Physics Letters B 679 (2009) 350–354

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Small-x behavior of the structure function F_2 and its slope $\partial \ln F_2/\partial \ln(1/x)$ for "frozen" and analytic strong-coupling constants

Gorazd Cvetič [a], Alexey Yu. Illarionov [b,c], Bernd A. Kniehl [d,*], Anatoly V. Kotikov [d,1]

[a] Department of Physics, Universidad Técnica Federico Santa María, Avenida España 1680, Casilla 110-V, Valparaío, Chile
[b] International School for Advanced Studies SISSA, via Beirut 2–4, 34014 Trieste, Italy
[c] INFN, Sezione di Trieste, Trieste, Italy
[d] II. Institut für Theoretische Physik, Universität Hamburg, Luruper Chaussee 149, 22761 Hamburg, Germany

ARTICLE INFO

Article history:
Received 10 June 2009
Received in revised form 12 July 2009
Accepted 28 July 2009
Available online 3 August 2009
Editor: A. Ringwald

PACS:
12.38.Bx
13.60.Hb

Keywords:
Deep-inelastic scattering
Proton structure function

ABSTRACT

Using the leading-twist approximation of the Wilson operator product expansion with "frozen" and analytic versions of the strong-coupling constant, we show that the Bessel-inspired behavior of the structure function F_2 and its slope $\partial \ln F_2/\partial \ln(1/x)$ at small values of x, obtained for a flat initial condition in the DGLAP evolution equations, leads to good agreement with experimental data of deep-inelastic scattering at DESY HERA.

1. Introduction

The experimental data from DESY HERA on the structure function F_2 of deep-inelastic scattering (DIS) [1–14] and its derivatives $\partial F_2/\partial \ln Q^2$ [4,6,15] and $\partial \ln F_2/\partial \ln(1/x)$ [15–18] bring us into a very interesting kinematic range for testing theoretical ideas on the behavior of quarks and gluons carrying a very small fraction of the proton's momentum, the so-called small-x region. In this limit, one expects that the conventional treatment based on the Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP) equations [19–22] does not account for contributions to the cross section which are leading in $\alpha_s \ln(1/x)$; moreover, the parton density functions (PDFs), in particular the one of the gluon, become large, and the need arises to apply a high-density formulation of QCD.

However, reasonable agreement between HERA data and the next-to-leading-order (NLO) approximation of perturbative QCD has been observed for $Q^2 \gtrsim 2\ \mathrm{GeV}^2$ (see reviews in Refs. [23,24] and references cited therein) indicating that perturbative QCD can describe the evolution of F_2 and its derivatives down to very small Q^2 values, traditionally characterized by soft processes.

The standard program to study the x dependence of quark and gluon PDFs is to compare the numerical solutions of the DGLAP equations with the data and so to fit the parameters of the x profiles of the PDFs at some initial factorization scale Q_0^2 and the asymptotic scale parameter Λ. However, for analyzing exclusively the small-x region, there is the alternative of doing a simpler analysis by using some of the existing analytical solutions of the DGLAP equations in the small-x limit [25–28]. This was done in Ref. [25], where it was pointed out that the small-x data from HERA can be interpreted in terms of the so-called double-asymptotic-scaling (DAS) phenomenon related to the asymptotic behavior of the DGLAP evolution discovered in Ref. [29] many years ago.

The study of Ref. [25] was extended in Refs. [26–28] to include the subasymptotic part of the Q^2 evolution. This led to predictions [27,28] of the small-x asymptotic PDF forms in the framework of DGLAP dynamics starting at some initial value Q_0^2 with flat x distributions:

$$x f_a(x, Q_0^2) = A_a \quad (a = q, g), \tag{1}$$

where $f_a(x, Q^2)$ are the PDFs and A_a are unknown constants to be determined from the data. We refer to the approach of

* Corresponding author.
E-mail addresses: gorazd.cvetic@usm.cl (G. Cvetič), illario@sissa.it (A.Yu. Illarionov), kniehl@desy.de (B.A. Kniehl), kotikov@theor.jinr.ru (A.V. Kotikov).
[1] On leave of absence from Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, 141980 Dubna, Moscow region, Russia.

0370-2693/$ – see front matter
doi:10.1016/j.physletb.2009.07.057

Refs. [26–28] as *generalized* DAS approximation. In this approach, the flat initial conditions in Eq. (1) play the basic role of the singular parts of the anomalous dimensions by determining the small-x asymptotics, as in the standard DAS case, while the contributions from the finite parts of the anomalous dimensions and from the Wilson coefficients can be considered as subasymptotic corrections, which are, however, important for better agreement with the experimental data. In the present Letter, similarly to Refs. [25–28], we neglect the contribution from the non-singlet quark component.

The use of the flat initial condition given in Eq. (1) is supported by the actual experimental situation: small-Q^2 data [4,6,11,15,30–32] are well described for $Q^2 \leqslant 0.4\ \mathrm{GeV}^2$ by Regge theory with Pomeron intercept $\alpha_P(0) = 1 + \lambda_P = 1.08$ (see Ref. [33] and references cited therein), close to the standard one, $\alpha_P(0) = 1$. The small rise of the HERA data [4,6,11,13,15] at small values of Q^2 can be explained, for instance, by contributions of higher-twist operators [28].

The purpose of this Letter is to compare the predictions for the structure function $F_2(x, Q^2)$ and its slope $\partial \ln F_2/\partial \ln(1/x)$ from the generalized DAS approach with H1 and ZEUS experimental data [1–18]. Detailed inspection of the H1 data points [4,6,16] reveals that, in the ranges $x < 0.01$ and $Q^2 \gtrsim 2\ \mathrm{GeV}^2$, they exhibit a power-like behaviour of the form

$$F_2(x, Q^2) = Cx^{-\lambda(Q^2)}, \tag{2}$$

where the slope $\lambda(Q^2)$ is, to good approximation, independent of x and scales logarithmically with Q^2, as $\lambda(Q^2) = a \ln(Q^2/\Lambda^2)$. A fit yields $C \approx 0.18$, $a \approx 0.048$, and $\Lambda = 292$ MeV [16]. The linear rise of $\lambda(Q^2)$ with $\ln Q^2$ is also indicated in Figs. 2 and 3, to be discussed below.

The rise of $\lambda(Q^2)$ linearly with $\ln Q^2$ can be traced to strong nonperturbative physics (see Ref. [34] and references cited therein), i.e. $\lambda(Q^2) \sim 1/\alpha_s(Q^2)$. However, the analysis of Ref. [35] demonstrated that this rise can be explained naturally in the framework of perturbative QCD (see also Section 3).

The H1 and ZEUS Collaborations [15,17,18] also presented new data for $\lambda(Q^2)$ at quite small values of Q^2. As may be seen from Fig. 8 of Ref. [15], the ZEUS value for $\lambda(Q^2)$ is consistent with a constant of about 0.1 at $Q^2 \lesssim 0.6\ \mathrm{GeV}^2$, as is expected under the assumption of single-soft-Pomeron exchange within the framework of Regge phenomenology.

It is interesting to extend the analysis of Ref. [35] to the small-Q^2 range with the help of the well-known infrared modifications of the strong-coupling constant. We shall adopt the "frozen" [36] and analytic [37] versions.

This Letter is organized as follows. Section 2 contains basic formulae for the structure function F_2 and its slope $\partial \ln F_2/\partial \ln(1/x)$ in the generalized DAS approximation [27,28,35], which are needed for the present study. In Section 3, we compare our results on F_2 and $\partial \ln F_2/\partial \ln(1/x)$ with experimental data. Our conclusions may be found in Section 4.

2. Generalized DAS approach

The flat initial conditions in Eq. (1) correspond to the case when the PDFs tend to constants as $x \to 0$ at some initial value Q_0^2. The main ingredients of the results at the leading order (LO) [27,28] include the following.[2] Both, the gluon and quark-singlet PDFs are presented in terms of two components ("+" and "−"),

$$F_2(x, Q^2) = e x f_q(x, Q^2),$$

$$f_a(x, Q^2) = f_a^+(x, Q^2) + f_a^-(x, Q^2) \quad (a = q, g), \tag{3}$$

which are obtained from the analytic Q^2-dependent expressions of the corresponding ("+" and "−") PDF moments. Here, $e = (\sum_{i=1}^{f} e_i^2)/f$ is the average charge square and f is the number of active quark flavors. The small-x asymptotic results for the PDFs $f_a^\pm$ are

$$\begin{aligned}
x f_q^+(x, Q^2) &= \frac{f}{9}\left(A_g + \frac{4}{9}A_q\right)\rho \tilde{I}_1(\sigma) e^{-\bar{d}_+(1)s} + O(\rho),\\
f_g^+(x, Q^2) &= \frac{9\tilde{I}_0(\sigma)}{f\rho\tilde{I}_1(\sigma)} f_q^+(x, Q^2),\\
x f_q^-(x, Q^2) &= A_q e^{-d_-(1)s} + O(x),\\
f_g^-(x, Q^2) &= -\frac{4}{9} f_q^-(x, Q^2),
\end{aligned} \tag{4}$$

where $\bar{d}_+(1) = 1 + 20f/(27\beta_0)$ and $d_-(1) = 16f/(27\beta_0)$ are the regular parts of the anomalous dimensions $d_+(n)$ and $d_-(n)$, respectively, in the limit $n \to 1$.[3] Here, n is the variable in Mellin space. The functions $\tilde{I}_\nu$ ($\nu = 0, 1$) are related to the modified Bessel function I_ν and the Bessel function J_ν by

$$\tilde{I}_\nu(\sigma) = \begin{cases} I_\nu(\sigma), & \text{if } s \geqslant 0,\\ i^{-\nu} J_\nu(i\sigma), & \text{if } s < 0. \end{cases} \tag{5}$$

The variables s, σ, and ρ are given by

$$s = \ln\frac{\alpha_s^{\mathrm{LO}}(Q_0^2)}{\alpha_s^{\mathrm{LO}}(Q^2)}, \qquad \sigma = 2\sqrt{\hat{d}_+(s - i\epsilon)\ln x},$$
$$\rho = \frac{\sigma}{2\ln(1/x)}, \tag{6}$$

where $\hat{d}_+ = -12/\beta_0$, $\alpha_s^{\mathrm{LO}}(Q^2)$ is the strong-coupling constant in the LO approximation, and β_0 is the first term of its β function.

Contrary to the approach of Refs. [25–28], various groups were able to fit the available data using a hard input at small values of x, of the form $x^{-\lambda}$, with different values $\lambda > 0$ at small and large values of Q^2 [33,38–48]. At small Q^2 values, there are well-known such results [33]. At large Q^2 values, this is not very surprising for the modern HERA data because they cannot distinguish between the behavior based on a steep PDF input at quite large Q^2 values and the steep form acquired after the dynamical evolution from a flat initial condition at quite small Q^2 values.

As has been shown in Refs. [27,28], the x dependencies of F_2 and the PDFs given by the Bessel-like forms in the generalized DAS approach can mimic power-law shapes over a limited region of x and Q^2 values:

$$F_2(x, Q^2) \sim x^{-\lambda_{F_2}^{\mathrm{eff}}(x,Q^2)}, \qquad x f_a(x, Q^2) \sim x^{-\lambda_a^{\mathrm{eff}}(x,Q^2)}. \tag{7}$$

In the twist-two LO approximation, the effective slopes have the following forms:

$$\begin{aligned}
\lambda_{F_2}^{\mathrm{eff}}(x, Q^2) &= \lambda_q^{\mathrm{eff}}(x, Q^2) = \frac{f_q^+(x, Q^2)}{f_q(x, Q^2)}\rho\frac{\tilde{I}_2(\sigma)}{\tilde{I}_1(\sigma)},\\
\lambda_g^{\mathrm{eff}}(x, Q^2) &= \frac{f_g^+(x, Q^2)}{f_g(x, Q^2)}\rho\frac{\tilde{I}_1(\sigma)}{\tilde{I}_0(\sigma)}.
\end{aligned} \tag{8}$$

The corresponding NLO expressions and the higher-twist terms may be found in Refs. [27,28].

[2] The NLO results may be found in Refs. [27,28].

[3] We denote the singular and regular parts of a given quantity $k(n)$ in the limit $n \to 1$ by $\hat{k}(n)$ and $\bar{k}(n)$, respectively.