



Infrared chiral anomaly at finite temperature

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ABSTRACT

We study the Schwinger model at finite temperature and show that a temperature dependent chiral anomaly may arise from the long distance behavior of the electric field. At high temperature this anomaly depends linearly on the temperature T and is present not only in the two point function, but also in all even point amplitudes.

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Quantum field theories are well known to have ultraviolet divergences which arise from the short distance behavior of the theory. These ultraviolet divergences need to be regularized (so that the theory makes sense) and one normally tries to choose a regularization scheme which would preserve the underlying symmetries of the quantum field theory. This requirement of choosing a regularization preserving the symmetry becomes particularly crucial when one is dealing with a gauge theory (with a local symmetry) in order to avoid possible inconsistencies in the theory. However, when there are several distinct symmetries present in a theory, it may not always be possible to find a regularization which respects all the symmetries of the theory in which case one tries to choose a regularization which respects the gauge symmetries of the theory. In this case, the regularization may not respect some of the global symmetries of the theory and may lead to nontrivial quantum corrections to the conservation laws for the currents associated with them. A deviation from the classical conservation law is normally called an anomaly and the chiral anomaly [1–5] is one of the well-studied examples of such phenomena. It is also well known that while ultraviolet divergences are present in a quantum field theory at zero temperature, temperature dependent parts of amplitudes are ultraviolet finite and do not require any additional

counterterm at finite temperature. Indeed since we do not need to regularize the temperature dependent parts of the amplitudes, it follows that there will not be any temperature dependent correction to anomalies and this has been explicitly checked in the case of the chiral anomaly (in various theories) [6–9].

Although there is no ultraviolet divergence in a quantum field theory at finite temperature, the infrared behavior is more pronounced and thermal infrared divergences are known to be severe. Therefore, it is natural to ask if the infrared behavior of a quantum field theory at finite temperature can lead to contributions for the chiral anomaly. We note that if this were to happen, it would clearly depend on the large distance behavior of a quantum field theory. In fact, since quite often one studies quantum fields in a constant background field (constant electric/magnetic field), such an investigation is quite meaningful. In this Letter we undertake a systematic investigation of this question for general electromagnetic background fields (coupled to massless fermions in $1+1$ dimensions) which do not vanish asymptotically and determine which backgrounds can contribute to the chiral anomaly at finite temperature. In $(1+1)$ -dimensional massless QED (the Schwinger model [10]) the chiral anomaly at zero temperature is normally associated with the two point function which is the only diagram in the theory with an ultraviolet divergence. In contrast, when the chiral anomaly has its origin in the infrared behavior of the theory, we show that higher point amplitudes can also become anomalous and we determine the complete temperature dependent anomaly functional (in the leading order at high temperature)

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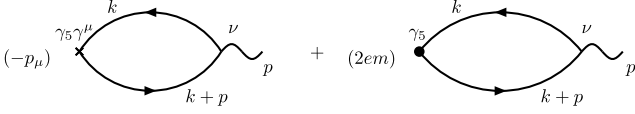


Fig. 1. The two diagrams contributing to (2) at the two point level.

for the Schwinger model. We work in the real time formalism of finite temperature known as the closed time path formalism due to Schwinger [9,11] and restrict our discussion, for simplicity, to the “+” branch of the contour (see [9] for details).

The Schwinger model [10] is described by (we are neglecting the Maxwell term for the photon) the (1 + 1)-dimensional Lagrangian density

$$\mathcal{L} = \bar{\psi} \gamma^\mu (i\partial_\mu - eA_\mu) \psi, \quad (1)$$

where $\mu = 0, 1$ and the Dirac matrices are identified with the Pauli matrices as $\gamma^0 = \sigma_1$, $\gamma^1 = -i\sigma_2$, $\gamma_5 = \gamma^0\gamma^1 = \sigma_3$. The fermions are massless in this theory and let us first show that if they had a mass, there would not be any temperature dependent contribution to the chiral anomaly even when the background fields do not vanish asymptotically. Therefore, the phenomenon that we are describing has a genuine infrared origin.

To see that the massive theory does not generate any temperature dependent contribution to the anomaly [12] (even when the background field does not vanish asymptotically), let us note that the chiral current in the massive Schwinger model satisfies the tree level identity

$$\partial_\mu J_5^\mu(x) + 2iem J_5(x) = 0, \quad (2)$$

where $J_5^\mu = e\bar{\psi}\gamma_5\gamma^\mu\psi$, $J_5 = \bar{\psi}\gamma_5\psi$ and m denotes the mass of the fermions. At one loop the contribution to this relation at the two point level comes from the two diagrams in Fig. 1.

In the closed time path formalism [9,11], the temperature dependent contribution from these diagrams (say, on the ++ branch) is proportional to

$$\begin{aligned} \epsilon^{\mu\nu} A_\nu(p) \int d^2k [(k^2 - m^2)(k+p)_\mu - ((k+p)^2 - m^2)k_\mu] \\ \times \left(\frac{i}{(k+p)^2 - m^2 + i\epsilon} - \pi n_F(|k^0 + p^0|) \delta((k+p)^2 - m^2) \right) \\ \times n_F(|k^0|) \delta(k^2 - m^2), \end{aligned} \quad (3)$$

where $\epsilon^{\mu\nu}$ denotes the two-dimensional Levi-Civita tensor and we use the convention $\epsilon^{01} = 1$. The single delta function terms in (3) vanish because of delta function constraints as well as antisymmetry. The nontrivial contribution, if present, can arise only from the two delta function terms in (3). The contributions from the two delta function terms would clearly vanish if the two delta function constraints truly restrict k_0 and k_1 because of the structure of the terms in the square brackets in (3). This is, in fact, true in the massive theory and there is no temperature dependent contribution to the anomaly. However, to appreciate why the contribution may not vanish in the massless theory (Schwinger model), let us analyze the constraints following from the product of two delta functions.

For example, in the massive theory we note that the two delta function constraints can be satisfied only if $p^2 \neq 0$ which follows from the kinematics of the diagrams since the loop involves propagation of massive particles. In fact, the diagrams contribute only if $1 - \frac{4m^2}{p^2} \geq 0$ which is the origin of the two branch cuts $p^2 \geq 4m^2$ and $p^2 \leq 0$ in the two point function at finite temperature [9,13]. In such a case, the product of the two delta functions can be factorized as

$$\begin{aligned} \delta(k^2 - m^2) \delta((k+p)^2 - m^2) \\ = \frac{\theta(\omega_+(p_0))}{2\omega_+(p_0)} \delta(k_0 - \omega_+(p_0)) \left[\frac{1}{|J_{+,+}(p_0)|} \delta(k_1 - X_+(p_0)) \right. \\ \left. + \frac{1}{|J_{+,-}(p_0)|} \delta(k_1 + X_+(p_0)) \right] \\ + \frac{\theta(\omega_-(p_0))}{2\omega_-(p_0)} \delta(k_0 - \omega_-(p_0)) \left[\frac{1}{|J_{-,+}(p_0)|} \delta(k_1 - X_-(p_0)) \right. \\ \left. + \frac{1}{|J_{-,-}(p_0)|} \delta(k_1 + X_-(p_0)) \right] \\ + (k_0, p_0 \rightarrow -k_0, -p_0), \end{aligned} \quad (4)$$

where we have identified

$$\begin{aligned} \omega_\pm(p_0) &= \frac{1}{2} \left(-p_0 \pm |p_1| \sqrt{1 - \frac{4m^2}{p^2}} \right), \\ X_\pm(p_0) &= \sqrt{\omega_\pm^2(p_0) - m^2}, \\ J_{+,\pm}(p_0) &= \frac{\mp X_+(p_0)p^2 - 2m^2 p_1}{\omega_+^2(p_0)}, \\ J_{-,\pm}(p_0) &= \frac{\mp X_-(p_0)p^2 - 2m^2 p_1}{\omega_-^2(p_0)}. \end{aligned} \quad (5)$$

The product of the two delta functions clearly constrains the integration variables k_0, k_1 and in such a case the terms in the square bracket in (3) make the integral vanish independent of the behavior of the background field $A_\nu(p)$.

In a massless theory ($m = 0$), on the other hand, the momenta naturally separate into light-cone components and the product of two delta functions in this case factorize as

$$\begin{aligned} \delta(k^2) \delta((k+p)^2) &= \frac{1}{|k_-||p_+|} \delta(k_+) \delta(k_- + p_-) \\ &+ \frac{1}{|k_+||p_-|} \delta(k_-) \delta(k_+ + p_+) \\ &+ \frac{1}{|k_-||k_- + p_-|} \delta(k_+) \delta(p_+) \\ &+ \frac{1}{|k_+||k_+ + p_+|} \delta(k_-) \delta(p_-). \end{aligned} \quad (6)$$

Comparing this with (4), we note that the first two terms in (6) constrain the variables of integration completely and lead to vanishing contribution as in the massive theory. However, the last two terms in (6) constrain only partially the variables of integration and as a result the second term in the square bracket leads to a nontrivial contribution for (3) of the form

$$(p_+ \delta(p_+) A_-(p) - p_- \delta(p_-) A_+(p)) I_2, \quad (7)$$

where as discussed in [14–16] ($\epsilon(x)$ denotes the alternating step function)

$$\begin{aligned} I_2 &= \frac{(2ie\pi)^2}{2} \int \frac{dk_1}{(2\pi)^2} \epsilon(k_1) \epsilon(k_1 + p_1) (n_F(|k_1|) + n_F(|k_1 + p_1|)) \\ &- 2n_F(|k_1|) n_F(|k_1 + p_1|). \end{aligned} \quad (8)$$

Normally, for background fields vanishing asymptotically, (7) vanishes leading to the conclusion that there is no temperature dependent contribution to the anomaly. However, we note that if $A_+(p) \sim \frac{1}{p_-}$ or $A_-(p) \sim \frac{1}{p_+}$ asymptotically (for, example), then (7)

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