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Correlations among elastic and inelastic cross-sections and slope parameter

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ABSTRACT

We discuss the unitarity motivated relations among the elastic cross-section, slope parameter and inelastic cross-section of the high energy *pp* interaction. In particular, the MacDowell-Martin unitarity bound is written down in another form to make a relation between the elastic and inelastic quantities more transparent. On the basis of an unitarity motivated relation we argue that the growth with energy of the elastic to total cross-section ratio is a consequence of the increasing with energy of the *inelastic interaction intensity*. The latter circumstance is an underlying reason for the acceleration of the slope parameter growth, for the slowing of the growth of the elastic to total cross-section ratio and for other interesting phenomena, which are observed in the TeV energy range. All of this confirms the old idea that the elastic scattering is a shadow of the particle production processes.

1. Introduction

A growth with energy of the *pp* total cross-section $\sigma_{\rm tot}(s) = \sigma_{\rm el}(s) + \sigma_{\rm inel}(s)$ is due to that of the elastic $\sigma_{\rm el}(s)$ and the inelastic $\sigma_{\rm inel}(s)$ cross-sections [1–7]. If the growth of $\sigma_{\rm inel}(s) = \sum_{n,\,{\rm inel}}^{N(s)} \sigma_n(s)$ can be formally attributed to a huge number of open inelastic channels $N(s)$ [8], the underlying reasons of the $\sigma_{\rm el}(s)$ growth are unknown. On the other hand, the unitarity condition relates the properties of the elastic scattering amplitude with the contribution from the inelastic channels and the elastic scattering can therefore be considered as a shadow of the particle production processes [9]. In other words, due to unitarity there are some correlations between behaviour of the characteristics of the elastic and inelastic scattering. Indeed, the MacDowell-Martin unitarity bound [10] gives such a relation among the total cross-section, the elastic cross-section and the slope parameter of the imaginary part of the elastic scattering amplitude

$$B_I(s) \equiv 2[\frac{d}{dt}\ln|{\rm Im}\,T(s,t)|]_{t=0} \geq \frac{\sigma^2_{\rm tot}(s)}{18\pi\sigma_{\rm el}(s)}. \tag{1}$$

In the present note we rewrite this inequality in another form to make a relation between the elastic and inelastic quantities more transparent.

According to the optical theorem (which is a consequence of the unitarity condition) the elastic differential cross-section at zero value of the square of the four-momentum transfer, t, is related to the total cross-section as

$$\frac{d\sigma}{dt}|_{t=0} = \frac{\sigma^2_{\rm tot}(s)(1+\rho^2(s))}{16\pi}, \quad \rho(s) = \frac{{\rm Re}\,T(s,0)}{{\rm Im}\,T(s,0)}. \tag{2}$$

The slope of the forward diffraction peak, $B(s)$, and $(d\sigma/dt)_{t=0}$ are determined experimentally by extrapolation of the nuclear elastic scattering differential cross-section data at small values of t to the forward direction $t = 0$ using the exponential form

$$\frac{d\sigma}{dt} = \frac{d\sigma}{dt}|_{t=0}\exp(Bt). \tag{3}$$

The experimental value of $\sigma_{\rm tot}(s)$ is then calculated from Eq. (2) (the ratio of the real to the imaginary part of the elastic scattering amplitude in the forward direction, $\rho(s)$, is taken in this method from the dispersion relations or from global model extrapolations). If the local slope parameter, $B(s,t) = d\ln(d\sigma/dt)/dt$, is approximately equal to $B(s)$ in the essential for the value of integral $\sigma_{\rm el}(s) = \int dt(d\sigma/dt)$ region $0 \leq |t| \leq |t_0|$, where $|t_0| \approx 0.4$ GeV^2, the elastic cross-section is given by the following formula [11]

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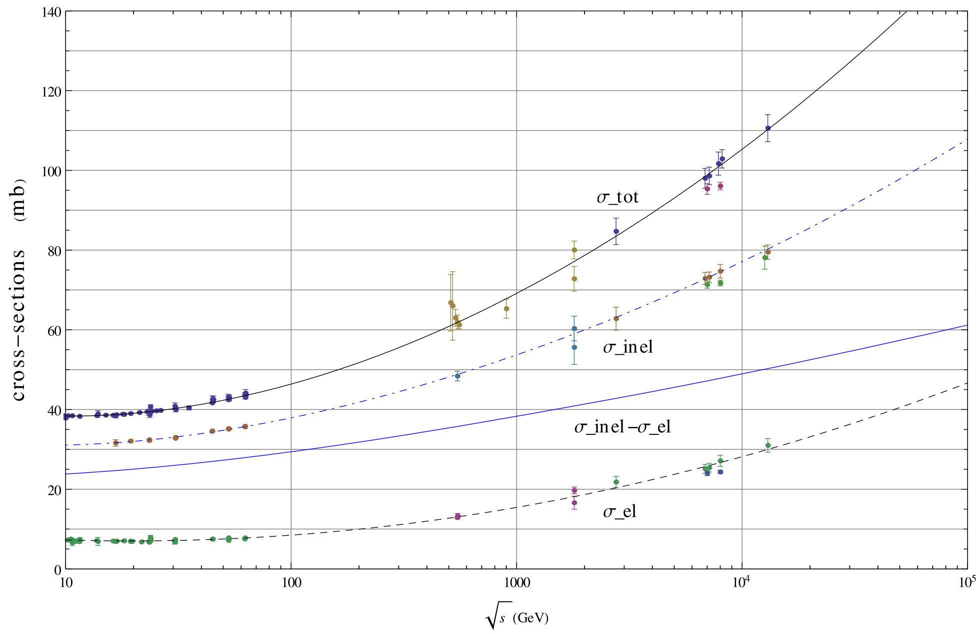

Fig. 1. The elastic (σ_{el}), inelastic (σ_{inel}), total (σ_{tot}) cross-section for pp collisions as a function of $\sqrt{s}$ [1–7], including the $\bar{p}p$ data at $\sqrt{s} = 546, 900, 1800$ GeV [14]. The first on top line is a fit of the total cross-section data by the COMPETE collaboration [33]. The dashed line is a fit of the elastic cross-section data by the TOTEM collaboration [7]. The dash-dotted and continuous lines refer to the $\sigma_{\text{inel}} = (\sigma_{\text{tot}} - \sigma_{\text{el}})$ and $(\sigma_{\text{inel}} - \sigma_{\text{el}}) = (\sigma_{\text{tot}} - 2\,\sigma_{\text{el}})$ respectively and are obtained as the differences between the σ_{tot} and σ_{el} fits.

$$\sigma_{\text{el}}(s) = \frac{\sigma_{\text{tot}}^2(s)(1+\rho^2(s))}{16\pi B(s)}. \tag{4}$$

At the ISR energies this formula gives a slightly underestimated value for $\sigma_{\text{el}}(s)$ because the local slope decreases noticeably with $|t|$ in the $0 \le |t| \le 0.4$ GeV^2 range [12], but beyond the ISR energies the relation (4) is practically exact [2–7]. The luminosity-independent measurements at 7 TeV [4] give much the same values for $\sigma_{\text{el}}(s)$, $\sigma_{\text{inel}}(s)$ and $\sigma_{\text{tot}}(s)$ as the discussed above method and confirm therefore the validity of formula (4). At the LHC energies the local slope reveals a trend to increase with $|t|$ at $|t| \gtrsim 0.2\,\text{GeV}^2$ [13] due to the nearness of the dip structure of the differential cross-section. For that reason, beyond the LHC energies the formula (4) will give a somewhat overestimated value for $\sigma_{\text{el}}(s)$. So, in the $10^2 \lesssim \sqrt{s} \lesssim 5 * 10^4$ GeV energy range Eq. (4) can be considered as a practically exact relation. We use it to see the connections between the elastic and inelastic quantities. Let us note that the MacDowell-Martin bound is close to Eq. (4) because according to the experimental data $\rho^2(s) \ll 1$ and $B_1(s) \approx B(s)$.

It is an astonishing experimental fact that the elastic cross-section grows faster than the total cross-section, that is the ratio $\sigma_{\text{el}}(s)/\sigma_{\text{tot}}(s)$ increases with energy [14,2–7]. According to Eq. (4) this growth is the same as the $\sigma_{\text{tot}}(s)/B(s)$ growth (the impact of the $(1+\rho^2(s))$ factor is negligible), which can be interpreted as an increase of the interaction intensity in the central part of the interaction region in the impact parameter space [15–17]. Indeed, if the cross-sections $\sigma_{\text{el}}(s)$, $\sigma_{\text{inel}}(s)$ grow only due to the increasing with energy of the radius of the interaction region $R(s)$, i.e. $\sigma_{\text{el}}, \sigma_{\text{inel}} \sim R^2$, then the ratios σ_{el}/B, σ_{inel}/B are energy-independent [1], since $B(s) = 0.5R^2(s)$. Hence, the energy growth of the σ_{el}/B, σ_{inel}/B ratios has a dynamical, non geometrical nature and will be referred to as the increasing of the elastic and inelastic interaction intensity respectively.

From $\sigma_{\text{tot}} = \sigma_{\text{el}} + \sigma_{\text{inel}}$ it is evident that the $\sigma_{\text{el}}/\sigma_{\text{tot}}$ growth is equivalent to the decreasing of the $\sigma_{\text{inel}}/\sigma_{\text{tot}}$ and $\sigma_{\text{inel}}/\sigma_{\text{el}}$ ratios. However, as can be seen from the experimental data [1–7], the ratio $\sigma_{\text{inel}}(s)/B(s)$ is an increasing function of energy. It means that the inelastic cross-section grows not only due to the growth of the radius of the interaction region but also due to the increasing of the inelastic interaction intensity in the expanding with energy central part of the interaction region [18–24]. As we will see, the relation (4) enables to get the ratios σ_{el}/B, $\sigma_{\text{el}}/\sigma_{\text{tot}}$ and $\sigma_{\text{el}}/\sigma_{\text{inel}}$ in the form of increasing functions of ratio (σ_{inel}/B). Therefore, the growth of these ratios with energy is a consequence (a "shadow") of the increasing of the inelastic interaction intensity.

According to the experimental data [1–7] the difference $(\sigma_{\text{inel}} - \sigma_{\text{el}}) = (\sigma_{\text{tot}} - 2\sigma_{\text{el}})$ monotonically increases with energy (see Fig. 1). Due to the $\sigma_{\text{el}}/\sigma_{\text{tot}}$ growth the ratios of this difference to the σ_{el}, σ_{inel} and σ_{tot} decrease with energy but the ratio $(\sigma_{\text{inel}} - \sigma_{\text{el}})/B$, as can be seen from Eq. (4), grows with energy up to $\sim$ 3 TeV (where $\sigma_{\text{el}}/\sigma_{\text{tot}} = 0.25$ [7]), reaches its maximum value and then decreases with energy. There is evidence that approximately at this energy the slope $B(s)$ begins to accelerate its growth [6,7] (the discussions of this new phenomenon in the context of different models can be found in Refs. [25–31]). As can be seen from Ref. [7], the curvature of the $\sigma_{\text{el}}/\sigma_{\text{tot}}$ growth changes its sign from positive to negative also in this energy range [16], and therefore the ratio $\sigma_{\text{el}}/\sigma_{\text{tot}}$ slows down its growth. As we will see, the unitarity motivated Eq. (4) enables to consider all these phenomena, as well as the $\sigma_{\text{el}}/\sigma_{\text{tot}}$ growth itself, as a consequence of the increasing of the intensity of the inelastic interaction (σ_{inel}/B).

In Section 2 we give a few new forms of the MacDowell-Martin unitarity bound, which relate the elastic and inelastic quantities. The consequences of Eq. (4) are studied in Section 3, where, in addition, we discuss some phenomenological arguments in favour of the shadow origin of the $\sigma_{\text{el}}(s)$ growth. A brief summary and discussion are given in Section 4.

2. The MacDowell-Martin bound

The MacDowell-Martin unitarity bound (1) can be written as

$$\sigma_{\text{tot}}^2(s) \le \tilde{\beta}(s)\sigma_{\text{el}}(s), \quad \tilde{\beta}(s) \equiv 18\pi B_1(s). \tag{5}$$

Taking into account a relation $\sigma_{\text{el}} = (\sigma_{\text{tot}} - \sigma_{\text{inel}})$, we can rewrite Eq. (5) in the following equivalent form