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A new method for resolving combinatorial ambiguities at hadron colliders

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ABSTRACT

We present a new method for resolving combinatorial ambiguities that arise in multi-particle decay chains at hadron colliders where the assignment of visible particles to the different decay chains has ambiguities. Our method, based on selection cuts favoring high transverse momentum and low invariant mass pairings, is shown to be significantly superior to the more traditional hemisphere method for a large class of decay chains, producing an increase in signal retention of up to a factor of 2. This new method can thus greatly reduce the combinatorial ambiguities of decay chain assignments.

1. Introduction

The turn-on of the Large Hadron Collider ushers in a data-driven era of particle physics. Major theoretical frameworks, such as supersymmetry (SUSY) and universal extra dimensions (UED), will be tested using present and upcoming collider data. In many such models, the lightest new physics particle is protected from decaying into Standard Model particles because of a discrete symmetry, e.g. R-parity or Kaluza–Klein parity for SUSY and UED, respectively. The resulting lightest supersymmetric particle (LSP) or lightest KK-odd particle (LKP), if neutral, is usually a viable dark matter candidate.

Measuring the properties of this dark matter candidate is critical for determining its cosmological behavior and its ultimate suitability as the dark matter. Because of the discrete symmetry, these particles must be produced in pairs at colliders, but since they escape, they manifest themselves in a collider event as large amounts of missing transverse momentum. Since only the total missing transverse momentum can be measured, the kinematics of these events cannot be fully reconstructed, and as a result, the dark matter candidate mass cannot in general be measured in any given event. It is therefore important to have methods of measuring this mass, which also serves to set the overall scale of new physics.

A large number of kinematic analysis techniques have been proposed in the literature to extract the mass of the dark matter candidate. Broadly speaking, these can be divided into three categories. The first is based on edge measurements of invariant mass distributions, using the fact that the algebraic expressions for such endpoints are related to the on-shell masses of the cascade decay chain [1–9]. The second class of analysis methods is known as the polynomial method, which solves the non-linear kinematic four-momentum conservation equations and thus determines the masses of the entire decay chain [10–14]. The last broad category of approaches uses new kinematic observables and functions such as m_{T2}, m_{CT}, and m_{CT2} [15–32].

While these methods work well under idealized conditions, in practice there can be serious obstacles to implementing them successfully. Perhaps the most important problem, common to all these approaches, is the presence of combinatoric backgrounds which occur because the new particles must be produced in pairs. For example, consider the process where two gluinos are produced and decay through a squark to a neutralino: $\tilde{g}\tilde{g} \to qqqq\tilde{\chi}_1^0\tilde{\chi}_1^0$. We will observe four jets, but we do not know which were emitted first nor from which gluino each was emitted. There is thus an ambiguity in reconstructing the event from the visible particles; we will refer to this as a combinatorial ambiguity. In general, with pair-produced parents that decay via identical long cascades, combinatorial ambiguities present a major hurdle in distinguishing the appropriate assignment of particles to each chain as well as the unique ordering of these particles, as discussed in a recent review on mass reconstruction techniques [30]. Wrong assignments can lead to significant suppression of mass peaks and cause large tails in distributions [14]. We also note that understanding these ambiguities is crucial in extracting the nature of the new physics.

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We emphasize that these kinematic reconstruction methods are not affected equally by wrong combinations. In fact, the most general transverse variable based on m_{T2}, known as m_{TGen} [17], does not suffer from combinatorial ambiguities since it explicitly includes a minimization over all possible decay chain assignments of the observed object momenta. For regular m_{T2} studies, the assignment ambiguity, *i.e.* assigning particles to separate decay chains, is relevant, while the ordering ambiguity, *i.e.* the sequence of the particles on the decay chain, is not important: by construction, the transverse mass of each decay chain is irrespective of decay chain placement, but is clearly dependent on decay chain assignments. The subsystem m_{T2} reconstruction method [24,26], however, is adversely affected by both the assignment ambiguity and the ordering ambiguity. Similarly, wrong assignment combinations and wrong ordering choices degrade the effectiveness of the polynomial method [10,13,14] and can also worsen results from the kinematic edges approach if the invariant mass distributions do not encapsulate entire decay chains.

In this Letter, we will focus on resolving the combinatorial ambiguity arising from decay chain assignments, leaving the question of resolving ordering ambiguities for the future. Thus, our results should improve the effectiveness of all of the aforementioned kinematic reconstruction methods except m_{TGen}, which, as noted above, does not suffer from assignment ambiguities. Henceforth, “combinatorial ambiguity” will refer solely to the decay chain assignment ambiguity discussed above.

In previous kinematic reconstruction studies, past authors have designed a variety of methods to address combinatorial ambiguities. For example, in [13,14], they apply the polynomial method to pairs of events, and they address combinatorial ambiguities by favoring, for a given event, the assignment that maximizes the number of event pairings that give algebraic solutions. In this way, they hoped to discard wrong combinations of a given event that would give unphysical solutions when paired with correct combinations of other events. In [31], which used m_{T2}- and subsystem m_{T2}-based methods, they chose the combinations of jet pairs with smaller invariant masses and smaller angular separation as well as discarding the largest sub-system m_{T2} values, arguing that correct jet pairings should be directionally focused, and high invariant mass jet pairings are more likely incorrect. While these representative approaches at reducing wrong combinations work on an individual analysis basis, we note that these methods are not interchangeable. Even though the combinatorial ambiguity for kinematic reconstruction studies is in general a common difficulty, many of the specific approaches to resolve such combinatorial ambiguities used in the literature cannot be generalized when using more than one kinematic reconstruction technique.

One exception is the hemisphere method, used in [21,22,24,29] (we will describe this method in detail below in Section 3). If the parent particles of each decay chain are strongly boosted, their decay products will be collimated along the initial momenta. By considering events with suitably large transverse momenta, one may hope to avoid combinatorial ambiguities (cf. Section 13.4 of [33]). However, this can lead to significant loss in signal statistics. On the other hand, the hemisphere method of reducing combinatorial ambiguities allows for the simultaneous application of multiple kinematic reconstruction methods, without a specialized approach.

In this Letter, we present a new method for resolving such combinatorial ambiguities in decay chains. Our method is described in Section 4. We show that our method can yield highly accurate assignments of particles to decay chains which have little contamination from wrong combinations. We contrast our method with a parallel study of the efficacy of the more familiar hemisphere method. Our results show that for cascade decay chains with on-shell mass resonances, our method improves signal retention up to a factor of 2 over the usual hemisphere method.

Table 1
Model spectra.

Model name	$\tilde{g}$ (GeV)	$\tilde{q}$ (GeV)	$\tilde{\chi}_1^0$ (GeV)	Diquark invariant mass edge (GeV)
Model A	600	400	100	433
Model B	600	800	100	500

We begin by presenting two toy models, which we shall use to compare the two methods. In Section 2, we present the model masses and aspects of the simulation. In Section 3, we review the hemisphere method and give our hemisphere method implementation details. We then discuss our new method (which we shall call the p_T v. M method) in Section 4 and describe the specifics of our procedure in Section 5. We compare the two methods in Section 6 and conclude in Section 7.

2. Models

In this section, we present the toy models used in our analysis. The masses of the two models considered are summarized in Table 1. Roughly, Model A mimics a SPS1a-style mass spectrum, while Model B represents an off-shell squark scenario.

Our aim is to isolate the combinatorial ambiguities arising from pure signal events when the final state particles are indistinguishable inside the detector. We focus solely on gluino pair production with each gluino decaying to the lightest neutralino and two quarks: $\tilde{g}\tilde{g} \to qqqq\,\tilde{\chi}_1^0\tilde{\chi}_1^0$. The intermediate squark is on-shell (Model A) or off-shell (Model B). All quarks are treated at parton level, and effects from initial state radiation or parton showering are not considered. These events have three distinct pair–pair combinations. Isolating which particles arise from the same decay chain is a first step in any kinematic event analysis. Since we are tackling the combinatorial ambiguity that arises even when dealing with only signal events, we shall not complicate our analysis by adding background events or multiple production or decay modes.

It is well known that cascade decays impose restrictions on the kinematics of the outgoing decay products. For our gluino cascade decay, the diquark invariant mass edge for on-shell intermediate squarks is

$$m_{qq}|_{\text{edge}} = \sqrt{\frac{(m_{\tilde{g}}^2 - m_{\tilde{q}}^2)(m_{\tilde{q}}^2 - m_{\tilde{\chi}_1^0}^2)}{m_{\tilde{q}}^2}}, \tag{1}$$

assuming the quarks are massless, and where $m_{\tilde{g}}$, $m_{\tilde{q}}$, and $m_{\tilde{\chi}}$ are the mass of the gluino, squark, and neutralino, respectively. For the off-shell squark case, the edge value is simply

$$m_{qq}|_{\text{edge}} = m_{\tilde{g}} - m_{\tilde{\chi}_1^0}. \tag{2}$$

Of necessity, a correct assignment of the jets to the two sides of the event will have the invariant mass of the jet–jet pair below the kinematic edge. Naturally, an incorrect pair assignment can produce an invariant mass beyond the relevant kinematic edge, since the two quarks are uncorrelated. Also, note that the diquark invariant mass distribution through an on-shell squark possesses a characteristic triangular shape, with its rising edge saturating the upper endpoint. On the other hand, the off-shell squark case has the upper invariant mass edge characteristically falling near the endpoint. Thus, the kinematic edge is easier to identify in on-shell scenarios, and in either case, rejection of pair assignments with invariant masses beyond the edge serves to reduce combinatorial ambiguities.