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Alephnull Anti-de Sitter supergravity with Lorentz Chern–Simons term in 3D

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ABSTRACT

We formulate supergravity in three-dimensional Anti-de Sitter (AdS) space–time with an arbitrary number of supersymmetries with a Lorentz Chern–Simons term. Our field content is $(e_\mu{}^m, \psi_\mu{}^A, A_\mu{}^{AB}, \omega_\mu{}^{mn}, \lambda^{mnA})$, where the gravitino $\psi_\mu{}^A$ and the gaugino $\lambda_{mn}{}^A$ are in the vectorial representation of $SO(N)$ for ${}^\forall N$, whose gauge field is $A_\mu{}^{AB}$. The $\omega_\mu{}^{mn}$ is a spin connection regarded as an independent field. Both $\omega_\mu{}^{mn}$ and $A_\mu{}^{AB}$ have their Chern–Simons (CS) terms. Local ${}^\forall N$ ($\aleph_0$) supersymmetry requires the coefficients of these CS terms be proportional to the gravitino mass. Differently from most conventional works, our supersymmetry transformation for $\omega_\mu{}^{mn}$ is proportional to the gaugino λ^{mnA}. The solution for the scalar curvature is a negative constant, and our space–time is AdS. Despite the Lorentz CS term, $\omega_\mu{}^{mn}$ can be algebraically solved. The Lorentz and $SO(N)$ CS terms serve as the gravitational and $SO(N)$ gauge anomalies for the two-dimensional boundary superconformal field theory. We also compute the charges for $\aleph_0$ supersymmetry, $SO(N)$ and Lorentz symmetries, and also show that Witten–Nester charges are positive definite.

1. Introduction

Topologically massive gravity theory with a cosmological constant in three-dimensional (3D) space–time [1–4] has been extensively studied, due to its importance associated with AdS/CFT correspondence with 2D theory, and with the peculiar black hole solution [5].

From the viewpoint of AdS/CFT correspondence, the gravitational Chern–Simons (CS) term in 3D plays an important role, corresponding to the gravitational anomaly on the 2D boundary of the bulk 3D Anti-de Sitter (AdS) space–time [6]. In particular, at the so-called 'chiral point' with $\mu = m$, the system becomes purely chiral, corresponding to the 'left-moving' conformal group [7]. The presence of the gravitational CS term with critical strength [2,3] may also provide resolutions to the obstructions [8] for solving the theory exactly.

More recently, Henneaux, Martinez and Troncoso (HMT) have shown that under their new asymptotic boundary condition for AdS solutions, one can get non-vanishing Virasoro generators on the 2D boundary [9], as desired for AdS/CFT correspondence. The first-order formalism for topologically massive $N = 1$ supergravity [1] has been also presented [10], leading to the conclusion that Witten–Nester energy [11,12] coincides with Abbott–Deser–Tetkin energy [13] for asymptotically AdS space–time under appropriate boundary conditions.

Rather independent of these developments, we have in our previous paper [14] formulated an N-extended supergravity with the spin connection as an independent field, with the gravitational and $SO(N)$ Chern–Simons terms added to the usual Hilbert action term $R(\omega)$. However, the drawback in [14] is that the spin connection $\omega_\mu{}^{mn}$ in the pure supergravity sector is equivalent to the usual spin connection $\widehat{\omega}_\mu{}^{mn}(e,\psi)$ in terms of $e_\mu{}^m$ and ψ_μ, as can be confirmed by the Palatini identity. Due to this fact, the formulation in [14] was not quite natural. The more natural system to be explored is the one with the spin connection as an independent field both in the pure supergravity sector and in its own CS term as well.

Our system given in the present Letter is also related to $N = p + q$ AdS supergravity with Chern–Simons terms [15]. However, there are four fundamental differences. First, we have the gaugino field $\lambda_{mn}{}^A$ as the superpartner for the spin connection $\omega_\mu{}^{mn}$. Second, our $\omega_\mu{}^{mn}$ is an independent field from the outset, different from [15]. Third, while we have the Lorentz CS term, it is absent in [15]. Fourth, our $\omega_\mu{}^{mn}$ is *algebraically* solved in terms of $e_\mu{}^m$ and $\psi_\mu{}^A$, despite the presence of the spin connection CS term with the first derivative. This is against the common wisdom with CS terms.

Even though there is similarity, our system is also different from the recent work on the 1-st order formulation of chiral topologically massive $N = 1$ supergravity [10]. The main difference is the absence of the higher-derivative gravitino term in our system compared with the latter. In this sense, our system is *not* so-called

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topological massive gravity [1–4,10]. Our system is also different from the recent study on the (curvature)^2-terms considered in massive gravity [16] and massive supergravity [17] in 3D, because we have *no* (curvature)^2-terms.

Our system is also related to $SO(N)$ conformal supergravity formulation in [18] with arbitrary number ($\aleph_0$) of supersymmetries. However, the lack of both conformal symmetry and the higher-derivative term for the gravitino, as well as the AdS space–time differentiates our system from [18].

In the present Letter, we first establish the Lagrangian and supersymmetry transformation rule for $\aleph_0$ AdS supergravity with a Lorentz CS term. We next derive field equations for all fields. As has been mentioned, our spin connection $\omega_\mu{}^{mn}$ is algebraically solved in terms of the supercovariant spin connection $\widehat{\omega}_\mu{}^{mn}$ and a constant torsion tensor. The effective scalar curvature $R(\widehat{\omega})$ is a negative constant, showing that our space–time is always 3D Anti-de Sitter (AdS_3). We next derive the currents and charges for local N-extended supersymmetry, $SO(N)$, and Lorentz transformations. We also compute the Witten–Nester charges [11], which turn out to be positive definite. Thanks to the simplification in terms of the independent spin connection $\omega_\mu{}^{mn}$, these computations are drastically simplified.

2. Action invariant under $\aleph_0$ local supersymmetries

As has been mentioned, our field content is $(e_\mu{}^m, \psi_\mu{}^A, A_\mu{}^{AB}, \omega_\mu{}^{mn}, \lambda^{mn\,A})$ with the indices $A, B, \ldots = 1, 2, \ldots, N$ for the **N** of $SO(N)$. We use the Greek indices $\mu, \nu, \ldots = 0, 1, 2$ or Latin indices $m, n, \ldots = (0), (1), (2)$ for the curved or local tangent coordinates, respectively. Our total action $I \equiv \int d^3x\, \mathcal{L}$ has the Lagrangian

$$\begin{aligned}\mathcal{L} = &+\frac{1}{4} e R(\omega) - \frac{1}{2}\epsilon^{\mu\nu\rho}\big[\overline{\psi}_\mu{}^A D_\nu(\omega, A)\psi_\rho{}^A\big] - \frac{1}{8} m\epsilon^{\mu\nu\rho} T_{\mu\nu\rho} \\ &+\frac{1}{8} m^{-1}\epsilon^{\mu\nu\rho}\left[R_{\mu\nu}{}^{mn}(\omega)\omega_{\rho mn} + \frac{2}{3}\omega_\mu{}^{mt}\omega_{\nu tu}\omega_\rho{}^u{}_m\right] \\ &-\frac{1}{4} m\epsilon^{\mu\nu\rho}\left(F_{\mu\nu}{}^{AB} A_\rho{}^{AB} + \frac{2}{3} A_\mu{}^{AB} A_\nu{}^{BC} A_\rho{}^{CA}\right) \\ &-\frac{1}{2} m^{-1} e\big(\bar{\lambda}_{mn}{}^A \lambda^{mn\,A}\big) + \frac{1}{4} m e\big(\overline{\psi}_\mu{}^A \gamma^{\mu\nu}\psi_\nu{}^A\big) \\ &+\frac{1}{2} m^2 e. \end{aligned} \tag{2.1}$$

We use the definitions, such as

$$R_{\mu\nu}{}^{mn}(\omega) \equiv +2\partial_{[\mu}\omega_{\nu]}{}^{mn} + 2\omega_{[\mu|}{}^{mt}\omega_{|\nu]t}{}^n, \tag{2.2a}$$

$$R_\mu{}^m(\omega) \equiv +R_{\mu\nu}{}^{m\nu}(\omega), \qquad R(\omega) \equiv R_m{}^m(\omega), \tag{2.2b}$$

$$\begin{aligned} D_\mu(\omega, A)\psi_\nu{}^A \equiv &+\partial_\mu\psi_\nu{}^A + \frac{1}{4}\omega_\mu{}^{mn}\big(\gamma_{mn}\psi_\nu{}^A\big) \\ &+ m A_\mu{}^{AB}\psi_\nu{}^B, \end{aligned} \tag{2.2c}$$

$$\begin{aligned} \widehat{T}_{\mu\nu}{}^m &\equiv +2\partial_{[\mu}e_{\nu]}{}^m + 2\omega_{[\mu}{}^{mt}e_{\nu]t} - \big(\overline{\psi}_\mu{}^A\gamma^m\psi_\nu{}^A\big) \\ &\equiv +2D_{[\mu}e_{\nu]}{}^m - \big(\overline{\psi}_\mu{}^A\gamma^m\psi_\nu{}^A\big) \\ &\equiv +T_{\mu\nu}{}^m - \big(\overline{\psi}_\mu{}^A\gamma^m\psi_\nu{}^A\big), \end{aligned} \tag{2.2d}$$

$$\begin{aligned} \widehat{F}_{\mu\nu}{}^{AB} &\equiv +\big[2\partial_{[\mu}A_{\nu]}{}^{AB} + 2mA_{[\mu}{}^{AC}A_{\nu]}{}^{CB}\big] - \big(\overline{\psi}_\mu{}^{[A}\psi_\nu{}^{B]}\big) \\ &= +F_{\mu\nu}{}^{AB} - \big(\overline{\psi}_\mu{}^{[A}\psi_\nu{}^{B]}\big), \end{aligned} \tag{2.2e}$$

$$\widehat{\mathcal{R}}_{\mu\nu}{}^A \equiv +2D_{[\mu|}(\omega, A)\psi_{|\nu]}{}^A + m\big(\gamma_{[\mu}\psi_{\nu]}{}^A\big), \tag{2.2f}$$

$$\widehat{R}_{\mu\nu}{}^{mn}(\omega) \equiv +R_{\mu\nu}{}^{mn}(\omega) - 2\big(\overline{\psi}_{[\mu}{}^A\gamma_{\nu]}\lambda^{mn\,A}\big). \tag{2.2g}$$

The supercovariantizations here are the usual ones in supergravity in 3D [15,19].

Our action is invariant under $\aleph_0$ local supersymmetry

$$\delta_Q e_\mu{}^m = +\big(\bar{\epsilon}^A\gamma^m\psi_\mu{}^A\big), \tag{2.3a}$$

$$\delta_Q \psi_\mu{}^A = +D_\mu(\omega, A)\epsilon^A + \frac{1}{2} m\big(\gamma_\mu\epsilon^A\big) \equiv \widehat{D}_\mu\epsilon^A, \tag{2.3b}$$

$$\delta_Q A_\mu{}^{AB} = +\big(\bar{\epsilon}^{[A}\psi_\mu{}^{B]}\big), \tag{2.3c}$$

$$\delta_Q \omega_\mu{}^{mn} = +\big(\bar{\epsilon}^A\gamma_\mu\lambda^{mn\,A}\big), \tag{2.3d}$$

$$\begin{aligned} \delta_Q \lambda_{mn}{}^A = &-\frac{1}{4}\big(\gamma^{\mu\nu}\epsilon^A\big)\widehat{R}_{\mu\nu\,mn}(\omega) \\ &-\frac{1}{4} m\big(\gamma_\mu\epsilon^A\big)\big(\widehat{T}_{mn}{}^\mu + 2e_{[m}{}^\mu\widehat{T}_{n]}\big) - \frac{1}{4} m^2\big(\gamma_{mn}\epsilon^A\big) \\ &-\big(\gamma^\mu\psi_\mu{}^B\big)\big(\bar{\epsilon}^{[A}\lambda_{mn}{}^{B]}\big) - \psi_\mu{}^B\big(\bar{\epsilon}^{[A|}\gamma_\mu\lambda_{mn}{}^{|B]}\big). \end{aligned} \tag{2.3e}$$

As the antisymmetrization shows, the last two terms in (2.3e) disappear in the case of $N = 1$. Our transformation rule (2.3d) is different from that in [15], because our spin connection is an independent field.

The closure of two supersymmetry transformations with the parameters ϵ_1^A and ϵ_2^B is

$$\big[\delta_Q(\epsilon_1), \delta_Q(\epsilon_2)\big] = \delta_\xi + \delta_\Lambda + \delta_\alpha + \delta_Q(\epsilon_3), \tag{2.4}$$

where the parameters $\xi^\mu, \Lambda^{mn}, \alpha^{AB}$ and ϵ_3^A are respectively for the general coordinate, local Lorentz, local $SO(N)$, and third local supersymmetry transformations:

$$\begin{aligned} &\xi^\mu \equiv \big(\bar{\epsilon}_2^A\gamma^\mu\epsilon_1^A\big), \qquad \Lambda^{mn} \equiv -\xi^\mu\omega_\mu{}^{mn}, \\ &\alpha^{AB} \equiv -\xi^\mu A_\mu{}^{AB} + \big(\bar{\epsilon}_2^{[A}\epsilon_1^{B]}\big), \qquad \epsilon_3^A \equiv -\xi^\mu\psi_\mu{}^A. \end{aligned} \tag{2.5}$$

Note that in Λ^{mn} there is only the $\xi\omega$-term, while the purely ϵ^2-term is absent.

It is to be mentioned that the coefficients $(1/8)m^{-1}$ and $-(1/4)m$ of Lorentz and $SO(N)$ gauge CS terms, and the gravitino mass are proportional to each other. This implies that 2D theory on $\partial\Sigma$ under the standard Brown–Henneaux boundary conditions [20], or HMT boundary conditions [9] corresponds to chiral conformal theory [3].

As has been well known [3], we can regard the system above as 'chiral' supergravity. This is because we can regard the AdS group as $SO(2,2) \approx SO(2,1) \times SO(2,1)$, where the first group as the 'left-handed' $SO(2,1)_L$, while the second group as the 'right-handed' $SO(2,1)_R$.

3. Supercovariant field equations

The field equations directly obtained from the Lagrangian are[1]

$$\begin{aligned} e^{-1}\frac{\delta\mathcal{L}}{\delta e_\mu{}^m} = &-\frac{1}{2}\left[\widehat{R}_m{}^\mu - \frac{1}{2}e_m{}^\mu\widehat{R}(\omega)\right] - \frac{1}{2}m^{-1}e_m{}^\mu\big(\bar{\lambda}_{rs}{}^A\lambda^{rs\,A}\big) \\ &-\frac{1}{4}me^{-1}\epsilon^{\mu\rho\sigma}\widehat{T}_{\rho\sigma m} + \frac{1}{2}m^2 e_m{}^\mu \doteq 0, \end{aligned} \tag{3.1a}$$

$$\begin{aligned} e^{-1}\frac{\delta\mathcal{L}}{\delta\omega_\mu{}^{mn}} = &+\frac{1}{4}\widehat{T}_{mn}{}^\mu + \frac{1}{2}e_{[m}{}^\mu\widehat{T}_{n]} + \frac{1}{4}me^{-1}\epsilon_{mn}{}^\mu \\ &+\frac{1}{4}m^{-1}e^{-1}\epsilon^{\mu\nu\rho}\widehat{R}_{\nu\rho mn}(\omega) \doteq 0, \end{aligned} \tag{3.1b}$$

$$e^{-1}\frac{\delta\mathcal{L}}{\delta\overline{\psi}_\mu{}^A} = -\frac{1}{2}\big(\gamma^{\mu\nu\rho}\widehat{\mathcal{R}}_{\nu\rho}{}^A\big) \doteq 0, \tag{3.1c}$$

[1] We use the symbol $\doteq$ for a field equation, distinguished from algebraic identities.