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The role of photon polarization modes in the magnetization and instability of the vacuum in a supercritical field

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A R T I C L E I N F O

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The response of the QED vacuum in an asymptotically large electromagnetic field is studied. In this regime the vacuum energy is strongly influenced by the vacuum polarization effect. The possible interaction between the virtual electromagnetic radiation and a superstrong magnetic field suggests that a background of virtual photons is a source of magnetization to the whole vacuum. The corresponding contribution to the vacuum magnetization density is determined by considering the individual contribution of each vacuum polarization eigenmode in the Euler–Heisenberg Lagrangian. Additional issues concerning the transverse pressures are analyzed. We also study the case in which the vacuum is occupied by a superstrong electric field. It is discussed that, in addition to the electron–positron pairs, the vacuum could create photons with different propagation modes. The possible relation between the emission of photons and the birefringent character of the vacuum is shown as well.

1. Introduction

Whilst there is some evidence that very large magnetic fields $|\mathbf{B}| \gg B_c$, $B_c = m^2/e = 4.42 \times 10^{13}$ G[1] exist in stellar objects identified as neutron stars [1–3], its origin and evolution remains poorly understood [4]. Some investigations in this area provide theoretical evidence that $|\mathbf{B}|$ might be generated due to gravitational and rotational effects, whereas other theories estimate that is self-consistent due to the Bose–Einstein condensation of charged and neutral boson gases in a superstrong magnetic field [5–8]. In this framework the nonlinear QED-vacuum possesses the properties of a paramagnetic medium and constitutes a source of magnetization, induced by the external magnetic field. Its properties are primarily determined by the vacuum energy of virtual electron–positron pairs. Because of this, a negative pressure transversal to the external field is generated [9] in similarity with the Casimir effect between metallic plates [10]. Moreover, the vacuum occupied by the external field turns out to be an "exotic" scenario in which processes like photon splitting [11,12] and photon capture [13–15] could take place. These two phenomena depend on the photon dispersion relation which differs from the light cone, due to vacuum polarization effects [16–19]. As a result, the issue of light propagation in empty space, in the presence of an external magnetic field, is similar to the dispersion of light in an anisotropic "medium".

The phenomenological aspects associated with this problem have been studied for a long time. In the meanwhile, other features of nonlinear electrodynamics in a superstrong magnetic field have been studied such as the dimensional reduction of the Coulomb potential [20–23] and the possible existence of a photon anomalous magnetic moment [24]. However, due to the vacuum polarization effect, virtual photons can carry a magnetization as well. As a consequence, they might be a source of magnetism to the whole vacuum. Motivated by this idea, we address the question in which way the virtual electromagnetic radiation contributes to the vacuum magnetization and therefore to increase the external field strength. The magnetic properties of the vacuum have been studied in [9,25–27] for weak ($|\mathbf{B}| \ll B_c$) and moderate fields ($|\mathbf{B}| \sim B_c$) in one-loop approximation of the Euler–Heisenberg Lagrangian [28] which involves the contribution from virtual electron–positron pairs. The contribution of virtual photons, created and annihilated spontaneously in the vacuum and interacting with $\mathbf{B}$ by means of $\Pi_{\mu\nu}$, is contained within the two-loop term of the Euler–Heisenberg Lagrangian (see Fig. 1). The latter was computed many years ago by Ritus [29,30] and has been recalculated by several authors as well [31–34]. In all these works, however, it is really cumbersome to discern the individual contributions given by each virtual photon propagation mode to the Euler–Heisenberg Lagrangian which should allow to determine the magnetism and pressure associated with each form of virtual mode. In this Letter we analyze these contributions separately for very large magnetic fields ($|\mathbf{B}| \gg B_c$) since these allow to establish relations between the birefringence of the vacuum [19,35,36] and the global properties of it.

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[1] Hereafter m and e are the electron mass and charge, respectively.

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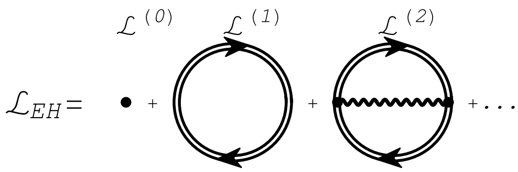

Fig. 1. Two-loop expansion of the Euler–Heisenberg Lagrangian. The double lines represent the electron–positron Green's functions, whereas the wavy line refers to the photon. Here $\mathcal{L}^{(0)}$ is the free Maxwell Lagrangian, $\mathcal{L}^{(1)}$ represents the one loop which gives the contribution of the virtual free electron–positron pairs created and annihilated spontaneously in vacuum and interacting with the external field. The radiative corrections (involved in $\mathcal{L}^{(2)}$) emerge from two loop due to exchange of the virtual photons.

Besides the strongly magnetized vacuum, there is another interesting external field configuration which deserves to be analyzed: a supercritical electric field $|\mathbf{E}| \gg \mathrm{E_c}$ with $\mathrm{E_c} = \mathrm{m}^2/\mathrm{e} = 1.3 \times 10^{16}$ V/cm. In this asymptotic region the Euler–Heisenberg Lagrangian acquires an imaginary term which characterizes the instability of the vacuum. This phenomenon is closely related to the production of observable particles from the own vacuum. Certainly, the creation of electron–positron pairs—the so-called Schwinger mechanism—turns out to be the most remarkable effect predicted through this procedure [28,37,38]. However, the imaginary part of this effective Lagrangian is just a measure of the vacuum decay and does neither give the actual rate of production of particles nor the accessible decay channels [39]. Thereby not only the creation of electron–positron pairs is a plausible effect but also the emission of observable photons [29,30]. The latter phenomenon was analyzed by Gitman, Fradkin and Shvartsman [50–52]. Their results showed that the total probability of photon emission from the vacuum, accompanied by the creation of an arbitrary number of electron–positron pairs, is connected to the decay probability of the vacuum and thus to the imaginary part arising from the two-loop term of the Euler–Heisenberg Lagrangian. In this case the corresponding decomposition in terms of the vacuum polarization modes is particularly illuminating because it reveals that only two of them contribute to the vacuum instability. It seems, therefore, that the vacuum could create photons with different propagation modes, an effect closely related to its own birefringence.

2. Preliminary remarks

In a magnetized vacuum the spatial symmetry is explicitly broken by the external field $\mathbf{B}$. In this context, there is a vectorial basis $\flat_\mu^{(i)}$ [20,40,41] which characterizes the vacuum symmetry properties and fulfills both the orthogonality condition: $\flat_\sigma^{(i)}\flat^{\sigma(j)} = \delta^{ij}(\flat^{(i)})^2$ and the completeness relation: $\delta^\mu{}_\nu - \frac{k^\mu k_\nu}{k^2} = \sum_{i=1}^{3}\flat^{\mu(i)}\flat_\nu^{(i)}/(\flat^{(i)})^2$. Explicitly, the basis vectors read $\flat_\mu^{(1)} = k^2\mathscr{F}^2_{\mu\lambda}k^\lambda - k_\mu(k\mathscr{F}^2k)$, $\flat_\mu^{(2)} = \tilde{\mathscr{F}}_{\mu\lambda}k^\lambda$, $\flat_\mu^{(3)} = \mathscr{F}_{\mu\lambda}k^\lambda$ and $\flat_\mu^{(4)} = k_\mu$. These expressions involve the external field tensor $\mathscr{F}_{\mu\nu}$ and its dual $\tilde{\mathscr{F}}^{\mu\nu} = 1/2\epsilon^{\mu\nu\rho\sigma}\mathscr{F}_{\rho\sigma}$. In this basis, the vacuum polarization tensor is diagonal *i.e.*

$$\Pi_{\mu\nu} = \sum_{i=0}^{4}\varkappa_i\frac{\flat_\mu^{(i)}\flat_\nu^{(i)}}{(\flat^{(i)})^2} \tag{1}$$

whereas the dressed photon Green function can be expressed as

$$\mathscr{D}_{\mu\nu} = \sum_{i=1}^{3}\frac{1}{k^2-\varkappa_i}\frac{\flat_\mu^{(i)}\flat_\nu^{(i)}}{(\flat^{(i)})^2} + \frac{\zeta}{k^2}\frac{k_\mu k_\nu}{k^2}. \tag{2}$$

Here the $\varkappa_i$ represent the $\Pi_{\mu\nu}$-eigenvalues and ζ is the gauge parameter. This diagonal decomposition of $\Pi_{\mu\nu}$ defines the energy spectrum of the electromagnetic field which differs from the isotropic vacuum ($\mathbf{B} = 0$).

Owing to the transversality property ($k^\mu\Pi_{\mu\nu} = 0$), the eigenvalue corresponding to the fourth eigenvector vanishes identically ($\varkappa^{(4)} = 0$). Furthermore, not all the remaining eigenmodes are physical. In general, this depend on the direction of wave propagation. To show this we consider $\flat_\mu^{(i)}(k)$ as the electromagnetic four vector describing a photon. The corresponding electric and magnetic fields of each mode are $\boldsymbol{e}^{(i)} = i(\boldsymbol{k}\flat_0^{(i)} - \omega^{(i)}\flat^{(i)})$, $\boldsymbol{b}^{(i)} = -i\boldsymbol{k}\times\flat^{(i)}$. It follows that the mode $i = 3$ is a wave polarized in the transverse plane to $\boldsymbol{k}$ whose electric $\boldsymbol{e}^{(3)} \sim \boldsymbol{k}_\perp \times \boldsymbol{n}_\parallel$ and magnetic $\boldsymbol{b}^{(3)} \sim \boldsymbol{n}_\parallel k_\perp^2 - \boldsymbol{k}_\perp k_\parallel$ fields are orthogonal to each other. Here the vectors $\boldsymbol{k}_\perp$ and $\boldsymbol{k}_\parallel$ are the components of $\boldsymbol{k}$ across and along $\mathbf{B}$ with $\boldsymbol{n}_\parallel = \mathbf{B}/|\mathbf{B}|$. For a pure longitudinal propagation to the external field $k_\perp = 0$, the mode $\flat_\mu^{(2)}$ is a longitudinal and nonphysical electric wave $\boldsymbol{e}^{(2)} \sim \boldsymbol{n}_\parallel$. On the other hand, $\flat_\mu^{(1)}$ is transverse since the associated electric field is $\boldsymbol{e}^{(1)} \sim \boldsymbol{k}_\perp$ whereas the magnetic one $\boldsymbol{b}^{(1)} \sim \boldsymbol{k}_\perp \times \boldsymbol{k}_\parallel$. As a consequence, both $\flat_\mu^{(1)}$ and $\flat_\mu^{(3)}$ represent physical waves which may be combined to form a circularly polarized transversal wave. In this case both modes propagate along $\mathbf{B}$ with a dispersion law independents of the magnetic field strength [17,19,41].

Now, if the photon propagation involves a nonvanishing transversal momentum component $k_\perp \neq 0$, we are allowed to perform the analysis in a Lorentz frame that does not change the value $k_\perp$, but gives $k_\parallel = 0$ and does not introduce an external electric field. In this Lorentz frame, the first eigenmode $\flat_\mu^{(1)}$ becomes purely electric longitudinal and a nonphysical mode whereas $\flat_\mu^{(2)}$ is transverse. Hence, for a photon whose three-momentum is directed at any nonzero angle with the external magnetic field, the two orthogonal polarization states $\flat_\mu^{(2)}$ and $\flat_\mu^{(3)}$ propagate. In this framework the analytical structures of the corresponding eigenvalues $\varkappa_{2,3}$ are different. As a matter of fact, the vacuum behaves like a birefringent medium with refraction indices [19,41]

$$\eta_2 = \frac{|\boldsymbol{k}|}{\omega_2} = \left(1+\frac{\varkappa_2}{\omega_2^2}\right)^{1/2} \quad\text{and}\quad \eta_3 = \frac{|\boldsymbol{k}|}{\omega_3} = \left(1+\frac{\varkappa_3}{\omega_3^2}\right)^{1/2}. \tag{3}$$

Here $\omega_{2,3}$ are the corresponding solution of the dispersion equations $k^2 = \varkappa_{2,3}$ arising from the poles of $\mathscr{D}_{\mu\nu}$.

Considering these aspects, we analyze the Euler–Heisenberg Lagrangian

$$\mathcal{L}_{\mathrm{EH}} = \mathcal{L}_{\mathrm{R}}^{(0)} + \mathcal{L}_{\mathrm{R}}^{(1)} + \cdots \tag{4}$$

where $\mathcal{L}_{\mathrm{R}}^{(0)} = -\frac{1}{2}\mathrm{B}^2$ is the free renormalized Maxwell Lagrangian, whereas $\mathcal{L}_{\mathrm{R}}^{(1)}$ denotes the one-loop regularized contribution of virtual electron–positron pairs created and annihilated spontaneously in vacuum and interacting with $\mathbf{B}$ [28]. In asymptotically large magnetic fields it reads

$$\mathcal{L}_{\mathrm{R}}^{(1)}(\mathfrak{b}) \approx \frac{\mathrm{m}^4\mathfrak{b}^2}{24\pi^2}\left\{\ln\left(\frac{\mathfrak{b}}{\gamma\pi}\right) + \frac{6}{\pi^2}\zeta'(2)\right\}. \tag{5}$$

Here $\mathfrak{b} = |\mathbf{B}|/\mathrm{B_c}$, $\ln(\gamma) = 0.577\ldots$ denotes the Euler constant whereas $6\pi^{-2}\zeta'(2) = -0.5699610\ldots$ and $\zeta(x)$ is the Riemann zeta-function.

The contribution of a virtual photon interacting with external field by means of the vacuum polarization tensor is expressed as [29,31,34]

$$\mathcal{L}^{(2)} = \frac{i}{2}\int\frac{d^4k}{(2\pi)^4}\Pi_{\mu\nu}(k)\mathscr{D}_0^{\mu\nu}(k), \tag{6}$$