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Hamiltonian analysis of linearized extension of Hořava–Lifshitz gravity

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ABSTRACT

We investigate the Hamiltonian structure of linearized extended Hořava–Lifshitz gravity in a flat cosmological background following the Faddeev–Jackiw's Hamiltonian reduction formalism. The Hamiltonian structure of extended Hořava–Lifshitz gravity is similar to that of the projectable version of original Hořava–Lifshitz gravity, in which there is one primary constraint and so there are *two* physical degrees of freedom. In the infrared (IR) limit, however, there is *one* propagating degree of freedom in the general cosmological background, and that is coupled to the scalar graviton mode. We find that extra scalar graviton mode in an inflationary background can be decoupled from the matter field in the IR limit. But it is necessary to go beyond linear order in order to draw any conclusion of the strong coupling problem.

The reconciliation of gravity and quantum theory, which is important to understand the very early stage of our Universe and the black hole, is a very challenging task in theoretical physics. Among the several proposals of quantum theory of gravity, recently Hořava [1] proposed a UV complete, non-relativistic gravity theory which is power-counting renormalizable giving up the Lorentz invariance. Since then, many paid attention to this scenario to apply to the black hole [2], cosmology [3,4] and observational tests [5]. In spite of its many appealing properties, it seems to suffer from many problems [6,7] such as instability, strong coupling, renormalizability etc.

In order to alleviate the original Hořava–Lifshitz gravity's problem, in [8] the extended version of Hořava–Lifshitz gravity is proposed, in which the new degree of freedom by taking the spatial gradient of a lapse function is introduced without violating the symmetry of the action. They argued that the strong coupling problem of the scalar graviton mode in the IR limit would be solved. There still remains some debate on strong coupling problem in extended version as well as on the physical degrees of freedom [9,10].

In this Letter, following the previous work [11] of one of the authors, we investigate the Hamiltonian structure of linearized extended Hořava–Lifshitz gravity in a cosmological background using the Faddeev–Jackiw approach [12]. First, we derive the quadratic action in the extended Hořava–Lifshitz gravity model and then obtain constraints and Hamiltonian. Through the investigation of the Poisson algebra, the physical degrees of freedom are exactly counted and we analyze the Hamiltonian structures. Next, we obtain the equations of motion of the physical degrees of freedom and finally we briefly comment about the strong coupling issues in our case in the IR limit.

We consider the Arnowitt–Deser–Misner (ADM) metric which is given by

$$ds^2 = \left(-N^2 + N_i N^i\right) dt^2 + 2N_i \, dt \, dx^i + \gamma_{ij} \, dx^i \, dx^j, \tag{1}$$

where N is the lapse function, N_i are shift vectors, and γ_{ij} is the spatial 3 metric. The Hořava–Lifshitz gravity action in the ADM metric with a single scalar field is

$$\mathcal{S} = \int d^4x \, N\sqrt{\gamma}\left[\frac{1}{2\kappa^2}\left(K_{ij}K^{ij} - \lambda K^2\right) - \mathcal{V} + \frac{1}{2N^2}\left(\dot{\phi} - N^i \partial_i \phi\right)^2 - Z(\phi) - V(\phi)\right]. \tag{2}$$

The extrinsic curvature K_{ij} and its trace are written in terms of the ADM metric (1) as

$$K_{ij} = \frac{1}{2N}\left(\partial_i N_j + \partial_j N_i - \frac{\partial \gamma_{ij}}{\partial t}\right), \tag{3}$$

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$$K = \gamma^{ij} K_{ij} = K^i_i. \tag{4}$$

For extended Hořava–Lifshitz gravity [8], the gravitational potential term $\mathcal{V}$ in the action (2) depends on γ_{ij}, its spatial derivative and on 3-dimensional vector a_i constructed from the lapse function $N(t, \mathbf{x})$ as

$$a_i = \frac{\partial_i N(t, \mathbf{x})}{N(t, \mathbf{x})}, \tag{5}$$

which represents the proper acceleration of the vector field of unit normals to the foliation surfaces [6]. Under the anisotropic scaling transformations

$$\mathbf{x} \to l\mathbf{x}, \qquad t \to l^z t, \tag{6}$$

the $z = 3$ theory in the UV is power-counting renormalizable, so the potential in the action (2) can have at most 6-th order spatial derivative terms. With these spirits, the gravitational potential for extended Hořava–Lifshitz gravity can take the form

$$\mathcal{V} = -\xi R - \alpha a_i a^i + f_1 R^2 + f_2 R_{ij} R^{ij} + f_3 R \partial_i a^i + f_4 a_i \partial^2 a^i + g_1 (\partial_i R)^2 + g_2 \partial_i R_{jk} \partial^i R^{jk} + g_3 \partial^2 R \partial_i a^i + g_4 a_i \partial^4 a^i, \tag{7}$$

where ξ, α, f_n, g_n are constants and $\partial^2 = \partial_i \partial^i$. R and R_{ij} are 3-dimensional Ricci scalar and Ricci tensor, respectively. Since we are interested in the linear analysis of extend Hořava–Lifshitz gravity, the only terms in (7) relevant to the linear analysis on a flat cosmological background are included [8,13]. Most general gravitational potential form in extended Hořava–Lifshitz gravity can be found in Ref. [8]. It is known that the action (2) with the gravitational potential (7) is invariant under the foliation conserving transformations

$$\mathbf{x} \to \mathbf{x} + g(t, \mathbf{x}), \quad t \to t + f(t). \tag{8}$$

The $Z(\phi)$ in (2) is the matter part potential constructed from the spatial derivative of a scalar field, which is given by

$$Z(\phi) = \sum_{n=1}^{3} \xi_n \partial_i^{(n)} \phi \partial^{i(n)} \phi. \tag{9}$$

The superscript (n) denotes the n-th spatial derivative.

In order to derive the background and linear perturbation equations of motion by varying the action, we expand the metric and the scalar field to the linear order as

$$N = a(\eta)(1 + \Phi), \qquad N_i = a(\eta)^2 \partial_i \beta, \qquad \gamma_{ij} = a(\eta)^2 (\delta_{ij} + h_{ij}) = a(\eta)^2 \left((1 - 2\mathcal{R})\delta_{ij} + 2\left(\partial_i \partial_j - \frac{1}{3} \delta_{ij} \partial^2 \right) E \right), \tag{10}$$

and

$$\phi(\eta, x) = \phi_0(\eta) + \delta\phi(\eta, x), \tag{11}$$

where the parameter $a(\eta)$ is a scale factor and η is a conformal time. In this Letter, we only consider the scalar mode perturbations.

From the linear order of the action (2)

$$\delta_1 \mathcal{S} = \int d^4x\, a^2 \left[\left\{ -\frac{3(1-3\lambda)}{2\kappa^2} \mathcal{H}^2 - \frac{1}{2} \phi_0'^2 - a^2 V_0 \right\} \Phi + \left\{ -\frac{(1-3\lambda)}{4\kappa^2} \left(\mathcal{H}^2 + 2\mathcal{H}' \right) + \frac{1}{2} \left(\frac{1}{2} \phi_0'^2 - a^2 V_0 \right) \right\} h^k_k \right.$$
$$\left. + \left\{ -\phi_0'' - 2\mathcal{H}\phi_0' - a^2 V_\phi \right\} \delta\phi \right], \tag{12}$$

where $\mathcal{H} = \frac{a'}{a}$ and the prime denotes the derivative with respect to η, we can obtain the background equation of motion in a flat cosmological background

$$\frac{3(1-3\lambda)}{2\kappa^2} \mathcal{H}^2 = -\left(\frac{1}{2} \phi_0'^2 + a^2 V_0 \right), \tag{13}$$

$$\frac{(1-3\lambda)}{2\kappa^2} \left(\mathcal{H}^2 + 2\mathcal{H}' \right) = \frac{1}{2} \phi_0'^2 - a^2 V_0, \tag{14}$$

$$\phi_0'' + 2\mathcal{H}\phi_0' + a^2 V_\phi = 0. \tag{15}$$

By expanding the action up to 2nd order in terms of the perturbed quantities, the quadratic action yields

$$\delta_2 \mathcal{S} = \int d^4x\, a^2 \left[\frac{1}{2\kappa^2} (1-3\lambda) \left\{ 3\mathcal{H}^2 \Phi^2 + 2\mathcal{H}\Phi \partial^2 (\beta - E') + 6\mathcal{H}\Phi\psi' + 3\psi'^2 + 2\psi' \partial^2 (\beta - E') \right\} + \frac{1}{2\kappa^2} (1-\lambda) \left[\partial^2 (\beta - E') \right]^2 \right.$$
$$- \alpha \Phi \partial^2 \Phi - \frac{1}{a^2} \left(\partial^2 \psi \right)^2 (16 f_1 + 6 f_2) - f_3 \frac{4}{a^2} \partial^2 \psi \partial^2 \Phi + \frac{f_4}{a^2} \partial^2 \Phi \partial^2 \Phi + \frac{1}{a^4} (16 g_1 + 6 g_2) \partial^2 \psi \left(\partial^2 \right)^2 \psi - g_3 \frac{4}{a^4} \left(\partial^2 \right)^2 \psi \partial^2 \Phi$$
$$+ g_4 \frac{1}{a^4} \partial^2 \Phi \partial^2 \partial^2 \Phi + 2\xi (2\Phi - \psi) \partial^2 \psi + \frac{1}{2} \delta\phi'^2 - \phi_0' \Phi \delta\phi' + \frac{1}{2} \phi_0'^2 \Phi^2 - a^2 \delta Z - \frac{1}{2} a^2 V_{\phi\phi} (\delta\phi)^2 - a^2 V_\phi \Phi \delta\phi + 3\phi_0' \psi' \delta\phi$$
$$\left. + \phi_0' \delta\phi \partial^2 \left(\beta - E' \right) \right], \tag{16}$$