'lm_j}(-1)^{\frac{l-l'}{2}}V_{ll'}\left(p, p'\right)\left(\mathbf{b}_{lm_j}(\mathbf{n})\hat{\boldsymbol{\sigma}}\hat{g}\right)_{\alpha\beta}\left(\hat{g}\hat{\boldsymbol{\sigma}}\mathbf{b}^*_{l'm_j}\left(\mathbf{n}'\right)\right)_{\gamma\delta}, \tag{4}$$

where $V_{ll'}(p, p')$ are the interaction amplitudes, and $l, l' = 1, 3$, in the case of tensor forces; $\varrho = p_F M^*/\pi^2$ is the density of states near the Fermi surface in the normal state. The effective mass of a neutron quasiparticle is defined as $M^* = p_F/\upsilon_F$, where $\upsilon_F \ll 1$ is the Fermi velocity of the non-relativistic neutrons.

The order parameter is of the following general form

$$\hat{D}(\mathbf{n}) = \sum_{lm_j}\Delta_{l,m_j}(\hat{\boldsymbol{\sigma}}\mathbf{b}_{l,m_j})\hat{g}. \tag{5}$$

The ground state occurring in neutron matter has a relatively simple structure (unitary triplet) [16,17], where

$$\sum_{lm_j}\Delta_{l,m_j}\mathbf{b}_{l,m_j}(\mathbf{n}) = \Delta\,\bar{\mathbf{b}}(\mathbf{n}). \tag{6}$$

On the Fermi surface, Δ is a complex constant, and $\bar{\mathbf{b}}(\mathbf{n})$ is a real vector which we normalize by the condition

$$\left\langle \bar{b}^2(\mathbf{n})\right\rangle = 1. \tag{7}$$

The following orthogonality relations are also valid:

$$\int\frac{d\varphi}{2\pi}\mathbf{b}^*_{l,m_j}\mathbf{b}_{l',m'_j} = \delta_{m_jm'_j}\mathbf{b}^*_{l,m_j}\mathbf{b}_{l',m_j}, \tag{8}$$

$$\int\frac{d\varphi}{2\pi}\left(\bar{\mathbf{b}}\mathbf{b}^*_{l,m_j}\right)\left(\bar{\mathbf{b}}\mathbf{b}_{l',m'_j}\right) = \delta_{m_jm'_j}\left(\bar{\mathbf{b}}\mathbf{b}^*_{l,m_j}\right)\left(\bar{\mathbf{b}}\mathbf{b}_{l',m_j}\right). \tag{9}$$

Thus the triplet order parameter can be written as

$$\hat{D}(\mathbf{n}) = \Delta\bar{\mathbf{b}}\hat{\boldsymbol{\sigma}}\hat{g}. \tag{10}$$

Making use of the adopted graphical notation for the ordinary and anomalous propagators, $\hat{G} = \longrightarrow$, $\hat{G}^-(p) = \longleftarrow$, $\hat{F}^{(1)} = \rightarrow\leftarrow$, and $\hat{F}^{(2)} = \leftarrow\rightarrow$, it is convenient to employ the Matsubara calculation technique for the system in thermal equilibrium. Then the analytic form of the propagators is as follows [18,19]

$$\hat{G}(p_\eta, \mathbf{p}) = G(p_\eta, \mathbf{p})\delta_{\alpha\beta}, \qquad \hat{G}^-(p_\eta, \mathbf{p}) = G^-(p_\eta, \mathbf{p})\delta_{\alpha\beta}, \qquad \hat{F}^{(1)}(p_\eta, \mathbf{p}) = F(p_\eta, \mathbf{p})\bar{\mathbf{b}}\hat{\boldsymbol{\sigma}}\hat{g}, \qquad \hat{F}^{(2)}(p_\eta, \mathbf{p}) = F(p_\eta, \mathbf{p})\hat{g}\hat{\boldsymbol{\sigma}}\bar{\mathbf{b}}, \tag{11}$$