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Isolating the Roper resonance in lattice QCD

M.S. Mahbub [a,b,c,*], Alan Ó Cais [a,b,d], Waseem Kamleh [a,b], Ben G. Lasscock [a,b], Derek B. Leinweber [a,b], Anthony G. Williams [a,b]

[a] *Special Research Centre for the Subatomic Structure of Matter, Adelaide, South Australia 5005, Australia*
[b] *Department of Physics, University of Adelaide, South Australia 5005, Australia*
[c] *Department of Physics, Rajshahi University, Rajshahi 6205, Bangladesh*
[d] *Cyprus Institute, Guy Ourisson Builiding, Athalassa Campus, PO Box 27456, 1645 Nicosia, Cyprus*

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ABSTRACT

We present results for the first positive parity excited state of the nucleon, namely, the Roper resonance ($N\frac{1}{2}^{+} = 1440$ MeV) from a variational analysis technique. The analysis is performed for pion masses as low as 224 MeV in quenched QCD with the FLIC fermion action. A wide variety of smeared–smeared correlation functions are used to construct correlation matrices. This is done in order to find a suitable basis of operators for the variational analysis such that eigenstates of the QCD Hamiltonian may be isolated. A lower lying Roper state is observed that approaches the physical Roper state. To the best of our knowledge, the first time this state has been identified at light quark masses using a variational approach.

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One of the long-standing puzzles in hadron spectroscopy has been the low mass of the first positive parity, $J^P = \frac{1}{2}^+$, excitation of the nucleon, known as the Roper resonance $N^*(1440$ MeV). In constituent or valence quark models with harmonic oscillator potentials, the lowest-lying odd parity state naturally occurs below the $N = \frac{1}{2}^+$ state (with principal quantum number $N = 2$) [1,2] whereas, in nature the Roper resonance is almost 100 MeV below the $N = \frac{1}{2}^-(1535$ MeV) state. Similar difficulties in the level orderings appear for the $J^P = \frac{3}{2}^+ \Delta^*(1600)$ and $\frac{1}{2}^+ \Sigma^*(1690)$ resonances, which have led to the speculation that the Roper resonance may be more appropriately viewed as a hybrid baryon state with explicitly excited gluon field configurations [3,4], or as a breathing mode of the ground state [5] or states which can be described in terms of meson–baryon dynamics alone [6].

The first detailed analysis of the positive parity excitation of the nucleon was performed in Ref. [7] using Wilson fermions and an operator product expansion spectral ansatz. Since then several attempts have been made to address these issues in the lattice framework [8–17], but in many cases no potential identification of the Roper state has been made [8–12]. Recently however, in the analysis of [13,14,18], a low-lying Roper state has been identified using Bayesian techniques.

Here, we use a 'variational method' [19–21], which is based on a correlation matrix analysis and has been used quite extensively in Refs. [11,16,21–37]. Though the ground state mass of the nucleon has been described successfully, an unambiguous determination of the Roper state with this method has not been achieved in the past, though significant amounts of research have been carried out in Ref. [23], by the CSSM Lattice Collaboration [11,16,25], the BGR [26–29,33] Collaboration and in Refs. [36,37].

In this Letter, we present evidence of a low-lying Roper state for the first time using a variational analysis. The observed state displays chiral curvature and approaches the physical mass of the Roper state. The standard nucleon interpolating field χ_1 is considered in this analysis. Various sweeps of Gaussian smearing [38] are used to construct a smeared–smeared correlation function basis from which we obtain the correlation matrices.

* Address for correspondence: Department of Physics, University of Adelaide, South Australia 5005, Australia.
E-mail address: md.mahbub@student.adelaide.edu.au (M.S. Mahbub).

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The two point correlation function matrix for $\vec{p}=0$ can be written as

$$G_{ij}(t) = \left(\sum_{\vec{x}} \mathrm{Tr}_{\mathrm{sp}}\left\{\Gamma_\pm \langle\Omega|\chi_i(x)\bar{\chi}_j(0)|\Omega\rangle\right\}\right) \quad (1)$$

$$= \sum_\alpha \lambda_i^\alpha \bar{\lambda}_j^\alpha e^{-m_\alpha t}, \quad (2)$$

where, Dirac indices are implicit. Here, λ_i and $\bar{\lambda}_j$ are the couplings of interpolators χ_i and $\bar{\chi}_j$ at the sink and source, respectively. α enumerates the energy eigenstates with mass m_α.

Since the only t dependence comes from the exponential term, one can seek a linear superposition of interpolators, $\bar{\chi}_j u_j^\alpha$, such that (more detail can be found in Refs. [11,16]),

$$G_{ij}(t+\Delta t)\, u_j^\alpha = e^{-m_\alpha \Delta t}\, G_{ij}(t)\, u_j^\alpha, \quad (3)$$

for sufficiently large t and $t+\Delta t$, see Refs. [22] and [16]. Multiplying the above equation by $[G_{ij}(t)]^{-1}$ from the left leads to an eigenvalue equation,

$$\left[\left(G(t)\right)^{-1} G(t+\Delta t)\right]_{ij} u_j^\alpha = c^\alpha\, u_i^\alpha, \quad (4)$$

where $c^\alpha = e^{-m_\alpha \Delta t}$ is the eigenvalue. Similar to Eq. (4), one can also solve the left eigenvalue equation to recover the v^α eigenvector,

$$v_i^\alpha \left[G(t+\Delta t)\left(G(t)\right)^{-1}\right]_{ij} = c^\alpha v_j^\alpha. \quad (5)$$

The vectors u_j^α and v_i^α diagonalize the correlation matrix at time t and $t+\Delta t$ making the projected correlation matrix,

$$v_i^\alpha G_{ij}(t) u_j^\beta \propto \delta^{\alpha\beta}. \quad (6)$$

The parity projected, eigenstate projected correlator,

$$v_i^\alpha G_{ij}^\pm(t) u_j^\alpha \equiv G_\pm^\alpha, \quad (7)$$

is then analyzed using standard techniques to obtain masses of different states.

Our analysis is exploratory, seeking to develop techniques to access the Roper state in lattice gauge theory. Our lattice ensemble consists of 200 quenched configurations with a lattice volume of $16^3 \times 32$. Gauge field configurations are generated by using the DBW2 gauge action [39,40] and an $\mathcal{O}(a)$-improved FLIC fermion action [41] is used to generate quark propagators. This action has excellent scaling properties and provides near continuum results at finite lattice spacing [42]. The lattice spacing is $a = 0.1273$ fm, as determined by the static quark potential, with the scale set using the Sommer scale, $r_0 = 0.49$ fm [43]. In the irrelevant operators of the fermion action we apply four sweeps of stout-link smearing to the gauge links to reduce the coupling with the high frequency modes of the theory [44] providing $\mathcal{O}(a)$ improvement [42]. We use the same method as in Refs. [16,45] to determine fixed boundary effects, and the effects are significant only after time slice 25 in the present analysis. Various sweeps (1, 3, 7, 12, 16, 26, 35, 48 sweeps corresponding to rms radii, in lattice units, of 0.6897, 1.0459, 1.5831, 2.0639, 2.3792, 3.0284, 3.5237, 4.1868) of gauge invariant Gaussian smearing [38] are applied symmetrically at the source (at $t=4$) and at the sink. This is to ensure a variety of overlaps of the interpolators with the lower-lying states. The analysis is performed on ten different quark masses corresponding to pion masses $m_\pi = \{0.797, 0.729, 0.641, 0.541, 0.430, 0.380, 0.327, 0.295, 0.249, 0.224\}$ GeV. Error analysis is performed using a second-order single elimination jackknife method, where the χ^2/dof is obtained via a covariance matrix analysis method. Our fitting method is discussed extensively in Ref. [16].

The nucleon interpolator we consider is the local scalar-diquark interpolator having a non-relativistic reduction [7,46],

$$\chi_1(x) = \epsilon^{abc}\left(u^{Ta}(x) C\gamma_5 d^b(x)\right) u^c(x). \quad (8)$$

We consider several 4×4 matrices. Each matrix is constructed with different sets of correlation functions, each set element corresponding to a different numbers of sweeps of gauge invariant Gaussian smearing at the source and sink of the $\chi_1\bar{\chi}_1$ correlators. This provides a large basis of operators with varieties of overlap among energy states. We consider seven combinations $\{1=(1,7,16,35),\ 2=(3,7,16,35),\ 3=(1,12,26,48),\ 4=(3,12,26,35),\ 5=(3,12,26,48),\ 6=(12,16,26,35),\ 7=(7,16,35,48)\}$ of 4×4 matrices. In Ref. [16] it was shown that one cannot isolate a low-lying excited eigenstate using a single fixed-size source smearing. The superposition of states manifested itself as a smearing dependence of the effective mass. In this Letter we exploit this sensitivity to isolate the energy eigenstates.

In Fig. 1, we show the mass from the projected correlation functions and from the eigenvalues (as shown in Ref. [16]) for the fourth combination $(3,12,26,35)$ of 4×4 matrices. We note that similar results in mass from the projected correlation functions and the eigenvalues are observed in this analysis as in Ref. [16]. Though the mass of the first excited state from projected correlation functions show little change with variational parameters, see Fig. 1, the second and third excited states start a little below which indicates t and $t+\Delta t$ are not sufficiently large. With larger Euclidean times fewer states will contribute significantly to the correlators. The robust aspect of fitting projected correlators is manifest in Fig. 1, and reflects the stability of the eigenvectors against changes in t and $t+\Delta t$. In contrast, the mass from the eigenvalue analysis shows significant dependence on the variational parameters. The same method as described in Ref. [16] is applied in this Letter to extract the mass from the projected correlation functions.

In Fig. 2, masses extracted from all the combinations of 4×4 matrices (from 1st to 7th) are shown for the pion mass of 797 MeV. Some dependence of the excited states on smearing count is also observed here as in Ref. [16] for a few of the interpolator basis smearing sets. However the ground and first excited states are robust against changes in the interpolator basis, providing evidence that an energy eigenstate has been isolated. It should be noted that the highest excited state (the third excited state) is influenced more by the level of smearing than the lower excited states. This is to be expected as this state must accommodate all remaining spectral strength.

The 1st combination in Fig. 2 provides heavier excited states as this basis begins with a low number of smearing sweeps (a sweep count of 1) and also contains another low smearing set of 7 sweeps. The second and third excited states, and more importantly, the first excited state sits a bit high in comparison with the other bases. Hence, extracting masses with this basis is not as reliable as other sets. The 2nd combination also contains elements with a small smearing sweep count (3 and 7), hence this basis also provides heavier excited states and shows some systematic drift in the second excited state. However, this basis has reduced contamination from the excited states when compared with the first basis. The 3rd combination also starts at the low smearing count, so the mass from this basis for the third excited state is a little high.

We can observe at this point that including basis elements with a low smearing count will increase the masses of excited states (for instance, consecutive low numbers of smearing sweeps 1, 7 and 3, 7, respectively). This is because the correlation functions with these low sweep counts have a large overlap with several heavier excited states in their sub-leading exponential. We also observe

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