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Thermodynamics in $F(R)$ gravity with phantom crossing

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ABSTRACT

We study thermodynamics of the apparent horizon in $F(R)$ gravity. In particular, we demonstrate that an $F(R)$ gravity model with realizing a crossing of the phantom divide can satisfy the second law of thermodynamics in the effective phantom phase as well as non-phantom one.

1. Introduction

Recently, there have been more and more evidences to support that the current expansion of the universe is accelerating [1,2]. The scenarios to explain the current accelerated expansion of the universe fall into two broad categories [3–12]. One is to introduce "dark energy" in the framework of general relativity. The other is to study a modified gravitational theory, such as $F(R)$ gravity, in which the action is described by an arbitrary function $F(R)$ of the scalar curvature R (for reviews, see [8–12]).

On the other hand, various observational data [13] imply that the ratio of the effective pressure to the effective energy of the universe, i.e., the effective equation of state (EoS) $w_{\mathrm{eff}} \equiv p_{\mathrm{eff}}/\rho_{\mathrm{eff}}$, may evolve from larger than -1 (non-phantom phase) to less than -1 (phantom phase [14]). Namely, it crosses -1 (the phantom divide) at the present time or in near future. Recently, an explicit model of $F(R)$ gravity with realizing a crossing of the phantom divide has been constructed in Ref. [15]. We note that the phantom crossing in the framework of general relativity has also been studied in the literature, e.g., "quintom" model [16].

It is believed that a modified gravitational theory must pass cosmological bounds and solar system tests because it corresponds to an alternative theory of gravitation to general relativity. However, at the initial studies of $F(R)$ gravity, models proposed in Refs. [17–20] with the powers of the scalar curvature are strongly constrained. In recent years, various investigations for viable models of $F(R)$ gravity [21–33] have been executed. A curvature singularity problem in $F(R)$ gravity has also been discussed in Refs. [34,35].

As another touchstone of modified gravity, it is interesting to examine whether the second law of thermodynamics can be satisfied in the models of $F(R)$ gravity. The connection between gravitation and thermodynamics was examined by following black hole thermodynamics (black hole entropy [36] and temperature [37]) and its application to the cosmological event horizon of de Sitter space [38]. It was shown that Einstein equation is derived from the proportionality of the entropy to the horizon area together with the fundamental thermodynamic relation, such as the Clausius relation [39]. This idea has been employed in a cosmological context [40–43]. It was demonstrated that if the entropy of the apparent horizon in the Friedmann–Robertson–Walker (FRW) spacetime is proportional to the apparent horizon area, Friedmann equations follow from the first law of thermodynamics [43]. The equivalent considerations for the FRW universe with the viscous fluid have also been studied [44].

In addition, it was proposed [45] that in $F(R)$ gravity, a non-equilibrium thermodynamic treatment should be required in order to derive the corresponding gravitational field equation by using the procedure in Ref. [39]. It was reconfirmed in Ref. [46] in $F(R)$ gravity as well as Ref. [47] in scalar–tensor theories. The first [48] and second [49] laws of thermodynamics on the apparent horizon in generalized theories of gravitation have recently been analyzed by taking into account the non-equilibrium thermodynamic treatment. Reinterpretations of the non-equilibrium correction [45] through the introduction of a mass-like function [50] and other ap-

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proaches [51,52] have also been explored. Incidentally, the horizon entropy in four-dimensional modified gravity [53] and a quantum logarithmic correction to the expression of the horizon entropy in a cosmological context [54–56] have been examined. Moreover, the first law of the ordinary equilibrium thermodynamics in $F(R)$ gravity, scalar–tensor theories, the Gauss–Bonnet gravity and more general Lovelock gravity have been discussed in Refs. [57–60], while the corresponding studies on the second law in the accelerating universe, $F(R)$ gravity, the Gauss–Bonnet gravity and the Lovelock gravity have been done in Refs. [61–63], respectively. Studies of thermodynamics in braneworld scenario [64–66] as well as its properties of dark energy [67] have also been performed.

It was pointed out in Ref. [68] that thermodynamics in the phantom phase usually leads to a negative entropy (see also [69]). Moreover, it was noted [70] that in the framework of general relativity the horizon entropy decreases in phantom models. However, the conditions that the black hole entropy can be positive in the $F(R)$ gravity models [21–23] with the solar-system tests have been analyzed in Ref. [71]. These recent studies have motivated us to explore whether in the framework of $F(R)$ gravity the second law of thermodynamics can be satisfied in the phantom phase. To illustrate the point, in the present Letter we consider an $F(R)$ gravity model with a crossing of the phantom divide [15] as it contains an effective phantom phase. We use units of $k_B = c = \hbar = 1$ and denote the gravitational constant $8\pi G$ by $\kappa^2 \equiv 8\pi/M_{Pl}{}^2$ with the Planck mass of $M_{Pl} = G^{-1/2} = 1.2 \times 10^{19}$ GeV.

The Letter is organized as follows. In Section 2, we explain the first and second laws of thermodynamics in $F(R)$ gravity. In Section 3, we demonstrate that the model of $F(R)$ gravity with the phantom crossing [15] can satisfy the generalized second law of the thermodynamics. Finally, conclusions are given in Section 4.

2. Thermodynamics in $F(R)$ gravity

In this section, we study the first and second laws of thermodynamics of the apparent horizon in $F(R)$ gravity. We consider the four-dimensional flat spacetime.

2.1. $F(R)$ gravity

The action of $F(R)$ gravity with matter is written as

$$I = \int d^4x \sqrt{-g}\left[\frac{F(R)}{2\kappa^2} + \mathcal{L}_{\text{matter}}\right], \tag{2.1}$$

where g is the determinant of the metric tensor $g_{\mu\nu}$ and $\mathcal{L}_{\text{matter}}$ is the matter Lagrangian. From the action in Eq. (2.1), the field equation of modified gravity is given by

$$F'(R)R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}F(R) + g_{\mu\nu}\Box F'(R) - \nabla_\mu\nabla_\nu F'(R) = \kappa^2 T^{(\text{matter})}_{\mu\nu}, \tag{2.2}$$

where the prime denotes differentiation with respect to R, ∇_μ is the covariant derivative operator associated with $g_{\mu\nu}$, $\Box \equiv g^{\mu\nu}\nabla_\mu\nabla_\nu$ is the covariant d'Alembertian for a scalar field, $R_{\mu\nu}$ is the Ricci curvature tensor, and $T^{(\text{matter})}_{\mu\nu} = \text{diag}(\rho, p, p, p)$ is the contribution to the energy–momentum tensor from all ordinary matters with ρ and p being the energy density and pressure of all ordinary matters, respectively.

We assume the flat Friedmann–Robertson–Walker (FRW) space-time with the metric,

$$ds^2 = -dt^2 + a^2(t)\gamma_{ij}\,dx^i\,dx^j, \tag{2.3}$$

$$\gamma_{ij}\,dx^i\,dx^j = dr^2 + r^2\,d\Omega^2, \tag{2.4}$$

where $a(t)$ is the scale factor and $d\Omega^2$ is the metric of two-dimensional sphere with unit radius. In the FRW background, from $(\mu,\nu) = (0,0)$ and the trace part of $(\mu,\nu) = (i,j)$ $(i,j = 1,\dots,3)$ components in Eq. (2.2), we obtain the gravitational field equations:

$$H^2 = \frac{\kappa^2}{3F'(R)}(\rho + \rho_c), \tag{2.5}$$

$$\dot{H} = -\frac{\kappa^2}{2F'(R)}(\rho + p + \rho_c + p_c), \tag{2.6}$$

where ρ_c and p_c can be regarded as the energy density and pressure generated due to the difference of $F(R)$ gravity from general relativity, given by

$$\rho_c = \frac{1}{\kappa^2}\left[\frac{1}{2}\big(-F(R) + RF'(R)\big) - 3H\dot{R}F''(R)\right], \tag{2.7}$$

$$p_c = \frac{1}{\kappa^2}\left[\frac{1}{2}\big(F(R) - RF'(R)\big) + (2H\dot{R} + \ddot{R})F''(R) + \dot{R}^2F'''(R)\right], \tag{2.8}$$

respectively, with the scalar curvature of $R = 6(\dot{H} + 2H^2)$. Here, $H = \dot{a}/a$ is the Hubble parameter and the dot denotes the time derivative of $\partial/\partial t$. We define the effective energy density and pressure of the universe as $\rho_{\text{eff}} \equiv \rho_t/F'(R)$ and $p_{\text{eff}} \equiv p_t/F'(R)$ with $\rho_t = \rho + \rho_c$ and $p_t = p + p_c$, respectively. Hence, from Eqs. (2.5) and (2.6) we see that even in $F(R)$ gravity, the gravitational field equations are expressed as $H^2 = \kappa^2\rho_{\text{eff}}/3$ and $\dot{H} = -\kappa^2(\rho_{\text{eff}} + p_{\text{eff}})/2$, which are the same as those in general relativity.

The continuity equation in terms of the effective energy density and pressure of the universe is given by

$$\dot{\rho}_{\text{eff}} + 3H(\rho_{\text{eff}} + p_{\text{eff}}) = 0. \tag{2.9}$$

Similarly, the (semi-)continuity equation of ordinary matters has the form

$$\dot{\rho} + 3H(\rho + p) = q. \tag{2.10}$$

One can take $q = 0$ because the gravity is determined only by ordinary matters. Assuming that the energy fluid, generated from the modification of gravity, behaves as a perfect fluid, we have similar semi-continuity equations as

$$\dot{\rho}_c + 3H(\rho_c + p_c) = q_c, \tag{2.11}$$

$$\dot{\rho}_t + 3H(\rho_t + p_t) = q_t, \tag{2.12}$$

where q_c and $q_t(= q + q_c)$ are quantities of expressing energy exchange. Using Eqs. (2.5), (2.6) and (2.12), we obtain

$$q_t = \frac{3}{\kappa^2}H^2\frac{\partial F'(R)}{\partial t}. \tag{2.13}$$

Clearly, from Eq. (2.13), we find that $q_t = 0$ for general relativity with $F(R) = R$, whereas q_t does not generally vanish in $F(R)$ gravity since there could exist some energy exchange with the horizon.

2.2. First law of thermodynamics

We now illustrate the first law of thermodynamics in $F(R)$ gravity. By using the spherical symmetry, the metric (2.3) can be written as

$$ds^2 = h_{\alpha\beta}\,dx^\alpha\,dx^\beta + \tilde{r}^2\,d\Omega^2, \tag{2.14}$$