Available online at www.sciencedirect.com

SCIENCE DIRECT®

Physics Letters B 633 (2006) 54–60

PHYSICS LETTERS B

www.elsevier.com/locate/physletb

Mixed QCD and weak corrections to top quark pair production at hadron colliders

W. Bernreuther [a,*], M. Fücker [a], Z.G. Si [b]

[a] *Institut für Theoretische Physik, RWTH Aachen, 52056 Aachen, Germany*
[b] *Department of Physics, Shandong University, Jinan, Shandong 250100, China*

Received 15 August 2005; received in revised form 14 November 2005; accepted 14 November 2005

Available online 29 November 2005

Editor: N. Glover

Abstract

The order $\alpha_s^2\alpha$ mixed QCD and weak corrections to top quark pair production by quark–antiquark annihilation are computed, keeping the full dependence on the t and $\bar{t}$ spins. We determine the contributions to the cross section and to single and double top spin asymmetries at the parton level. These results are necessary ingredients for precise standard model predictions of top quark observables, in particular of top-spin induced parity-violating angular correlations and asymmetries at hadron colliders.

PACS: 12.15.Lk; 12.38.Bx; 13.88.+e; 14.65.Ha

Keywords: Hadron collider physics; Top quarks; QCD and electroweak corrections; Parity violation; Spin effects

One promising tool for investigating the so-far relatively unexplored dynamics of top quark production and decay, once high statistics samples of t and/or $\bar{t}$ quarks are available, are observables associated with the spins of these quarks. As far as QCD-induced $t\bar{t}$ production and decay at hadron colliders is concerned, theoretical predictions for differential distributions including the full dependence on the t, $\bar{t}$ spins are available at NLO in the QCD coupling [1,2].

For full exploration of sizable, respectively large $t\bar{t}$ data samples that are expected at the Tevatron and at the LHC the standard model (SM) predictions should be as precise as possible. Specifically weak interaction contributions to $t\bar{t}$ production should be taken into account. Although they are nominally subdominant with respect to the QCD contributions they can become important at large $t\bar{t}$ invariant mass due to large Sudakov logarithms (for reviews and references, see, e.g., [3,4]).

SM weak interaction effects in hadronic production of heavy quark pairs were considered previously. The parity-even and parity-odd order $\alpha_s^2\alpha$ vertex corrections[1] were determined in [5] and in [7], respectively (see also [6]). In Ref. [7] also parity-violating non-SM effects were analysed. The box contributions to $q\bar{q} \to t\bar{t}$ and, apparently, the quark triangle diagrams $gg \to Z^* \to t\bar{t}$ were not taken into account in these papers. In [8] the weak contributions to the hadronic $b\bar{b}$ cross section, including these box contributions, were computed.

In this Letter we report on the calculation of the mixed QCD and weak radiative corrections of order $\alpha_s^2\alpha$ to the (differential) cross section of $t\bar{t}$ production by quark–antiquark annihilation, keeping the full information on the spin state of the $t\bar{t}$ system. These results are necessary ingredients for definite SM predictions, in particular of parity-violating observables associated with the spin of the (anti)top quark.

---

[*] Corresponding author.
E-mail addresses: breuther@physik.rwth-aachen.de (W. Bernreuther), fuecker@physik.rwth-aachen.de (M. Fücker), zgsi@sdu.edu.cn (Z.G. Si).

[1] Here α_s and α denote the strong and electromagnetic couplings, and the weak coupling is $\alpha_W = \alpha/\sin^2\theta_W$.

0370-2693/$ – see front matter © 2005 Elsevier B.V. All rights reserved.
doi:10.1016/j.physletb.2005.11.056

In the following we first give some details of our calculation. Then we present numerical results for the cross section and for several single spin and spin–spin correlation observables.

Top quark pair production both at the Tevatron and at the LHC is dominated by the QCD contributions to $q\bar{q} \to t\bar{t}$ and $gg \to t\bar{t}$, which are known to order α_s^3. Due to color conservation there are no $\alpha_s\alpha$ Born level contributions to these processes. The leading contributions involving electroweak interactions are the order α^2 Born terms for $q\bar{q} \to t\bar{t}$ and the mixed contributions of order $\alpha_s^2\alpha$. For the quark–antiquark annihilation processes, which we analyze in the following, this amounts to studying the reactions

$$q(p_1) + \bar{q}(p_2) \to t(k_1, s_t) + \bar{t}(k_2, s_{\bar{t}}), \tag{1}$$

$$q(p_1) + \bar{q}(p_2) \to t(k_1, s_t) + \bar{t}(k_2, s_{\bar{t}}) + g(k_3). \tag{2}$$

Here p_1, p_2, k_1, k_2, and k_3 denote the parton momenta. The vectors s_t, $s_{\bar{t}}$, with $s_t^2 = s_{\bar{t}}^2 = -1$ and $k_1 \cdot s_t = k_2 \cdot s_{\bar{t}} = 0$ describe the spin of the top and antitop quarks. All quarks but the top quark are taken to be massless.

The respective contributions to the differential cross section of (1) are of the form

$$\alpha^2 \left|\mathcal{M}_2(p, k, s_t, s_{\bar{t}})\right|^2 + \alpha_s^2 \alpha\, \delta\mathcal{M}_2(p, k, s_t, s_{\bar{t}}), \tag{3}$$

where $\mathcal{M}_2$ corresponds to the γ and Z exchange diagrams. As we are interested in this Letter in particular in parity-violating effects, we take into account only the mixed QCD and weak contributions to $\delta\mathcal{M}_2$ and to (2) in the following. The photonic contributions form a gauge invariant set and can be straightforwardly obtained separately. The contributions to $\delta\mathcal{M}_2$ are the order α_s^2 two-gluon box diagrams interfering with the Born Z-exchange diagram, and the Z gluon (g) box diagrams and the diagrams with the weak corrections to the $q\bar{q}g$ and $gt\bar{t}$ vertices interfering with the Born gluon exchange diagram. The ultraviolet divergences in the vertex corrections are removed using the on-shell scheme for defining the wave function renormalizations of the quarks and the top quark mass m_t.

The respective contributions to the differential cross section of (2) are of the form $\alpha_s^2 \alpha\, \delta\mathcal{M}_3(p, k, s_t, s_{\bar{t}})$ and result from the interference of the order g_s^3 with the order $g_s e^2$ gluon bremsstrahlung diagrams.

The box diagram contributions to (3) contain infrared divergences due to virtual soft gluons. They are canceled against terms from soft gluon bremsstrahlung. As a consequence of color conservation both the sum of the box diagram contributions to $\delta\mathcal{M}_2$ and $\delta\mathcal{M}_3$ are free of collinear divergences.

We have extracted the IR divergences, using dimensional regularization, with two different methods: a phase space slicing procedure (as in [2]) and, alternatively as a check, we have constructed subtraction terms that render the three particle phase space integral over the subtracted term $[\delta\mathcal{M}_3]_{\text{subtr}}$ finite. When calculating observables, in particular those given below, both methods led to results which numerically agree to high precision.

We have determined (3) and $\delta\mathcal{M}_3$, respectively their infrared-finite counterparts, analytically for arbitrary t and $\bar{t}$ spin states. From these expressions one may extract the respective production spin density matrices. These matrices, combined with the decay density matrices describing semi- and non-leptonic t and $\bar{t}$ decay [9] then yield, in the $t\bar{t}$ leading pole or narrow width approximation, standard model predictions for distributions of the reactions $q\bar{q} \to t\bar{t} \to b\bar{b} + 4f(+g)$ ($f = q, \ell, \nu_\ell$) with the t and $\bar{t}$ spin degrees of freedom fully taken into account.

The expressions for (3) and $\delta\mathcal{M}_3$ are rather lengthy when the full dependence on the t and $\bar{t}$ spins is kept, and we do not reproduce them here. We represent these contributions to the partonic cross sections and to several single and double spin asymmetries, which we believe are of interest to phenomenology, in terms of dimensionless scaling functions depending on the kinematic variable $\eta = \frac{\hat{s}}{4m_t^2} - 1$, where $\hat{s}$ is the $q\bar{q}$ c.m. energy squared. The inclusive, spin-summed $q\bar{q}$ cross sections for (1), (2) may be written, to NLO in the SM couplings, in the form

$$\sigma_{q\bar{q}} = \sigma_{q\bar{q}}^{(0)\text{QCD}} + \delta\sigma_{q\bar{q}}^{\text{QCD}} + \delta\sigma_{q\bar{q}}^{W}, \tag{4}$$

where the first and second term are the LO (order α_s^2) and NLO (order α_s^3) QCD contributions [10–12], and the third term is generated by the electroweak contributions (3) and $\delta\mathcal{M}_3$ described above. We decompose this term as follows:

$$\delta\sigma_{q\bar{q}}^{W}\left(\hat{s}, m_t^2\right) = \frac{4\pi\alpha}{m_t^2}\left[\alpha f_{q\bar{q}}^{(0)}(\eta) + \alpha_s^2 f_{q\bar{q}}^{(1)}(\eta)\right]. \tag{5}$$

We have numerically evaluated the scaling functions $f^{(i)}(\eta)$—and those defined below—and parameterized them in terms of fits which allow for a quick use in applications. In the following we use $m_Z = 91.188$ GeV, $\sin^2\theta_W = 0.231$, and $m_t = 178$ GeV. In several figures below also $m_t = 173$ GeV is employed, which corresponds to the recent CDF and D0 combined central value [13]. In Figs. 1–11 below we use $m_H = 114$ GeV for the mass of the standard model Higgs boson. The dependence on the Higgs boson mass is shown in Fig. 3 in the case of $f_{d\bar{d}}^{(1)}$ for two values of m_H [14].

In Fig. 1 the functions $f_{q\bar{q}}^{(i)}$ are displayed as functions of η for annihilation of initial massless partons $q\bar{q}$ of the first and sec-

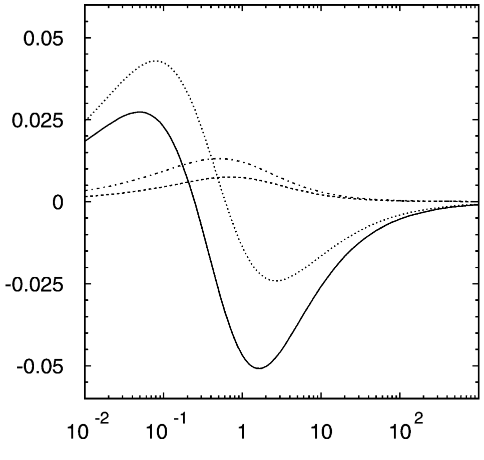

Fig. 1. Dimensionless scaling functions $f_{q\bar{q}}^{(0)}(\eta)$ (dashed), $f_{q\bar{q}}^{(1)}(\eta)$ (solid) that determine the parton cross section (5) for $q = d$ type quarks. The dash-dotted and dotted lines correspond to the respective functions for $q = u$ type quarks. The Higgs boson mass is put to 114 GeV and $m_t = 178$ GeV.