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Topological insulators and superconductors from D-brane

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ABSTRACT

Realization of topological insulators (TIs) and superconductors (TSCs), such as the quantum spin Hall effect and the $\mathbb{Z}_2$ topological insulator, in terms of D-branes in string theory is proposed. We establish a one-to-one correspondence between the K-theory classification of TIs/TSCs and D-brane charges. The string theory realization of TIs and TSCs comes naturally with gauge interactions, and the Wess–Zumino term of the D-branes gives rise to a gauge field theory of topological nature. This sheds light on TIs and TSCs beyond non-interacting systems, and the underlying topological field theory description thereof.

1. Introduction

A gapped state of quantum condensed matter is called topological phase when it supports stable gapless boundary modes, such as an edge or a surface state. The integer quantum Hall effect (QHE), which exists in $d=2$ spatial dimensions and under a strong magnetic field, is the best known example of such a phase. The recent discovery of the quantum spin Hall effect (QSHE) in $d=2$ and the $\mathbb{Z}_2$ topological insulator in $d=3$ [1–8] shows topological phases can exist even in $d>2$ spatial dimensions, and can be protected by some discrete symmetries such as time-reversal symmetry (TRS, T), particle–hole symmetry (PHS, C), and chiral (or sublattice) symmetry (SLS, S).

For non-interacting fermions, an exhaustive classification of topological insulators (TIs) and superconductors (TSCs) is proposed in Refs. [9,10]: TIs/TSCs are classified in terms of spatial dimensions d and the $10=2+8$ symmetry classes (two "complex" and eight "real" classes) (Table 1). The ten symmetry classes are in one-to-one correspondence to the Riemannian symmetric spaces (without exceptional series) and, as pointed out in [10], they are equivalent to K-theory classifying spaces [11]. For example, the IQHE, QSHE, and $\mathbb{Z}_2$ TI are a topologically non-trivial state belonging to class A ($d=2$), AII ($d=2$), and AII ($d=3$), respectively.

The complete classification of non-interacting TIs and TSCs opens up a number of further questions, most interesting among which are interaction effects: Do non-interacting topological phases continue to exist in the presence of interactions? Can interactions give rise to novel topological phases other than non-interacting TIs/TSCs? What is a topological field theory underlying TIs/TSCs, which can potentially describe TIs/TSCs beyond non-interacting examples, etc.

Table 1
Classification of topological insulators and superconductors [9,10]; d is the space dimension; the left-most column (A, AIII, ..., CI) denotes the ten symmetry classes of fermionic Hamiltonians, which are characterized by the presence/absence of time-reversal (T), particle–hole (C), and chiral (or sublattice) (S) symmetries of different types denoted by ± 1 in the right most three columns. The entries "$\mathbb{Z}$", "$\mathbb{Z}_2$", "$2\mathbb{Z}$", and "0" represent the presence/absence of topological insulators and superconductors, and when they exist, types of these states (see Ref. [9] for detailed descriptions).

class\\d	0	1	2	3	4	5	6	7	T	C	S
A	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	0	0	0
AIII	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	0	1
AI	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	+	0	0
BDI	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	+	+	1
D	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	0	+	0
DIII	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	−	+	1
AII	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	−	0	0
CII	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	−	−	1
C	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	−	0
CI	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	+	−	1

On the other hand, the ten-fold classification of TIs/TSCs reminds us of D-branes, which are fundamental objects in string theory, and are also classified by K-theory [12] (Table 2) via the open string tachyon condensation [13]. It is then natural to speculate a possible connection between TIs/TSCs and of D-branes. In this Letter, we propose a systematic construction of TIs/TSCs in

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Table 2
Dp-brane charges from K-theory, classified by $\mathrm{K}(\mathbb{S}^{9-p})$, $\mathrm{KO}(\mathbb{S}^{9-p})$ and $\mathrm{KSp}(\mathbb{S}^{9-p})$ [12]. A $\mathbb{Z}_2$ charged Dp-brane with p even or p odd represents a non-BPS Dp-brane or a bound state of a Dp and an anti-Dp brane, respectively [13].

	D(−1)	D0	D1	D2	D3	D4	D5	D6	D7	D8	D9
type IIB	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
$\mathrm{O9}^-$ (type I)	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
$\mathrm{O9}^+$	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$

Table 3
External G (left-most column) and internal $\tilde{G}$ gauge groups for each spatial dimension d and symmetry class; U, O, Sp, represents U(1), $\mathrm{O}(1)=\mathbb{Z}_2$, and $\mathrm{Sp}(1)=\mathrm{SU}(2)$, respectively.

G	class\\d	0	1	2	3	4	5	6	7
U	A	U	–	U	–	U	–	U	–
U	AIII	–	U	–	U	–	U	–	U
O	AI	O	–	–	–	Sp	–	U	O
O	BDI	O	O	–	–	–	Sp	–	U
O	D	U	O	O	–	–	–	Sp	–
O	DIII	–	U	O	O	–	–	–	Sp
Sp	AII	Sp	–	U	O	O	–	–	–
Sp	CII	–	Sp	–	U	O	O	–	–
Sp	C	–	–	Sp	–	U	O	O	–
Sp	CI	–	–	–	Sp	–	U	O	O

Table 4
Dp–Dq systems for class A and AIII where $p=5$ and $q=3,5,7$ for A, and $p=4$ and $q=4,6$ for AIII. The D-branes extend in the μ-th direction denoted by "×" in the ten-dimensional space-time ($\mu=0,\dots,9$); $d+1$ is the number of common directions of Dp- and Dq-branes; The last column shows the Dq-brane charge, together with fermion spectra per Dq-brane, where "N_f Mj" or "N_f Di" represents N_f flavor of Majorana and Dirac spinor, respectively.

	0	1	2	3	4	5	6	7	8	9	d	A
D5	×	×	×	×	×	×						
D3	×						×	×	×		0	$\mathbb{Z}$ (2 Mj)
D5	×	×	×				×	×	×		2	$\mathbb{Z}$ (2 Mj)
D7	×	×	×	×	×		×	×	×		4	$\mathbb{Z}$ (1 Di)
	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**d**	**AIII**
D4	×		×	×	×	×						
D4	×		×				×	×	×		1	$\mathbb{Z}$ (2 Mj)
D6	×		×	×	×		×	×	×		3	$\mathbb{Z}$ (2 Mj)

terms of two D-branes (Dp- and Dq-branes), possibly with an orientifold plane (O-plane). Besides the appealing mathematical similarity between TIs/TSCs and D-branes, realizing TIs/TSCs in string theory has a number of merits, since string theory and D-branes are believed to be rich enough to reproduce many types of field theories and interactions in a fully consistent and UV complete way. Indeed, our string theory realizations of TIs/TSCs give rise to massive fermion spectra, which are in one-to-one correspondence with the ten-fold classification of TIs/TSCs, and come quite naturally with gauge interactions. These systems, while interacting, are all topologically stable, as protected by the K-theory charge of D-branes. We thus make a first step toward understanding interacting TIs/TSCs [14]. We are also separately preparing a regular paper with more details and expanded results in [15].

In Dp–Dq systems, massive fermions arise as an open string excitation between the two D-branes. The distance between the branes corresponds to the mass of fermions. Open strings ending on the same D-branes give rise to a gauge field, which we call A_μ (Dp) and $\tilde{A}_\mu$ (Dq) with gauge group G and $\tilde{G}$, respectively, and couple to the fermions. These two gauge fields play different roles in our construction: The gauge field A_μ "measures" K-theory charge of the Dq-brane, and in that sense it can be interpreted as an "external" gauge field. In this picture, the Dq-brane charge is identified with the topological (K-theory) charge of TIs/TSCs. On the other hand, $\tilde{A}_\mu$ is an internal degree of freedom on the Dq-brane. For example, in the integer/fractional QHE, the external gauge field is the electromagnetic U(1) gauge field, which measures the Hall conductivity, while the internal gauge field is the Chern–Simons (CS) gauge field describing the dynamics of the droplet itself.

The massive fermions can be integrated out, yielding the description of the topological phase in terms of the gauge fields. The resulting effective field theory comes with terms of topological nature, such as the CS or the θ-terms. In our string theory setup, they can be read off from the Wess–Zumino (WZ) action of the D-branes, by taking one of the D-branes as a background for the other. One can view these gauge-interacting TIs/TSCs from Dp–Dq systems as an analogue of the projective (parton) construction of the (fractional) QHE [16]. Our string theory realization of TIs/TSCs sheds light on extending the projective construction of the QHE to more generic TIs/TSCs; it tells us what type of gauge field is "natural" to couple with fermions in topological phases, and guarantees the topological stability of the system.

2. Complex case

Let us start with the most familiar example of the QHE (class A in $d=2$). We fix the value of p to be $p=5$ by T-duality, and consider a D5-brane in type IIB string theory which extends in the $x^{0,1,2,3,4,5}$ directions in ten-dimensional space–time. We take the Dq-brane with $q=5$ in the $x^{0,1,2,6,7,8}$ directions (Table 4). By T-duality, this setup is equivalent to the D3–D7 system studied in [19–21]. Since the number of Neumann–Dirichlet (ND) directions is six, open string excitations between the D5-branes give rise to two Majorana fermions (Mj) [= one two-component Dirac fermion (Di), ψ] and no bosons. The distance between the D-branes in x^9 direction (Δx^9) is proportional to the mass m of the fermions. The low-energy effective theory is schematically summarized by the effective Lagrangian in the $(2+1)$-dimensional common direction of the two D5-branes,

$$\mathcal{L}=\bar{\psi}\big[\gamma^\mu(i\partial_\mu-A_\mu-\tilde{A}_\mu)-m\big]\psi+\cdots. \tag{1}$$

Integrating the massive fermions yields the CS terms $\frac{k}{4\pi}\int A\wedge dA$ and $\frac{k}{4\pi}\int\tilde{A}\wedge d\tilde{A}$ with $k=\pm 1/2$ (parity anomaly). The Hall conductivity is read off from the CS term for A_μ as $\sigma_{xy}=k/(2\pi)$. Alternatively, the presence of the CS terms can be read off from the WZ action of either one of D5-branes, e.g.,

$$S^{\mathrm{WZ}}_{\mathrm{D5}}\propto\int_{\mathrm{D5}}F\wedge F\wedge C_2=\int_{\mathrm{D5}}A\wedge F\wedge(dC)_3 \tag{2}$$

for the external gauge field A_μ, where C_2 is the RR 2-form from the Dq-brane. When we change the sign of m by passing the Dq-brane through the Dp-brane, the value of k jumps from $\pm 1/2$ to $\mp 1/2$. If we instead put N_f Dq-branes, we have N_f copies of massive Dirac fermions ψ_i which couple with $\mathrm{U}(N_f)$ gauge fields A_μ and $\tilde{A}_\mu$ (when all Dq are coincident).

This brane construction can be extended to other even space dimensions $d=2n$ by considering D5–Dq systems with $q=5,7$ (Table 4). This setup gives rise to the fermion spectrum consisting