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Extended general relativity: Large-scale antigravity and short-scale gravity with $\omega = -1$ from five-dimensional vacuum

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ABSTRACT

Considering a five-dimensional (5D) Riemannian spacetime with a particular stationary Ricci-flat metric, we obtain in the framework of the induced matter theory an effective 4D static and spherically symmetric metric which give us ordinary gravitational solutions on small (planetary and astrophysical) scales, but repulsive (anti gravitational) forces on very large (cosmological) scales with $\omega = -1$. Our approach is an unified manner to describe dark energy, dark matter and ordinary matter. We illustrate the theory with two examples, the solar system and the great attractor. From the geometrical point of view, these results follow from the assumption that exists a confining force that make possible that test particles move on a given 4D hypersurface.

1. Introduction

In much the term antigravity has come to present all those physical phenomena in which the usual gravitational potential is modified to accommodate repulsive gravitational forces. This is a fascinating subject which has many implications from the possible check of antigravity against experiments to the several theoretical issues that are involved, the 4D principle of equivalence and energy conservation among others [1]. The idea of antigravity has been subject of different approaches through the last decades. Scherk [2] considered this phenomenon in the framework of supergravity related to fermionic generators. In essence, antigravity can be treated as one where the gravitational and other forces between certain objects in a field theory can mutually cancel. Antigravity has been studied from five-dimensional Kaluza–Klein (KK) theory, where the extra dimension is compact [3]. The original version of the KK theory assures, as a postulate, that the fifth dimension is compact. A few years ago, a non-compactified approach to KK gravity known as Induced Matter (IM) theory was proposed by Wesson and collaborators [4]. In this theory all classical physical quantities, such as matter density and pressure, are susceptible of a geometrical interpretation. Wesson's proposal also assumes that the fundamental 5D space in which our usual spacetime is embedded, should be a solution of the classical 5D vacuum Einstein equations: $R_{AB} = 0$. The mathematical basis of it is the Campbell–Magaard theorem [5], which ensures an embedding of 4D general relativity with sources in a 5D theory whose field equations are apparently empty. That is, the Einstein equations $G_{\alpha\beta} = (8\pi G/c^4)T_{\alpha\beta}$ are embedded perfectly in the Ricci-flat equations $R_{AB} = 0$. In simple terms, Wesson uses the fifth dimension to model matter. More recently, has been suggested that antigravity can be originated as the repulsion effect of matter and antimatter [6]. The relationship between antigravity and antimatter has been studied also in [7]. In the framework of brane world models, has been suggested that antigravity effects could be important for very large scales when gravitons are metastable [8]. The possibility of obtaining an infrared modification to gravity on cosmological scales from extra dimensions has been considered in [9]. Strong antigravity has been obtained by compactification on a manifold with flat directions from a higher-dimensional model [10]. Very

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recently was proposed an experiment to measure antigravity with an antihydrogen beam [11].

On the other hand, when applied to cosmic structure on galactic and larger scales, standard 4D general relativity and its Newtonian weak-field limit fail at describing the observed phenomenology. To reconcile the theory with observations we need to assume that $\sim 85\%$ of the mass is seen only through its observational effect and that $\sim 74\%$ of the energy content of the universe is due to either to an arbitrary cosmological constant or to a not well defined dark energy fluid. The cosmological constant problem appears to be so serious as the dark matter problem. The Einstein equations admit the presence of an arbitrary constant Λ. The Friedmann solutions with a positive Λ fit very satisfactorily the observational evidence of an accelerating universe. The problem arises when one wishes to attach a physical interpretation to Λ. Since observations indicate $\Lambda > 0$, the dark energy fluid has negative pressure. Current observations suggest $\omega = -1$ at all probed epochs [12], so models more sophisticated than a simple Λ could seem in principle unnecessary. However, in the context of quantum field theory, the Λ problem translates into an extreme fine-tuning problem, because $\rho_v(t_P)/(\sum \Delta\rho_v) = (1 + 10^{-108})$ is extremely close to 1, but not exactly 1. This problem would disappear if Λ were exactly zero [15]. An alternative conclusion we can draw from this failure is that standard 4D General Relativity must be modified on these cosmic scales, or, in other words that the equation of state for matter is not $\omega_m = 0$, but could be $\omega_m = -1$. In this Letter we explore this idea from a 5D vacuum state using some ideas of the STM theory.

2. The field equations on 4D hypersurfaces

We start by considering a 5D spacetime with a Ricci-flat metric g_{ab} determined by the line element [13]

$$dS^2 = \left(\frac{\psi}{\psi_0}\right)^2 \left[c^2 f(r)\, dt^2 - \frac{dr^2}{f(r)} - r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right] - d\psi^2, \tag{1}$$

where $f(r) = 1 - (2G\zeta\psi_0/(rc^2))[1 + c^2 r^3/(2G\zeta\psi_0^3)]$ is a dimensionless function, $\{t, r, \theta, \phi\}$ are the usual local spacetime spherical coordinates employed in general relativity and ψ is the space-like extra dimension that following the approach of the induced matter theory, will be considered as non-compact. In this line element ψ and r have length units, θ and ϕ are angular coordinates, t is a time-like coordinate, c denotes the speed of light, ψ_0 is an arbitrary constant with length units and the constant parameter ζ has units of $(\text{mass})(\text{length})^{-1}$.

Now let us to assume that the 5D spacetime can be foliated by the family of hypersurfaces $\{\Sigma_0\colon\ \psi = \psi_0\}$. On every generic hypersurface Σ_0 the induced metric is given by the 4D line element

$$dS^2_{\text{ind}} = c^2 f(r)\, dt^2 - \frac{dr^2}{f(r)} - r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right). \tag{2}$$

Given the symmetry properties of the 5D spacetime, it seems natural to assume that the induced matter on Σ_0 can be globally described by a 4D energy momentum tensor of a perfect fluid $T_{\alpha\beta} = (\rho c^2 + P)U_\alpha U_\beta - P g_{\alpha\beta}$ where $\rho(t,r)$ and $P(t,r)$ are respectively the energy density and pressure of the induced matter. From the relativistic point of view, observers that are on Σ_0 move with $U^\psi = 0$ [see Section 3]. The Einstein field equations on the hypersurface Σ_0 for the metric in (2), read

$$r\frac{df}{dr} - 1 + f = -\frac{8\pi G}{c^2} r^2 \rho, \tag{3}$$

$$r\frac{df}{dr} - 1 + f = \frac{8\pi G}{c^4} r^2 P. \tag{4}$$

The resulting equation of state is

$$P = -\rho c^2 = -\frac{3c^4}{8\pi G}\frac{1}{\psi_0^2}, \tag{5}$$

which technically corresponds to a vacuum equation of state. On the other hand, regarding that the metric in (2) has spherical symmetry, we can associate the energy density of induced matter ρ to a mass density of a sphere of physical mass $m \equiv \zeta\psi_0$ and radius r_0. If we do that, it follows that M and the radius r_0 of such a sphere are related by the expression $\zeta = r_0^3/(2G\psi_0^3)$. An immediate consequence of this expression is that in principle given an specific value of r_0, depending of the value of ψ_0 we could induce a large massive object or a mini-massive object. This way we can say that is possible in this case to treat the induced matter as a massive compact object embedded in a 5D vacuum. Some information that we can obtain by simple inspection of the metric (1) is that when $G\zeta = \sqrt{3}/9$ there is a single Schwarzschild radius. In this case the Schwarzschild radius is $r_{\text{Sch}} = \psi_0/\sqrt{3} \geqslant r_0$. When it is greater than the radius of the sphere of parameter ζ, the compact object has properties very close to those of a black hole on distances $1 \gg r/\psi_0 > r_{\text{Sch}}/\psi_0$, this condition holds when $G\zeta \leqslant 1/(2\sqrt{27}) \simeq 0.096225$. For $G\zeta > \sqrt{3}/9$ one obtains that $f(r) < 0$ and there is not Schwarzschild radius. When $G\zeta \leqslant \sqrt{3}/9$ there are two Schwarzschild radius, an interior r_{S_i} and an exterior one r_{S_e}, such that by definition $f(r_{S_i}) = f(r_{S_e}) = 0$. This last case has very interesting properties and we will focus on the study of that properties in some scenarios at astrophysical and cosmological scales. When we assume that the present universe we live in can be modeled on the 4D hypersurface $\Sigma_{H_0}\colon\ \psi_0 = cH_0^{-1}$, H_0 being $H_0 = 73\ \frac{\text{km}}{\text{s}}\text{Mpc}^{-1}$ the present Hubble constant, we found that the exterior Schwarszchild radius r_{S_e} becomes of the order of size of the Hubble radius which is the size of the present observable universe. On the other hand as the interior Schwarszchild radius r_{S_i} depends strongly of the value of $G\zeta$ then when $G\zeta \ll 1$, the interior Schwarszchild radius r_{S_i} approximates to zero. What makes interesting this case is that an observer located between these two Schwarzschild radius could be able to see a compact object with a horizon event determined by r_{S_i} immersed in our observable universe whose size is determined by the Hubble horizon given by r_{S_e}. This particular case is our interest and in the preceding sections we will study in detail more properties of it.

3. Particle trajectories

In order to study with more detail properties of the metric (2), we shall describe the geodesic trajectories of non-massive and massive test particles. All of these are described by the equation

$$\frac{dU^a}{dS} + \Gamma^a{}_{bc} U^b U^c = 0, \tag{6}$$

where $U^a = \frac{dX^a}{dS}$ and the five-dimensional velocity conditions are fulfilled respectively

$$g_{ab} U^a U^b = 0, \tag{7}$$

$$g_{ab} U^a U^b = c^2, \tag{8}$$

for non-massive and massive test particles that moves on the 5D Ricci-flat metric (1).

3.1. Non-massive particles

For non-massive particles on the metric (1) the condition of 5D null-geodesics: $g_{ab}U^a U^b = 0$, can be written as