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Neutrino mass operators of dimension up to nine in two-Higgs-doublet model

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ABSTRACT

We study higher-dimensional neutrino mass operators in a low energy theory that contains a second Higgs doublet, the two Higgs doublet model. The operators are relevant to underlying theories in which the lowest dimension-five mass operators would not be induced. We list the independent operators with dimension up to nine with the help of Young tableau. Also listed are the lowest dimension-seven operators that involve gauge bosons and violate the lepton number by two units. We briefly mention some of possible phenomenological implications.

1. Introduction

The tiny neutrino mass can be accommodated at low energies by nonrenormalizable, higher-dimensional mass operators. With the lepton fields as established in the standard model (SM) and the Higgs fields assumed to be a doublet, such operators first appear at dimension five [1]:

$$\mathcal{O}^{\alpha\beta}_{xy}=\overline{F^C_{Lx}}\tilde{H}^*_\alpha\tilde{H}^\dagger_\beta F_{Ly},\qquad \mathcal{P}^{\alpha\beta}_{xy}=\overline{F^C_{Lx}}\tilde{F}^*_{Ly}\tilde{H}^\dagger_\beta H_\alpha. \tag{1}$$

Here H_α is the α-th Higgs doublet with hypercharge $Y=+1$, and F_{Lx} is the x-th left-handed leptonic doublet with $Y=-1$. A tilde denotes the complex-conjugated field that transforms under $SU(2)_L$ exactly as the original one, e.g., $\tilde{F}_L=i\sigma^2F^*_L$, while the superscript C denotes charge conjugation with the convention $F^C_L=(F_L)^C$.

Both operators $\mathcal{O}$ and $\mathcal{P}$ break the lepton number by two units. When the neutral components of the scalar doublets develop a vacuum expectation value (VEV), $\mathcal{O}$ generates a mass for neutrinos that is inversely proportional to the energy scale Λ of some underlying theory responsible for the operator. Although the operator $\mathcal{P}$ does not generate a mass but involves interactions amongst leptons and scalars of different charge, it may arise from the same mechanism that induces $\mathcal{O}$ due to the similar structure. With a single Higgs doublet as in SM, the operator $\mathcal{O}$ is unique while $\mathcal{P}$ does not exist.

It is interesting to realize that the unique operator $\mathcal{O}$ in SM may be written in three apparently different ways [2]. This amounts to forming a singlet in three ways out of four factors of the two half-isospin fields, and suggests its possible origin from three types of seesaw mechanisms [3–5]. A phenomenological issue with those mechanisms is that the energy scale Λ is so high that it would not be possible to detect any other effects pertained to the origin of neutrino mass. From the viewpoint of effective field theory, the scale may be lowered if the mass is induced not from a dimension-five operator but from those of even higher dimensions. It is conceivable that there will be more and more mechanisms that can induce a mass operator as its dimension increases, see Refs. [6,7] for some recent examples. However, it has been established recently that the mass operator at each higher dimension is always unique [8]. This implies that as far as the neutrino mass is concerned different mechanisms are completely equivalent. But with a lowered scale, it becomes possible to distinguish them through other effects.

In this work we will address the neutrino mass operators in an effective field theory that contains two Higgs doublets. Although the two Higgs doublet model (2HDM) is interesting in itself, the main motivation comes from supersymmetry which is a leading candidate for physics beyond SM and is under examination at high energy colliders. It would be also tempting to see how those higher-dimensional operators are induced in a supersymmetric framework. We will show that with two Higgs doublets the operators are no more unique but increase quickly in number with their dimension. We will list all mass operators of dimension up to nine as well as related dimension-seven operators involving

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the SM gauge fields. We will also discuss briefly some of the phenomenological implications of these operators at low energies.

2. Mass operators up to dimension nine

We assume that the low energy theory contains the SM fields and an additional Higgs doublet that also develops a VEV. The neutrinos can only have a Majorana mass in this case. We are interested in the high-dimensional operators that can yield a neutrino mass (called $\mathscr{O}$-type) when the Higgs fields assume their VEV's, as well as those that do not give a mass but have a similar structure ($\mathscr{P}$-type). We can therefore restrict ourselves to the two-lepton sector that violates the lepton number by two units. The relevant fields are the lepton doublet F_L, the two Higgs doublets H_1 and H_2 plus their properly complex-conjugated fields which also transform as a doublet under $SU(2)_L$:

$$
\begin{aligned}
&a=\overline{F_L^C}(-1), \qquad b=F_L(-1), \qquad c=H_1(+1),\\
&d=H_2(-1), \qquad e=\tilde{H}_2(+1), \qquad f=\tilde{H}_1(-1),
\end{aligned}
\tag{2}
$$

where the number in parentheses indicates hypercharge. Our notation is such that we always use column spinors in isospin space though $\overline{F_L^C}$ is a row spinor in Dirac space and should appear on the left of F_L to form an appropriate Dirac bilinear. The lepton generation index is generally inessential and can be easily recovered when necessary. We note the following features that are useful to exhaust all possibilities. First, since the pair ab appears once, there are two more factors of c or e than d or f to balance hypercharge. The dimension of mass operators is thus $2n+5$, where n denotes the number of copies of d or f. Second, the occurrence of c may be replaced by e if this yields a different and nonvanishing result, and similarly with d and f. Finally, the SM case is recovered by the identifications $e=c$ and $d=f$.

With an even number of fields with nonzero isospin one may imagine to form higher isospin products before building a singlet out of them. But this is unnecessary when all the fields are in the fundamental representation (spinor for short) of $SU(2)$: all isospin invariants of a given mass dimension can be exhausted by first forming singlets from any two spinors and then multiplying them. This is the group-theoretical reason that the three types of seesaws reduce to the unique dimension-five Weinberg operator $\mathscr{O}$ in SM [2] and that its higher-dimensional generalizations are also unique at each dimension [8].

The above point can be best seen in the tensor method in terms of Young tableau. For $SU(2)$ a Young tableau has at most two rows, and each column with two rows is a separate invariant. This is especially convenient when only spinors appear, because in that case each box represents an individual field and a two-row column is an antisymmetric, invariant product of the two spinors involved. This has a few immediate consequences. First, there can be no bare mass term from $\overline{F_L^C}F_L$ even if F_L had a zero hypercharge. Second, denoting a spinor by its index in the box, we have the basic relation:

$$
\begin{array}{|c|c|}\hline i & m \\ \hline j & n \\ \hline\end{array}
-
\begin{array}{|c|c|}\hline i & m \\ \hline n & j \\ \hline\end{array}
=
\begin{array}{|c|c|}\hline i & j \\ \hline m & n \\ \hline\end{array}
\tag{3}
$$

which is equivalent to the relation $(i,j,m,n=1,2)$

$$
\epsilon_{ij}\epsilon_{mn}-\epsilon_{in}\epsilon_{mj}=\epsilon_{im}\epsilon_{jn}.
\tag{4}
$$

Applied to the dimension-five Weinberg operators in Eq. (1), we have

$$
\mathscr{O}^{\alpha\beta}_{xy}-\mathscr{O}^{\alpha\beta}_{yx}=\mathscr{P}^{\alpha\beta}_{xy},
\tag{5}
$$

which means that only one group of dimension-five operators (type $\mathscr{O}$) listed in Ref. [1] are actually independent. (Be careful not to mix the generation indices with the spinor indices.) More generally, putting spinors directly in boxes we have

$$
\begin{array}{|c|c|c|}\hline a & \kappa & \\ \cline{1-2} b & \sigma & \mathscr{Y} \\ \hline\end{array}
=
\begin{array}{|c|c|c|}\hline a & b & \\ \cline{1-2} \kappa & \sigma & \mathscr{Y} \\ \hline\end{array}
-
\begin{array}{|c|c|c|}\hline a & b & \\ \cline{1-2} \sigma & \kappa & \mathscr{Y} \\ \hline\end{array}
\tag{6}
$$

where $\mathscr{Y}$ is any Young tableau. Namely, the $\mathscr{P}$-type operators that contain as a factor an invariant formed out of a, b are linear compositions of the $\mathscr{O}$-type operators. By making a complete list of all mass operators (of type-$\mathscr{O}$), all non-mass operators (of type-$\mathscr{P}$) with a similar structure are automatically covered. In the language of Young tableau, we will never put a, b in the same column.

It is easy to figure out all dimension-five operators since d, f cannot appear while c/e appears twice. They are

$$
\begin{array}{|c|c|}\hline a & b \\ \hline c & c \\ \hline\end{array}
\qquad
\begin{array}{|c|c|}\hline a & b \\ \hline c & e \\ \hline\end{array}
\tag{7}
$$

plus those obtained by $c\leftrightarrow e$, or

$$
\begin{aligned}
&S_5=(a,c)_0(b,c)_0, \qquad T_5=(a,c)_0(b,e)_0,\\
&\bar{S}_5=S_5|_{c\leftrightarrow e}, \qquad \bar{T}_5=T_5|_{c\leftrightarrow e},
\end{aligned}
\tag{8}
$$

where the subscript 0 denotes an isospin invariant formed by antisymmetrizing the fields inside the parentheses; for instance, denoting the upper (lower) component of a spinor by a subscript plus (minus) sign, we have $\sqrt{2}(a,c)_0=a_+c_--a_-c_+$. Since a and b are essentially the same field, the list of operators may be further reduced. To see this clearly, we reserve the lepton generation index by putting $a=\overline{F^C_{Lx}}$, $b=F_{Ly}$. Then,

$$
\begin{aligned}
2\bar{T}_5^{xy}&=\big(\overline{\nu^C_{Lx}}e_--\overline{f^C_{Lx}}e_+\big)(\nu_{Ly}c_--f_{Ly}c_+)\\
&=\big(\overline{\nu^C_{Ly}}c_--\overline{f^C_{Ly}}c_+\big)(\nu_{Lx}e_--f_{Lx}e_+)=2T_5^{yx},
\end{aligned}
\tag{9}
$$

where $\overline{\psi^C_i}\psi_j=\overline{\psi^C_j}\psi_i$ is used. We can thus choose S_5, $\bar{S}_5$, T_5 as the complete and independent list of dimension-five operators.

At dimension seven, the operators contain three copies of c or e and one copy of d or f, and can be classified as S: c^3d, T: c^2ed, plus those obtained by $c\leftrightarrow e$, or $d\leftrightarrow f$, or both interchanges. The first one is easy to write down:

$$
S_7=(a,c)_0(b,c)_0(d,c)_0.
\tag{10}
$$

For the second one, there are following possibilities to distribute the spinors in the boxes of a 2×3 Young tableau:

$$
\underset{\text{1st}}{\begin{array}{|c|c|c|}\hline a & b & d \\ \hline c & c & e \\ \hline\end{array}}
\quad
\underset{\text{2nd}}{\begin{array}{|c|c|c|}\hline a & b & d \\ \hline c & e & c \\ \hline\end{array}}
\quad
\underset{\text{3rd}}{\begin{array}{|c|c|c|}\hline a & b & e \\ \hline c & d & c \\ \hline\end{array}}
\quad
\underset{\text{4th}}{\begin{array}{|c|c|c|}\hline a & b & d \\ \hline e & c & c \\ \hline\end{array}}
\quad
\underset{\text{5th}}{\begin{array}{|c|c|c|}\hline a & b & e \\ \hline d & c & c \\ \hline\end{array}}
\tag{11}
$$

But the basic relation in Eq. (3) implies

$$
\text{1st}-\text{2nd}+\text{3rd}=0, \qquad \text{1st}-\text{4th}+\text{5th}=0,
\tag{12}
$$

which eliminate two operators. We choose the 1st, 3rd, and 5th ones to be independent:

$$
\begin{aligned}
&T_7^1=(a,c)_0(b,c)_0(d,e)_0, \qquad T_7^2=(a,c)_0(b,d)_0(e,c)_0,\\
&T_7^3=(a,d)_0(b,c)_0(e,c)_0.
\end{aligned}
\tag{13}
$$

But for the same reason as for $\bar{T}_5$, T_7^3 is covered by T_7^2 when the lepton generation indices are reserved, and may thus be excluded as redundant. The remaining operators are obtained by interchanges: