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Annihilation rates of heavy 1^{--} S-wave quarkonia in Salpeter method

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ABSTRACT

Annihilation rates of vector 1^{--} charmonium and bottomonium $^3S_1$ states $V \to e^+e^-$ and $V \to 3\gamma$, $V \to \gamma gg$ and $V \to 3g$ are estimated in the relativistic Salpeter method. In our calculations, special attention is paid to the relativistic correction, which is important and cannot be ignored for excited $2S$, $3S$ and higher excited states. We obtain $\Gamma(J/\psi \to 3\gamma) = 6.8 \times 10^{-4}$ keV, $\Gamma(\psi(2S) \to 3\gamma) = 2.5 \times 10^{-4}$ keV, $\Gamma(\psi(3S) \to 3\gamma) = 1.7 \times 10^{-4}$ keV, $\Gamma(\Upsilon(1S) \to 3\gamma) = 1.5 \times 10^{-5}$ keV, $\Gamma(\Upsilon(2S) \to 3\gamma) = 5.7 \times 10^{-6}$ keV, $\Gamma(\Upsilon(3S) \to 3\gamma) = 3.5 \times 10^{-6}$ keV and $\Gamma(\Upsilon(4S) \to 3\gamma) = 2.6 \times 10^{-6}$ keV.

1. Introduction

Annihilation decay of 1^{--} S-wave heavy quarkonia has been extensively studied [1–7]. The interest in this study comes from several sources. First, the annihilation amplitudes are related to the behavior of wave function, enabling an understanding of the formalism of inter-quark interactions. Further more, it can be a sensitive test of potential models [8]. Finally, the ratio of the decay widths, e.g. $\Gamma(V \to e^+e^-)/\Gamma(V \to 3g)$, is sensitive to the running coupling constant $\alpha_s(\mu)$, where V is a heavy quarkonium vector state and μ is the scale ($\mu = m_c$ for J/ψ and $\mu = m_b$ for Υ), and may provide very useful information for α_s at the heavy quark mass scale [9,10].

In our previous Letters [11], two-photon and two-gluon annihilation rates of $J^{PC} = 0^{-+}$, 0^{++} and 2^{++} $c\bar{c}$ and $b\bar{b}$ states are computed with the relativistic Salpeter method. Good agreement of the predictions with other theoretical calculations and the available experimental data is found. In the calculations, we found the relativistic corrections are large and not negligible, especially for high excited states, such as, the $2S$ and $3S$ states, because there are node structures in wave functions of $2S$ and $3S$ states, these cause large relativistic corrections even for heavy quarkonium like bottomonium. So in the theoretical studies concerning the highly excited states, a relativistic model is required.

The annihilation decays of the vector 1^{--} states are different from the C even states. Basically there are two types of annihilation decay modes, which are $V \to \gamma^* \to l^+l^-$ and $V \to 3\gamma$, γgg, $3g$. These decay widths have been studied in non-relativistic limit and found to be proportional to the square of the wave function at the origin $|\psi(0)|^2$ [1,12]. However, the decay rates of many processes are subject to substantial relativistic corrections [10,13,14]. In this Letter, we will continue to study the annihilation decays of $J^{PC} = 1^{--}$ $c\bar{c}$ and $b\bar{b}$ states with the relativistic Salpeter method.

There are two sources of relativistic corrections [4,11], one is the correction in relativistic kinematics which appears in the decay amplitudes through a well-defined form of relativistic wave function (i.e., not merely through the wave function at origin); the other relativistic correction comes via the relativistic inter-quark dynamics, which requires a relativistic formalism to describe the interactions among quarks and relativistic formalism to consider the transition amplitude. To consider the relativistic corrections, we choose the Salpeter method [15], which is an instantaneous version of Bethe–Salpeter method [16]. For the equal-mass quarkonium, the non-instantaneous correction is very small [17]. For the annihilation amplitude, we choose Mandelstam formalism [18], which is well suited for the computation of relativistic transition amplitude with Bethe–Salpeter wave functions as input.

In Section 2, we give theoretical details for the annihilation amplitude in Mandelstam formalism and the corresponding wave function with a well-defined relativistic form. The decay width of $V \to \gamma^* \to e^+e^-$ and $V \to 3\gamma$, γgg, $3g$ are formulated in this

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section. We will show the numerical results and give discussions in Section 3.

2. Theoretical details

2.1. The $V \to e^+e^-$ decay

According to Mandelstam formalism [18], the transition amplitude of a quarkonium decaying into an electron and a positron (see Fig. 1) can be written as

$$T_{e^+e^-} = i\sqrt{3}e^2 e_q \int \frac{d^4q}{(2\pi)^4} \mathrm{tr}\big[\chi(q)\gamma_\mu\big] \frac{g^{\mu\nu}}{M^2} \bar{u}_{r_1}(\vec{k}_1)\gamma_\nu v_{r_2}(\vec{k}_2), \quad (1)$$

where $e_q = \frac{2}{3}$ for charm quark and $e_q = -\frac{1}{3}$ for bottom quark; $\vec{k}_1$ and $\vec{k}_2$ are the momenta of electron and positron respectively; M is the meson mass; $\chi(q)$ is the Bethe–Salpeter wave function with the total momentum P and relative momentum q, related by

$$p_1 = \alpha_1 P + q, \quad \alpha_1 \equiv \frac{m_1}{m_1 + m_2},$$
$$p_2 = \alpha_2 P - q, \quad \alpha_2 \equiv \frac{m_2}{m_1 + m_2},$$

where $m_1 = m_2$ is the constitute quark mass of charm or bottom.

After performing the integration over q^0, one reduce the expression, with the notation of Salpeter wave function $\Psi(\vec{q}) = i\int \frac{dq_0}{2\pi}\chi(q)$, to

$$T_{e^+e^-} = \sqrt{3}e^2 e_q \int \frac{d\vec{q}}{(2\pi)^3} \mathrm{tr}\big[\Psi(\vec{q})\gamma_\mu\big] \frac{g^{\mu\nu}}{M^2} \bar{u}_{r_1}(\vec{k}_1)\gamma_\nu v_{r_2}(\vec{k}_2). \quad (2)$$

We note that the form of the wave function is also important in the calculation, since the corrections of the relativistic kinetics come mainly through it. By analyzing the parity and charge conjugation, the general form of relativistic wave function of 1^- state (1^{--} for equal mass systems) can be written as [19]

$$\begin{aligned}
\Psi^{\lambda}_{1^-}(q_\perp) &= q_\perp \cdot \epsilon^{\lambda}_\perp \left[f_1(q_\perp) + \frac{\not{P}}{M} f_2(q_\perp) + \frac{\not{q}_\perp}{M} f_3(q_\perp) + \frac{\not{P}\not{q}_\perp}{M^2} f_4(q_\perp) \right] \\
&\quad + M\not{\epsilon}^{\lambda}_\perp f_5(q_\perp) + \not{\epsilon}^{\lambda}_\perp \not{P} f_6(q_\perp) + \big(\not{q}_\perp \not{\epsilon}^{\lambda}_\perp - q_\perp \cdot \epsilon^{\lambda}_\perp\big) f_7(q_\perp) \\
&\quad + \frac{1}{M}\big(\not{P}\not{\epsilon}^{\lambda}_\perp \not{q}_\perp - \not{P} q_\perp \cdot \epsilon^{\lambda}_\perp\big) f_8(q_\perp), \qquad (3)
\end{aligned}$$

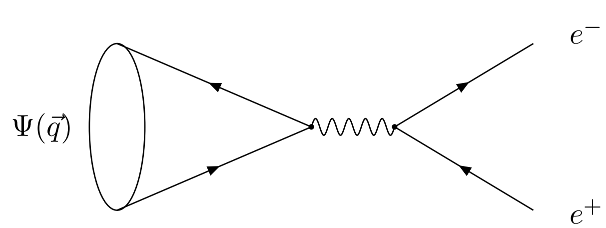

Fig. 1. Leptonic decay diagram of quarkonium.

where P and $\epsilon^{\lambda}_\perp$ are the momentum and polarization vector of the vector meson; $q_\perp = (0, \vec{q})$. The 8 wave functions f_i are not independent due to the equations $\varphi^{+-}_{1^-}(q_\perp) = \varphi^{-+}_{1^-}(q_\perp) = 0$. For quarkonium states we get the constraints on the components of the wave functions [19]:

$$f_1(q_\perp) = \frac{q_\perp^2 f_3(q_\perp) + M^2 f_5(q_\perp)}{M m_1}, \qquad f_7(q_\perp) = 0,$$
$$f_8(q_\perp) = -\frac{M f_6(q_\perp)}{m_1}, \qquad f_2(q_\perp) = 0.$$

With these constraints, only four independent components f_3, f_4, f_5 and f_6 are left. Namely

$$\begin{aligned}
\Psi^{\lambda}_{1^{--}}(q_\perp) &= q_\perp \cdot \epsilon^{\lambda}_\perp \left(\frac{q_\perp^2}{M m_1} + \frac{\not{q}_\perp}{M} \right) f_3(q_\perp) + q_\perp \cdot \epsilon^{\lambda}_\perp \frac{\not{P}\not{q}_\perp}{M^2} f_4(q_\perp) \\
&\quad + \left(M\not{\epsilon}^{\lambda}_\perp + q_\perp \cdot \epsilon^{\lambda}_\perp \frac{M}{m_1} \right) f_5(q_\perp) \\
&\quad + \left[\not{\epsilon}^{\lambda}_\perp \not{P} + \frac{\not{P}(q_\perp \cdot \epsilon^{\lambda}_\perp)}{m_1} - \frac{\not{P}\not{\epsilon}^{\lambda}_\perp \not{q}_\perp}{m_1} \right] f_6(q_\perp). \qquad (4)
\end{aligned}$$

These wave functions and the bound state mass M can be obtained by solving the full Salpeter equation with the constituent quark mass as input. We will not show the details of how to solve the full Salpeter equation, only give the final results. Interested readers can find the detail technique in Refs. [19,20].

Defining the decay constant f_V by

$$f_V M \epsilon^{\lambda}_\mu \equiv \langle 0|\bar{q}_1 \gamma_\mu q_2|V, \epsilon\rangle = \sqrt{3}\int \frac{d^3q}{(2\pi)^3} \mathrm{tr}\big[\varphi(\vec{q})\gamma_\mu\big], \qquad (5)$$

and with Eq. (4) we can easily obtain

$$f_V = 4\sqrt{3}\int \frac{d\vec{q}}{(2\pi)^3} \left[f_5(\vec{q}) - \frac{\vec{q}^2}{3M^2} f_3(\vec{q}) \right]. \qquad (6)$$

Summing over the polarizations of the final states and averaging over that of the initial state, neglecting the electron mass, it is easy to get the decay width

$$\Gamma_{e^+e^-} = \frac{4\pi}{3}\alpha^2 e_q^2 f_V^2 / M. \qquad (7)$$

2.2. $V \to 3\gamma$, $V \to \gamma gg$ and $V \to 3g$ decays

With the notation and definition used in the previous subsection, the relativistic transition amplitude of a quarkonium decaying into three photons (see Fig. 2) can be written as

$$\begin{aligned}
T_{3\gamma} &= \sqrt{3}(iee_q)^3 \int \frac{d^4q}{(2\pi)^4} \\
&\quad \times \mathrm{tr}\left\{ \chi(q) \left[\not{\epsilon}_3 \frac{1}{\not{k}_3 - \not{p}_2 - m} \not{\epsilon}_2 \frac{1}{\not{p}_1 - \not{k}_1 - m} \not{\epsilon}_1 \right.\right. \\
&\quad \left.\left. + \text{all other permutations of } 1, 2, 3 \right] \right\}, \qquad (8)
\end{aligned}$$

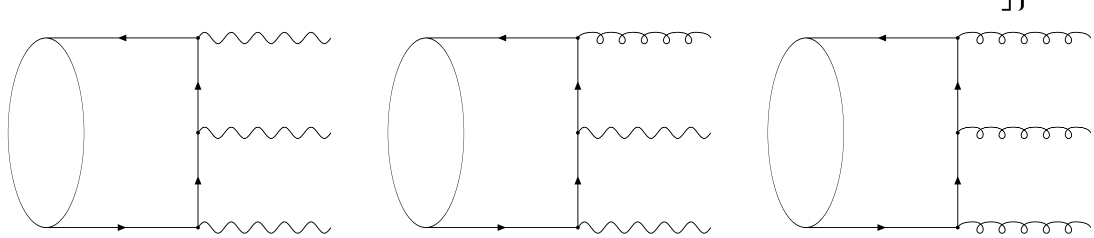

Fig. 2. 3γ, γgg, $3g$ annihilation diagrams of quarkonium. We do not show other diagrams with all the possible permutations of photons and gluons.