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$g_{B^*B\pi}$-coupling in the static heavy quark limit

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ABSTRACT

By means of QCD simulations on the lattice, we compute the coupling of the heavy–light mesons to a soft pion in the static heavy quark limit. The gauge field configurations used in this calculations include the effect of $N_f = 2$ dynamical Wilson quarks, while for the static quark propagator we use its improved form (so-called HYP). On the basis of our results we obtain that the coupling $\hat{g} = 0.44 \pm 0.03^{+0.07}_{-0.00}$, where the second error is flat (not Gaussian).

1. Introduction

The static quark limit of QCD offers a simplified framework to solving the non-perturbative dynamics of light degrees of freedom in the heavy–light systems. That dynamics is constrained by heavy quark symmetry (HQS): it is blind to the heavy quark flavor and its spin. As a result the total angular momentum of the light degrees of freedom becomes a good quantum number (j_ℓ^P), and therefore the physical heavy–light mesons come in mass-degenerate doublets. In phenomenological applications the most interesting information involves the lowest lying doublet, the one with $j_\ell^P = (1/2)^-$, consisting of a pseudoscalar and a vector meson, such as (B_q, B_q^*) or (D_q, D_q^*) states, where $q \in \{u, d, s\}$. When studying any phenomenologically interesting quantity from the QCD simulations on the lattice that includes heavy–light mesons (decay constants, various form factors, bag parameters and so on), one of the major sources of systematic uncertainty is related to the necessity to make chiral extrapolations. The reason is that the physical light quarks, which are expected to most significantly modify the structure of the QCD vacuum, are much lighter than the ones that are directly simulated on the lattice, $m_q \gg m_{u,d}$. Here by "q" we label the light quark masses that are attainable from the lattice. Since the QCD dynamics with very light quarks is bound to be strongly affected by the effects of spontaneous chiral symmetry breaking, a more suitable (theoretically more controllable) way to guide such extrapolations is by using the expressions derived in heavy meson chiral perturbation theory (HMChPT), which is an effective theory built on the combination of HQS and the spontaneous chiral symmetry breaking $[SU(N_f)_L \otimes SU(N_f)_R \to SU(N_f)_V]$. Its Lagrangian is given by [1]

$$\mathcal{L}_{\rm heavy} = -\,{\rm tr}_a\,{\rm Tr}[\bar{H}_a i v\cdot D_{ba} H_b] + \hat{g}\,{\rm tr}_a\,{\rm Tr}\big[\bar{H}_a H_b \gamma_\mu \mathbf{A}^\mu_{ba}\gamma_5\big],$$
$$D^\mu_{ba}H_b = \partial^\mu H_a - H_b\frac{1}{2}\big[\xi^\dagger\partial_\mu\xi + \xi\partial_\mu\xi^\dagger\big]_{ba},$$
$$\mathbf{A}^{ab}_\mu = \frac{i}{2}\big[\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger\big]_{ab}, \tag{1}$$

where

$$H_a(v) = \frac{1+\not{v}}{2}\big[P^{*a}_\mu(v)\gamma_\mu - P^a(v)\gamma_5\big], \tag{2}$$

is the heavy meson doublet field containing the pseudoscalar, $P^a(v)$, and the vector meson field, $P^{*a}(v)$. In the above formulas, the indices a, b run over the light quark flavors, $\xi = \exp(i\Phi/f)$, with Φ being the matrix of $(N_f^2 - 1)$ pseudo-Goldstone bosons, and "f" is the pion decay constant in the chiral limit. We see that the term connecting the Goldstone boson ($\mathbf{A}_\mu$) with the heavy-meson doublet $[H(v)]$ is proportional to the coupling $\hat{g}$, which will therefore enter into every expression related to physics of heavy–light mesons with $j_\ell^P = (1/2)^-$ when the chiral loop corrections are included.[1] Being the parameter of effective theory, its value cannot be predicted but should be fixed in some other way. It can

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[1] A special attention should be given to the problem related to the presence of the nearby excited states as discussed in Ref. [2]. Any precision lattice calculation

be related to the measured decay width $\Gamma(D^*\to D\pi)$ [3], with the resulting value $\hat{g}_{\rm charm}=0.61(7)$. That value turned out to be much larger than predicted by all of the QCD sum rule approaches [4], but consistent with some model predictions such as the one in Ref. [5], in which a more detailed list of predictions with their references can be found. The large value for $g_{D^*D\pi}$-coupling was confirmed by the quenched lattice QCD study in Ref. [6], and recently also in the unquenched case [7]. Since the charm quark is not very heavy, the use of $g_{D^*D\pi}$ to fix the value of $\hat{g}$-coupling, via

$$\hat{g}=\frac{g_{D^*D\pi}}{2\sqrt{m_D m_{D^*}}}f_\pi\,, \tag{3}$$

and its use in chiral extrapolations of the quantities relevant to B-physics phenomenology may be dangerous mainly because of the potentially large $\mathcal{O}(1/m_c^n)$-corrections. Unfortunately the decay $B^*\to B\pi$ is kinematically forbidden and therefore, to determine the size of $\hat{g}$, we have to resort to a non-perturbative approach to QCD. Unlike for the computation of the heavy-to-light form factors, QCD sum rules proved to be inadequate when computing $g_{D^*D\pi}$, most likely because of the use of double dispersion relations when the radial excitations should be explicitly included in the analysis, as claimed in Ref. [8]. In this Letter, instead, we compute the $\hat{g}$-coupling on the lattice by using the unquenched gauge field configurations, with $N_f=2$ dynamical light quarks, and in the static heavy quark limit. The attempts to compute this coupling in this limit were made in Ref. [9], and very recently in Ref. [10]. On the basis of the currently available information, the coupling $\hat{g}$ in the static limit is indeed smaller than the one obtained in the charmed heavy quark case.

In the remainder of this Letter we will briefly describe the standard strategy to compute this coupling, list the correlation functions that are being computed to extract the bare coupling $\hat{g}_q$, as well as the axial vector renormalization constants. We then give details concerning the gauge field configurations used in this work, and present our results.

2. Definitions and correlation functions to be computed

In the limit in which the heavy quark is infinitely heavy and the light quarks massless, the axial coupling of the charged pion to the lowest lying doublet of heavy–light mesons, $\hat{g}$, is defined via [9]

$$\langle B|\vec{A}|B^*(\varepsilon)\rangle=\hat{g}\vec{\varepsilon}_\lambda, \tag{4}$$

where the non-relativistic normalisation of states $|B^{(*)}\rangle$ is assumed, $\langle B_a(v)|B_b(v')\rangle=\delta_{ab}\delta(v-v')$. For the heavy–light hadrons at rest ($\vec{v}=\vec{v}\,'=\vec{0}$), the soft pion that couples to the axial current, $A_\mu=\bar{u}\gamma_\mu\gamma_5 d$, is at rest too, $|\vec{q}|=0$. ε_μ^λ is the polarisation of the vector static-light meson. In the typical situation on the lattice we are away from the chiral limit ($\hat{g}\to\hat{g}_q$), and the coupling $\hat{g}_q$ becomes the axial form factor whose value should be extrapolated to the chiral limit, in which the soft pion theorem relating the matrix element of the axial current to the pionic coupling applies [9].

The standard strategy to compute the above matrix element on the lattice consists in evaluating the following correlation functions:

$$C_2(t)=\left\langle\sum_{\vec{x}}P(x)P^\dagger(0)\right\rangle_U\overset{\rm HQS}{=}\frac{1}{3}\left\langle\sum_{i,\vec{x}}V_i(x)V_i^\dagger(0)\right\rangle_U$$

$$=\left\langle\sum_{\vec{x}}{\rm Tr}\left[\frac{1+\gamma_0}{2}W_x^0\gamma_5\mathcal{S}_{u,d}(0,x)\gamma_5\right]\right\rangle_U,$$

$$C_3(t_y,t_x)=\left\langle\sum_{i,\vec{x},\vec{y}}V_i(y)A_i(x)P^\dagger(0)\right\rangle_U$$

$$=\left\langle\sum_{\vec{x},\vec{y}}{\rm Tr}\left[\frac{1+\gamma_0}{2}W_0^y\gamma_i\mathcal{S}_u(y,x)\gamma_i\gamma_5\mathcal{S}_d(x,0)\gamma_5\right]\right\rangle_U, \tag{5}$$

where $\langle\cdots\rangle_U$ denotes the average over independent gauge field configurations, the interpolating fields are $P=\bar{h}\gamma_5 q$, $V_i=\bar{h}\gamma_i q$, with $h(x)$ and $q(x)$ the static heavy and the light quark field, respectively. In what follows, we drop the dependence on t_y. In practice its value is fixed to one or several values as it will be specified in the text. In Eq. (5) we also expressed the correlation functions in terms of quark propagators: the light ones, $\mathcal{S}_q(x,y)$, and the static heavy one, which becomes a Wilson line,

$$W_x^y=\delta(\vec{x}-\vec{y})\prod_{\tau=t_y}^{t_x-1}U_0^{\rm impr.}(\tau,\vec{x}). \tag{6}$$

The latter is merely obtained from the discretized static heavy quark action [11]

$$\mathcal{L}_{\rm HQET}=\sum_x h^\dagger(x)\big[h(x)-U_0^{\rm impr.}(x-\hat{0})^\dagger h(x-\hat{0})\big], \tag{7}$$

where for $U_0^{\rm impr.}$, the time component of the link variable, we use its improved form, obtained after applying the hyper-cubic blocking procedure on the original link variable, with the parameters optimized in a way described in Ref. [12], namely with $\vec{\alpha}=(0.75,0.6,0.3)$. That step is essential as it ensures the exponential improvement of the signal to noise ratio in the correlation functions with respect to what is obtained by using the simple product of link variables [13].

The spectral decomposition of the three point function, given in Eq. (5), reads

$$C_3(t_x)=\sum_{m,n}\big[\mathcal{Z}_n e^{-\mathcal{E}_n^q t_y}\langle B_n|A_i|B_m^*\rangle e^{-(\mathcal{E}_m^q-\mathcal{E}_n^q)t_x}\mathcal{Z}_m\varepsilon_i^{(m)}\big],$$

where the sum includes not only the ground states ($m=n=0$) but also their radial excitations ($m,n>0$), which are heavier and thus exponentially suppressed. Note a shorthand notation, $\mathcal{Z}_n=|\langle 0|h^\dagger\gamma_5 q|B_q\rangle|$, and the fact that we do not distinguish $\mathcal{Z}_n$ from couplings to the vector interpolating operator because of the HQS. If the non-diagonal terms in the above sum were important ($n\neq m$) the correlation function $C_3(t_x)$ would exhibit some exponential dependence in t_x. In practice, it appears that the correlation functions $C_3(t_x)$, as defined in Eq. (5) are very flat (t_x-independent) for all the data sets that we use in this work and the details of which will be given in the next section (c.f. Fig. 1). This observation in fact agrees with what one can deduce from various quark models, and in particular from the one in Ref. [5]. We will therefore discard the non-diagonal terms in the spectral decomposition of $C_3(t_x)$. We are still left with the problem of contamination of the desired signal ($n=0$) by the axial transitions among radial excitations, $n=m>0$. To solve that problem we should employ some smearing procedure and suppress the couplings of the source operators to the radial excitations. To that purpose we use the smearing technique proposed in Ref. [14], which essentially means that – in Eq. (5) – the interpolating fields are replaced by $\bar{h}(x)\gamma_5 q(x)\to\bar{h}(x)\gamma_5 q^S(x)$, and similarly for the source of the heavy–light vector mesons, where

cannot be fully trusted if the chiral extrapolations are made without discussing the problem of discerning the mixing with the $j_\ell^P=(1/2)^+$ states in the chiral loop diagrams.