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CPT violation does not lead to violation of Lorentz invariance and vice versa

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ABSTRACT

We present a class of interacting nonlocal quantum field theories, in which the *CPT* invariance is violated while the Lorentz invariance is present. This result rules out a previous claim in the literature that the *CPT* violation implies the violation of Lorentz invariance. Furthermore, there exists the reciprocal of this theorem, namely that the violation of Lorentz invariance does not lead to the *CPT* violation, provided that the residual symmetry of Lorentz invariance admits the proper representation theory for the particles. The latter occurs in the case of quantum field theories on a noncommutative space–time, which in place of the broken Lorentz symmetry possesses the twisted Poincaré invariance. With such a *CPT*-violating interaction and the addition of a *C*-violating (e.g., electroweak) interaction, the quantum corrections due to the combined interactions could lead to different properties for the particle and antiparticle, including their masses.

1. Introduction

Lorentz symmetry and the *CPT* invariance are two of the most fundamental symmetries of Nature, whose violation has not yet been observed. While the Lorentz invariance is a continuous symmetry of space–time, the *CPT* involves the discrete space- and time-inversions, *P*, *T*, and the charge conjugation operation on the fields, *C*. Although the individual symmetries, *C*, *P* and *T* have been observed to be violated in various interactions, their combined product, *CPT*, remarkably remains still as an exact symmetry. The first proof of *CPT* theorem was given by Lüders and Pauli [1,2] based on the Hamiltonian formulation of quantum field theory, which involves locality of the interaction, Lorentz invariance and Hermiticity of the Hamiltonian. Later on the theorem was proven by Jost [3] (see also [4–6]) within the axiomatic formulation of quantum field theory without reference to any specific form of interaction. This proof of *CPT* theorem relaxes the requirement of locality or "local commutativity" condition to the so-called "weak local commutativity". Lorentz symmetry has been an essential ingredient of the proof, both in the Hamiltonian and in the axiomatic proofs.

A simple phenomenological classification of possible *C*-, *P*-, *T*-, *CP*-, *PT*-, *TC*- and *CPT*-violating effects is presented in [7]. For consequences of *CPT* and their experimental tests, as well as some theoretical considerations on the possibilities of violation of Lorentz invariance and *CPT* in the known interactions, we refer to [8–13] and references therein.

It is important to clarify the relation between the *CPT* and Lorentz invariance and in particular to see whether the violation of any of them implies the violation of the other. This issue has recently become a topical one due to the growing phenomenological importance of *CPT* violating scenarios, namely in neutrino physics as well as its cosmological and astrophysical consequences. Indeed, the relation between the *CPT* and Lorentz invariance has acquired a prominent place in nowadays particle physics with the attempts of explaining in a unified manner the contradictory results, "anomalies", in the interpretation of various neutrino physics experiments, without enlarging the neutrino sector. The idea was first suggested by Murayama and Yanagida [14] in the form of different masses for neutrino and antineutrino, based on phenomenological considerations. This proposal was formalized as a *CPT*-violating quantum field theory with a mass difference between neutrino and antineutrino in [15] (see also [16]). The issue was taken up in relation with the Lorentz symmetry by Greenberg [17], the conclusion of Greenberg's analysis being that *CPT* violation implies violation of Lorentz invariance. This result was given as a "theorem", the dispute on the validity of which is the subject of this Letter.

We should emphasize that a theorem which states that *CPT* violation implies violation of Lorentz invariance has to be explicit, first of all about what is meant by the charge conjugation in a

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Lorentz violating theory. Is the violation complete or is any subgroup of Lorentz symmetry left, which should have the needed spin-representations to which the particles are assigned? Does the corresponding theory which violates both *CPT* and Lorentz invariance contain fields with a plausible description in terms of equations of motion?

2. *CPT*-violating free field model

A free field model in which particle and antiparticle have different masses was proposed in [15]. Although the model was hoped to be Lorentz-invariant, a closer examination [17] showed that it is not – the propagator is not Lorentz-covariant, unless the masses of particle and antiparticle coincide. The model is also nonlocal and acausal: the $\Delta(x, y)$-function, i.e. the commutator of two fields, does not vanish for space-like separation, unless the two masses are the same, thus violating the Lorentz invariance. This was considered in [17] as supporting a general "theorem" that interacting fields that violate *CPT* symmetry necessarily violate Lorentz invariance.

We would like to point out that the model taken in [17] is utmost pathological and cannot be considered as a quantum field theory. There, the claim was that the model represents a free complex scalar field, quantized in such a way that the mass of the antiparticle differs from that of the particle. However, there is no definite equation of motion that this "field" satisfies, and no quantization procedure that would support the claim that the mode expansion with different masses for "particle" and "antiparticle" really represents a free quantized field. Also, two such "free fields" separated by a space-like distance do not commute, i.e. the theory is acausal at the free level without invoking interaction.

Moreover, by requiring that the classical symmetries and in particular the global $U(1)$ symmetry for a free complex scalar field, i.e. the conservation of electric charge, be preserved at the quantum level, one can show that using the expansion for a free "field" as proposed in [17], would bring it back uniquely to the usual field expansion in terms of creation and annihilation operators with $m = \bar{m}$ – otherwise, the electric charge is not conserved.

Furthermore, in a quantum field theory with acausal free fields, as taken in [17], observables, which are functions of those fields, do not commute when separated by space-like distances. This, according to Pauli's proof of the spin–statistics theorem, implies that there is no spin–statistics relation already for *the free fields*. Thus, one has no rule whether to apply commutation or anticommutation relations in quantizing the fields. But the worst is that in such a model, where Lorentz invariance is violated by the free fields, there is *no concept of spin* to start with altogether.

3. *CPT*-violating but Lorentz-invariant nonlocal model

Here, as an example, we propose a model which preserves Lorentz invariance while breaking the *CPT* symmetry through a (nonlocal) interaction. The latter attitude is taken as responsible for the violations of a symmetry, based on our experience that all the discrete, C, P and T invariances, as well as other symmetries, are broken in our description of Nature by means of interaction. We also know that nonlocal field theories appear, in general, as effective field theories of a larger theory.

Consider a field theory with the nonlocal interaction Hamiltonian of the type

$$\mathcal{H}_{int}(x) = \lambda \int d^4y\, \phi^*(x)\phi(x)\phi^*(x)\theta(x_0 - y_0) \times \theta\big((x-y)^2\big)\phi(y) + \text{h.c.}, \tag{3.1}$$

where λ is a coupling constant with dimension appropriate for the Hamiltonian density, $\phi(x)$ is a Lorentz-scalar field in the interaction picture and θ is the Heaviside step function, with values 0 or 1, for its negative and positive argument, respectively. The combination $\theta(x_0 - y_0)\theta((x-y)^2)$ in (3.1) ensures the Lorentz invariance, i.e. invariance under the proper orthochronous Lorentz transformations, since the order of the times x_0 and y_0 remains unchanged for time-like intervals, while for space-like distances the interaction vanishes. Also, the same combination makes the nonlocal interaction causal at the tree level, which dictates that there is no interaction when the fields are separated by space-like distances and thus there is a maximum speed of $c = 1$ for the propagation of information.

On the other hand, it is clear that C and P invariance are trivially satisfied in (3.1), while T invariance is broken due to the presence of $\theta(x_0 - y_0)$ in the integrand.

One can always insert into the Hamiltonian (3.1), without changing its symmetry properties, a weight function or form-factor $F((x-y)^2)$, for instance of a Gaussian type:

$$F = \exp\left(-\frac{(x-y)^2}{l^2}\right), \tag{3.2}$$

with l being a nonlocality length in the considered theory. Such a weight function would smear out the interaction and would guarantee the desired behaviour of the integrand in (3.1); in the limit of fundamental length $l \to 0$ in (3.2), the Hamiltonian (3.1) would correspond to a local, *CPT*- and Lorentz-invariant theory. A weight function such as (3.2) would make the acausality of the model (see the next section) restricted only to very small distances, of the order of l. The latter could be looked upon as being a characteristic parameter relating the effective field theory to its parent one, for instance the radius of a compactified dimension when the parent theory is a higher-dimensional one. Furthermore, with such a weight function, the interaction vanishes at infinite $(x-y)^2$ separations and thus one can envisage the existence of in- and out-fields.

There exists a whole class of such *CPT*-violating, Lorentz-invariant field theories involving different, scalar, spinor or higher-spin interacting fields. Typical simplest examples are:

$$\mathcal{H}_{int}(x) = \lambda \int d^4y\, \phi_1^*(x)\phi_1(x)\theta(x_0 - y_0) \times \theta\big((x-y)^2\big)\phi_2(y) + \text{h.c.}, \tag{3.3}$$

$$\mathcal{H}_{int}(x) = \lambda \int d^4y\, \bar{\psi}(x)\psi(x)\theta(x_0 - y_0) \times \theta\big((x-y)^2\big)\phi(y) + \text{h.c.}, \tag{3.4}$$

$$\mathcal{H}_{int}(x) = \lambda \int d^4y\, \phi(x)\theta(x_0 - y_0)\theta\big((x-y)^2\big)\phi^2(y) + \text{h.c.} \tag{3.5}$$

4. Quantum theory of such nonlocal interactions

The S-matrix in the interaction picture is obtained as solution of the Lorentz-covariant Tomonaga–Schwinger equation [18, 19] (see also [20,21]):

$$i\frac{\delta}{\delta\sigma(x)}\Psi[\sigma] = \mathcal{H}_{int}(x)\Psi[\sigma], \tag{4.1}$$

with σ a space-like hypersurface, and the boundary condition:

$$\Psi[\sigma_0] = \Psi, \tag{4.2}$$

where $\mathcal{H}_{int}$ is for instance the Hamiltonian (3.5) with the fields in the interaction picture. Then Eq. (4.1) with the boundary condition (4.2) represent a well-posed Cauchy problem.