Physics Letters B 698 (2011) 297–305

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Threshold effects on renormalization group running of neutrino parameters in the low-scale seesaw model

Johannes Bergström, Tommy Ohlsson, He Zhang *

Department of Theoretical Physics, School of Engineering Sciences, Royal Institute of Technology (KTH) – AlbaNova University Center, Roslagstullsbacken 21, 106 91 Stockholm, Sweden

ARTICLE INFO

Article history:
Received 22 September 2010
Received in revised form 18 February 2011
Accepted 15 March 2011
Available online 21 March 2011
Editor: T. Yanagida

Keywords:
Renormalization group equations
Threshold effects
TeV seesaw models

ABSTRACT

We show that, in the low-scale type-I seesaw model, renormalization group running of neutrino parameters may lead to significant modifications of the leptonic mixing angles in view of so-called seesaw threshold effects. Especially, we derive analytical formulas for radiative corrections to neutrino parameters in crossing the different seesaw thresholds, and show that there may exist enhancement factors efficiently boosting the renormalization group running of the leptonic mixing angles. We find that, as a result of the seesaw threshold corrections to the leptonic mixing angles, various flavor symmetric mixing patterns (e.g., bi-maximal and tri-bimaximal mixing patterns) can be easily accommodated at relatively low energy scales, which is well within the reach of running and forthcoming experiments (e.g., the LHC).

1. Introduction

Experiments on neutrino oscillations have opened up a new window in searching for new physics beyond the Standard Model (SM) during the past decade. Since neutrinos are massless particles within the renormalizable SM, one often extends the SM particle content in order to accommodate massive neutrinos. Among various theories giving rise to neutrino masses, the seesaw mechanism attracts a lot of attention in virtue of its natural and simple description of tiny neutrino masses. In the conventional type-I seesaw model [1–4], three right-handed neutrinos are introduced with super-heavy Majorana masses far away from the electroweak scale $\Lambda_{\rm EW} = \mathcal{O}(100)$ GeV. Small neutrino masses are strongly suppressed by the ratio between the electroweak scale and the large mass of the right-handed neutrinos, which on the other hand, leaves the theory lacking in experimental testability. However, there are alternatives that allow us to realize the seesaw mechanism at an experimentally accessible level, e.g., the TeV scale. One popular way to lower the seesaw scale is to introduce additional singlet fermions, which have the same masses as the right-handed neutrinos but different CP parity [5]. They can be combined with the right-handed neutrinos to form four-component Dirac fields, while the lepton number is broken by a small Majorana mass insertion. In such a scenario, the masses of the light neutrinos are strongly suppressed by the small Majorana mass insertion instead of the seesaw scale, while the non-unitarity effects in the lepton flavor mixing can also be boosted to an observable level [6]. Other possibilities like structural cancellation [7–10] or the minimal flavor seesaw mechanism [11,12] may be employed to construct low-scale seesaw models.

In principle, the current neutrino parameters are observed via low-energy neutrino oscillation experiments. On the other hand, the seesaw-induced neutrino mass operator is usually given at the seesaw scale. Therefore, neutrino parameters are subject to radiative corrections, i.e., they are modified by renormalization group (RG) running effects. Typically, at energy scales lower than the seesaw threshold (i.e., the mass scale of the corresponding seesaw particle), the RG running behavior of neutrino masses and leptonic mixing parameters should be described in an effective theory, which is essentially the same for different seesaw models. However, at energy scales higher than the seesaw threshold, full seesaw theories have to be considered, and the interplay between the heavy and light sectors could make the RG running effects particularly different compared to those in the effective theory. However, in the spirit of some grand unified theories (GUTs), a unified description of fermion masses and flavor mixing depends on the lepton flavor structure at the GUT scale, which inevitably requires the RG running between seesaw particle thresholds and above. The full sets of renormalization group equations (RGEs) in the type-I [1–4], type-II [13–18] and type-III [19] seesaw models have been derived, both in the SM, and in the Minimal Supersymmetric Standard Model (MSSM) [20–26]. The general feature of the running parameters have also been intensively studied in the literature, and

* Corresponding author.
E-mail addresses: johbergs@kth.se (J. Bergström), tommy@theophys.kth.se (T. Ohlsson), zhanghe@kth.se (H. Zhang).

0370-2693/$ – see front matter
doi:10.1016/j.physletb.2011.03.028

it has been shown that there could be sizable radiative corrections to the leptonic mixing parameters at superhigh energy scales (see, e.g., Refs. [27,28] and references therein). In particular, certain flavor symmetric mixing patterns can be achieved at the GUT scale indicating that there might exist some flavor symmetries similar to the gauge symmetry (see, e.g., Ref. [29] and references therein).

Since the RG evolution together with the threshold effects of the heavy seesaw particles may result in visible corrections to the leptonic mixing parameters, there is a need to look into the RG running effects on the leptonic mixing parameters in the TeV seesaw model. In this work, we will explore in detail the RG evolution of neutrino masses and leptonic mixing parameters in the TeV type-I seesaw model. In particular, we will show that the threshold effects play a crucial role in the RG running of neutrino parameters, and some phenomenologically interesting flavor symmetric mixing patterns may be feasible even at an observable energy scale.

The remaining part of this work is organized as follows. In Section 2, we first present general RGEs of the neutrino parameters in the type-I seesaw model; in particular, the matching conditions in crossing the seesaw thresholds. Then, the threshold corrections to the neutrino parameters are discussed in detail in Section 3. Based on the analytical results, we illustrate a phenomenologically interesting numerical example in Section 4, in which the bi-maximal leptonic mixing pattern at TeV scale is shown to be compatible with experimental data when the seesaw threshold corrections are properly taken into account. Finally, a brief summary and our conclusions are given in Section 5. In addition, there are two appendices including the detailed analytical treatment of the threshold effects.

2. RGEs in the type-I seesaw model

The simplest type-I seesaw model is constructed by extending the SM particle content with three right-handed neutrinos $\nu_R = (\nu_{R1}, \nu_{R2}, \nu_{R3})$ together with a Majorana mass term of right-handed neutrinos. The mass part of the lepton sector Lagrangian reads

$$-\mathcal{L} = \overline{\ell_L}\phi Y_e e_R + \overline{\ell_L}\tilde{\phi} Y_\nu \nu_R + \frac{1}{2}\overline{\nu^c_R} M_R \nu_R + \text{h.c.}, \tag{1}$$

where ϕ is the SM Higgs boson, while ℓ_L and e_R denote lepton doublets and right-handed charged leptons, respectively. Here Y_e and Y_ν are the corresponding Yukawa coupling matrices with M_R being the Majorana mass matrix of the right-handed neutrinos. Without loss of generality, one can always perform a basis transformation and work in the basis in which M_R is diagonal, i.e., $M_R = \text{diag}(M_1, M_2, M_3)$. At energy scales below the right-handed neutrino masses, the heavy right-handed neutrinos should be integrated out from the theory and the masses of the light neutrinos are effectively given by a non-renormalizable dimension-five operator

$$-\mathcal{L}^{d=5}_\nu = \frac{1}{2}(\overline{\ell_L}\phi)\cdot\kappa\cdot(\phi^T \ell^c_L) + \text{h.c.}, \tag{2}$$

where the effective coupling matrix κ can be obtained from the type-I seesaw formula

$$\kappa = Y_\nu M_R^{-1} Y_\nu^T, \tag{3}$$

and the Majorana mass matrix of the left-handed neutrinos is

$$m_\nu \equiv \kappa v^2, \tag{4}$$

with $v \simeq 174$ GeV being the vacuum expectation value of the Higgs field.

Because of the seesaw thresholds, the RG running of neutrino parameters needs to be treated separately. At energies above the seesaw thresholds, the full theory should be considered and the relevant beta-functions are given by [20–23]

$$16\pi^2\mu\frac{\text{d}Y_e}{\text{d}\mu} = \left(\alpha_e + C^e_e H_e + C^\nu_e H_\nu\right)Y_e, \tag{5}$$

$$16\pi^2\mu\frac{\text{d}Y_\nu}{\text{d}\mu} = \left(\alpha_\nu + C^e_\nu H_e + C^\nu_\nu H_\nu\right)Y_\nu, \tag{6}$$

$$16\pi^2\mu\frac{\text{d}M_R}{\text{d}\mu} = C_R M_R\left(Y_\nu^\dagger Y_\nu\right) + C_R\left(Y_\nu^\dagger Y_\nu\right)^T M_R, \tag{7}$$

where $H_f = Y_f Y_f^\dagger$ for $f = e, \nu, u, d$ and the coefficients $(C^e_e, C^\nu_e, C^e_\nu, C^\nu_\nu, C_R) = (3/2, -3/2, -3/2, 3/2, 1)$ in the SM. The coefficient α_ν is flavor blind and reads

$$\alpha_\nu = \text{tr}(3H_u + 3H_d + H_e + H_\nu) - \left(\frac{9}{20}g_1^2 + \frac{9}{4}g_2^2\right). \tag{8}$$

If we make use of m_ν at energy scales above the seesaw threshold, we can derive from Eqs. (3) and (5)–(7)

$$\frac{\text{d}m_\nu}{\text{d}t} \equiv \dot{m}_\nu = 2\alpha_\nu m_\nu + \left(C^e_\nu H_e + C_m H_\nu\right)m_\nu + m_\nu\left(C^e_\nu H_e + C_m H_\nu\right)^T, \tag{9}$$

with $C_m = 1/2$. Here, for simplicity, we have defined $t = \ln\mu/(16\pi^2)$.

At energies below the seesaw thresholds, i.e., in the effective theory, neutrino masses are attached to the dimension-five operator, and the RGE of κ reads

$$\dot{\kappa} = \alpha_\kappa\kappa + \left(C^e_\nu H_e\right)\kappa + \kappa\left(C^e_\nu H_e\right)^T, \tag{10}$$

where

$$\alpha_\kappa = 2\,\text{tr}(3H_u + 3H_d + H_e) + \lambda - 3g_2^2, \tag{11}$$

with λ denoting the SM Higgs self-coupling constant.

In crossing the seesaw thresholds, one should ensure that the full and effective theories give identical predictions for physical quantities at low energy scales, and therefore, the physical parameters of both theories have to be related to each other. In the case of the neutrino mass matrix, this means relations between the effective coupling matrix κ and the parameters Y_ν and M_R of the full theory. This is technically called *matching* between the full and effective theories. For the simplest case, if the mass spectrum of the heavy singlets is degenerate, namely $M_1 = M_2 = M_3 = M_0$, one can simply make use of the tree-level matching condition at the scale $\mu = M_0$

$$\kappa|_{M_0} = Y_\nu M_R^{-1} Y_\nu^T\big|_{M_0}. \tag{12}$$

In the most general case with non-degenerate heavy singlets, i.e., $M_1 < M_2 < M_3$, the situation becomes more complicated and the heavy singlets have to be sequentially decoupled from the theory [30]. For instance, at energy scales between the n-th and $(n-1)$-th thresholds, the heavy singlets are partially integrated out, leaving only a $3\times(n-1)$ sub-matrix in Y_ν, which is non-vanishing in the basis, where the heavy singlet mass matrix is diagonal. The decoupling of the n-th heavy singlet leads to the appearance of an effective dimension-five operator similar to that in Eq. (2), and the effective neutrino mass matrix below M_n is described by two parts

$$m_\nu^{(n)} = v^2\left[\kappa^{(n)} + Y_\nu^{(n)}\left(M_R^{(n)}\right)^{-1} Y_\nu^{(n)T}\right], \tag{13}$$