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Dispersive e.m. corrections to πN scattering lengths

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Abstract

We investigate the dispersive contribution by radiative processes such as $\pi^- p \to n\gamma$ and $\pi^- p \to \Delta\gamma$ to the πN scattering lengths of charged pions in the heavy baryon limit. They give a large isospin violating contribution in the corresponding isoscalar scattering length, but only a small violation in the isovector one. These terms contribute 7.1(3)% to the 1s level shift of pionic hydrogen and give a chiral constant $F_\pi^2 f_1 = -28.0(8)$ MeV.

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1. Introduction

The observed 1s energy shift and width in the $\pi^- p$ atom as compared to the purely electromagnetic bound state energies have recently reached the remarkable precision of ±0.2% and ±4%, respectively [1] (see also [2]). These quantities are proportional to the corresponding real and imaginary part of the threshold $\pi^- p$ scattering amplitude (the scattering length) [3–7], which therefore are determined to the same precision provided electromagnetic corrections on the several % level can be properly understood and accounted for.

In the approximation that the electromagnetic interaction is switched off, but with the observable hadron masses, the real and imaginary parts of the corresponding scattering lengths and of the real and imaginary parts of the S-wave amplitude $f^{\pi^- p}(0)$ at threshold ($q = 0$, where q is the relative momentum of the $\pi^- p$ pair) have the relation:

$$\mathcal{R}\mathrm{e}\, f^{\pi^- p}(0) = a_{cc}, \qquad \mathcal{I}\mathrm{m}\, f^{\pi^- p}(0) = q_{\pi^0 n} a_{nc}^2. \tag{1}$$

Here $a_{cc} = a_{\pi^- p \to \pi^- p}$ and $a_{nc} = a_{\pi^- p \to \pi^0 n}$ are the S-wave scattering lengths of the reactions $\pi^- p \to \pi^- p$ (the charged channel (cc)) and of $\pi^- p \to \pi^0 n$ (the charge exchange one), respectively, and $q_{\pi^0 n} = 28.040$ MeV is the relative momentum of the $\pi^0 n$ system.[2]

One has then, in principle, an exceptional source for the scattering lengths which can be directly compared to the large body of phenomenological πN phase shift data as well as to predictions of chiral perturbation theory (ChPT).

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[2] We have removed the physical contribution to the imaginary part from the radiative channel $\pi^- p \to \gamma n$, which is accurately known from the Panofsky ratio $P = \sigma(\pi^- p \to \pi^0 n)/\sigma(\pi^- p \to \gamma n) = 1.546(10)$ [8].

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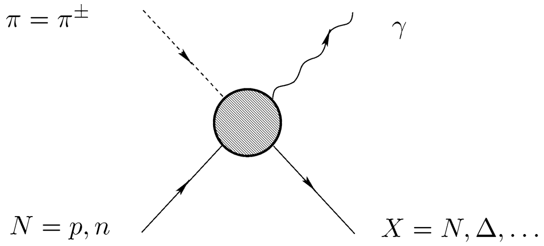

Fig. 1. Diagram of $\pi N \to X\gamma$ reactions with one photon in the intermediate state generating a contribution to low-energy elastic πN scattering.

The electromagnetic corrections to the observed energy level shift, caused by one-photon exchange, are of two types: (a) corrections with the intermediate $\pi^- p$ state remaining in its ground state and (b) corrections corresponding to inelastic intermediate states. In the ladder approximation, taking into account multi-photon exchanges, the corrections of the first class have been investigated in Ref. [7]. These moderate corrections were found to correspond to well understood physical effects to a precision commensurate with the present experimental one. While such effects violate isospin, it is not usual to consider them as genuine dynamic isospin breaking. To leading order such a term is generated by gauging the Tomozawa–Weinberg interaction for an extended charge distribution. This gives a well defined contribution corresponding to the effective ChPT (EFT) constant f_2 [7].

The second group of e.m. corrections are intrinsic to the scattering processes. They produce genuine isospin breaking together with the isospin breaking from the strong interaction itself. In order to reduce the data to pure hadronic interactions one must therefore control these intrinsic e.m. contributions to sufficient accuracy and understand their physics.[3]

Here we investigate this second class of electromagnetic corrections. There are excellent reasons to believe that such electromagnetic processes contribute substantially to the $\pi^- p$ scattering length. In fact, the dominant contribution (65%) to the 1s width of pionic hydrogen is the radiative capture by the electric dipole transition $\pi^- p \to n\gamma$, the so-called Kroll–Ruderman process [10] (Fig. 1). The corresponding radiative width is no less than 8% of the strong interaction shift as observed in the Panofsky ratio [8]. This indicates that the corresponding dispersive term may contribute 4% and even more to the energy shift in pionic hydrogen. This is indeed the case as we will show.

Since the Kroll–Ruderman contribution is generated from the pion P-wave part of the nucleon pole term, one must inevitably consider also other important aspects of the well understood P-wave πN physics. Here the Δ isobar and the $N\Delta$ mass difference are central features with the Δ pole as important as the nucleon one [11]. We therefore include it on an equal footing with the nucleon. Since the main purpose of the present Letter is to clarify the physical mechanisms of e.m. isospin breaking in the πN scattering lengths in simple terms, we rely on the heavy baryon limit. This is in accordance with P-wave πN phenomenology for which this approximation describes well the main aspects of the interaction. It will become apparent that the bulk of the isospin breaking occurs already in the limit of a vanishing pion mass. The finite pion mass introduces small, but characteristic, additional terms.

The heavy baryon limit, used in chiral perturbation expansion as well, and the threshold condition lead to great simplifications of the problem. The relevant contribution becomes that of an electric dipole process (E1) due to transition radiation by the absorption or emission of the charged pion. In particular, there is no coupling to the baryon convection current nor to its magnetic moment in this limit, while the threshold condition suppresses radiation produced by changes in the pion convection current.

The leading e.m. isospin breaking effects in low-energy πN scattering have previously been discussed in particular using heavy baryon ChPT [see Ref. [12] and references therein] as well as using a heavy-quark model [13]. Such isospin breaking is implicit in several theoretical studies of the contributions to the 1s energy shift of the $\pi^- p$ atom to leading [14] and next to leading order power counting in an effective field theory (EFT) of QCD + QED [9]. Additional contributions are of order αm_π^2 [7,9].

The Letter is organized as follows. In Section 2 we derive the general expression for the dispersive e.m. contributions to the S-wave amplitude of elastic $\pi^- p$ scattering at threshold, saturated by intermediate $X\gamma$ states related to the reactions the $\pi^- p \to X\gamma$, where X is a hadronic state (see Fig. 1). In Section 3 we apply the results specifically to the calculation of the dispersive contributions from the reactions $\pi^- p \to n\gamma$ and $\pi^- p \to \Delta\gamma$, respectively, and generalize this to the elastic $\pi^c N$ threshold amplitude for any charged pion. In Section 4 we discuss the individual contributions to the isospin breaking under different assumptions and discuss the relation of our results to other investigations. In the Conclusion we summarize the results.

2. One-photon exchange contributions to the elastic $\pi^- p$ threshold amplitude

For concreteness we will illustrate these contributions from inelastic intermediate states for the case of $\pi^- p$ elastic scattering at threshold, but the argument is nearly identical for any elastic πN channel with a charged pion. The S-wave amplitude of the $\pi^- p$

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[3] Another use of the relation (1) is the description of the energy shift in pionic hydrogen in terms of ChPT [9]. In this case terms of different origin are not separated. The e.m. contributions include implicitly the Coulomb terms.