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Photon pair production at flavour factories with per mille accuracy

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ABSTRACT

We present a high-precision QED calculation, with 0.1% theoretical accuracy, of two photon production in e^+e^- annihilation, as required by more and more accurate luminosity monitoring at flavour factories. The accuracy of the approach, which is based on the matching of exact next-to-leading order corrections with a QED Parton Shower algorithm, is demonstrated through a detailed analysis of the impact of the various sources of radiative corrections to the experimentally relevant observables. The calculation is implemented in the latest version of the event generator BabaYaga, available for precision simulations of photon pair production at e^+e^- colliders of moderately high energies.

1. Introduction

The precision measurement of the hadron production cross section in e^+e^- annihilation at flavour factories, such as Φ, τ-charm and B-factories, requires a detailed knowledge of the collider luminosity [1]. It can be derived by counting the number of events of a given reference process and normalizing this number to the corresponding theoretical cross section [2]. It follows that, in order to maintain small the total luminosity error given by the sum in quadrature of the relative experimental and theoretical uncertainty, the reference process must be a reaction with high statistics and calculable with an accuracy as high as possible. For this reason, the luminosity monitoring processes employed at flavour factories are QED processes, namely Bhabha scattering, two photon and muon pair production. In particular, at DAΦNE, VEPP-2M and PEP-II the large-angle Bhabha process is primarily used and the other reactions are measured as cross checks [2–4], while at CESR [5] all the three processes are considered and the luminosity is derived as an appropriate average of the measurements of the three QED reactions.

At DAΦNE, a comparison between the luminosity measurement using Bhabha events and the process $e^+e^- \to \gamma\gamma$ shows very good agreement, the average difference in a run-by-run comparison being 0.3% [6]. This precision necessarily demands progress on the theory side, since the Monte Carlo (MC) programs used for the simulation of photon pair production, i.e. BabaYaga v3.5 [7,8] and BKQED [9], have a theoretical precision of about 1%. Actually, the original formulation of BabaYaga is based on a QED Parton Shower (PS) approach for the treatment of leading logarithmic (LL) QED corrections and, as such, it lacks the effect of $\mathcal{O}(\alpha)$ non-log contributions, which are important to achieve a precision at the per mille level. On the other hand, the generator BKQED relies on an exact $\mathcal{O}(\alpha)$ diagrammatic calculation, therefore neglecting the contribution of higher-order LL corrections, which have been already demonstrated to be necessary for $\mathcal{O}(0.1\%)$ luminosity monitoring at flavour factories [3,7,10]. Because of this motivation, the aim of the present Letter is to describe a high-precision calculation of photon pair production in QED, based on the matching of exact next-to-leading-order (NLO) corrections with the QED PS algorithm, along the lines of the approach already developed for the Bhabha process in Ref. [10]. This will allow a reduction of the theoretical error in luminosity measurements at flavour factories, as demanded, in addition to precision measurements of the hadronic cross section, by improved experimental determinations of the $e^+e^- \to \tau^+\tau^-$ cross section at low energies [11], impor-

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tant for precision calculations of the anomalous magnetic moment of the muon. Furthermore, a precise knowledge of e^+e^- annihilation in two photons is of interest for estimates of the background to neutral meson production. We do not include in our calculation pure weak corrections, which have been computed in Ref. [12] and turn out to be important at very high energies, well above the energy range explored by flavour factories. For completeness, it is worth mentioning that an independent calculation, including exact $\mathcal{O}(\alpha)$ contributions supplemented with higher-order LL terms through collinear QED structure functions [13], of the relevant corrections to $e^+e^- \to \gamma\gamma$ at moderately high energies was performed in Ref. [14], recently revisited in Ref. [15].

The outline of the Letter is as follows. In Section 2 we describe the matching algorithm for the $e^+e^- \to \gamma\gamma$ process, while in Section 3 we provide numerical results, both for integrated cross sections and differential distributions of experimental interest, in order to discuss the effects of the various sources of radiative corrections and provide evidence for the per mille accuracy of the approach. Conclusions and possible perspectives are drawn in Section 4.

2. Theoretical formulation

The cross section of the photon pair production process, with the additional emission of an arbitrary number of photons, can be written in the LL approximation as follows

$$d\sigma_{\rm LL}^{\infty} = \Pi^2\big(Q^2,\epsilon\big)\sum_{n=0}^{\infty}\frac{1}{n!}|\mathcal{M}_{n,\rm LL}|^2\, d\Phi_n, \tag{1}$$

where $\Pi(Q^2,\epsilon)$ is the Sudakov form factor accounting for the soft-photon (up to an energy equal to ϵ in units of the incoming fermion energy E) and virtual emission, ϵ is an infrared separator dividing soft and hard radiation and Q^2 is related to the energy scale of the hard-scattering process. In our calculation, Q^2 is fixed to be equal to the squared centre of mass (c.m.) energy s, by comparing with the exact $\mathcal{O}(\alpha)$ calculation of Ref. [9]. $|\mathcal{M}_{n,\rm LL}|^2$ is the squared amplitude in LL approximation describing the process with the emission of n additional hard photons, with energy larger than ϵ in units of E, with respect to the lowest-order approximation $e^+e^- \to \gamma\gamma$. $d\Phi_n$ is the exact phase space element of the process (divided by the incoming flux factor), with the emission of n additional photons with respect to the Born-like final state configuration. The Sudakov form factor, which is defined as

$$\Pi\big(Q^2,\epsilon\big) = \exp\left(-\frac{\alpha}{2\pi}\mathcal{I}_+ L\right), \tag{2}$$

where

$$L = \log\frac{Q^2}{m^2}, \qquad \mathcal{I}_+ = \int_0^{1-\epsilon} dz\,\mathcal{P}(z), \tag{3}$$

appears in Eq. (1) to the second power to account for the presence of two charged particles in the initial state. In Eq. (3) $\mathcal{P}(z)$ is the electron $\to$ electron + photon splitting function $\mathcal{P}(z) = (1+z^2)/(1-z)$.

The cross section as calculated in Eq. (1) has the advantage that the photonic corrections, in LL approximation, are resummed up to all orders in perturbation theory. On the other hand, the weak point of Eq. (1) is that its $\mathcal{O}(\alpha)$ expansion does not coincide with the exact $\mathcal{O}(\alpha)$ (NLO) result. Actually, we have

$$\begin{aligned} d\sigma_{\rm LL}^{\alpha} &= \left(1-\frac{\alpha}{\pi}\mathcal{I}_+\ln\frac{Q^2}{m^2}\right)|\mathcal{M}_0|^2\, d\Phi_0 + |\mathcal{M}_{1,\rm LL}|^2\, d\Phi_1 \\ &\equiv (1+C_{\alpha,\rm LL})|\mathcal{M}_0|^2\, d\Phi_0 + |\mathcal{M}_{1,\rm LL}|^2\, d\Phi_1, \end{aligned} \tag{4}$$

whereas an exact NLO can be always cast in the form

$$d\sigma^{\alpha} = (1+C_{\alpha,\rm SV})|\mathcal{M}_0|^2\, d\Phi_0 + |\mathcal{M}_1|^2\, d\Phi_1, \tag{5}$$

where the coefficient $C_{\alpha,\rm SV}$ is equal to the exact squared amplitude of the annihilation process, in the presence of soft and virtual radiative corrections [9,14], in units of the exact Born squared amplitude $|\mathcal{M}_0|^2$, and $|\mathcal{M}_1|^2$ is the exact squared matrix element of the radiative process $e^+e^- \to \gamma\gamma\gamma$ [16]. The matching of the LL and NLO calculation can be obtained considering the correction factors (free of infrared and collinear singularities)

$$F_{\rm SV} = 1 + (C_{\alpha,\rm SV} - C_{\alpha,\rm LL}), \qquad F_{\rm H} = 1 + \frac{|\mathcal{M}_1|^2 - |\mathcal{M}_{1,\rm LL}|^2}{|\mathcal{M}_{1,\rm LL}|^2}. \tag{6}$$

As can be seen, the exact $\mathcal{O}(\alpha)$ cross section as in Eq. (5) can be expressed, up to terms of $\mathcal{O}(\alpha^2)$ and in terms of its LL approximation, as

$$d\sigma^{\alpha} = F_{\rm SV}(1+C_{\alpha,\rm LL})|\mathcal{M}_0|^2\, d\Phi_0 + F_{\rm H}|\mathcal{M}_{1,\rm LL}|^2\, d\Phi_1. \tag{7}$$

A similar procedure, repeated to all orders in α, leads to the correction of Eq. (1), which becomes

$$d\sigma_{\rm matched}^{\infty} = F_{\rm SV}\Pi^2\big(Q^2,\epsilon\big)\sum_{n=0}^{\infty}\frac{1}{n!}\left(\prod_{i=0}^{n}F_{{\rm H},i}\right)|\mathcal{M}_{n,\rm LL}|^2\, d\Phi_n, \tag{8}$$

where

$$F_{{\rm H},i} = 1 + \frac{|\mathcal{M}_i|^2 - |\mathcal{M}_{i,\rm LL}|^2}{|\mathcal{M}_{i,\rm LL}|^2}, \tag{9}$$

with $|\mathcal{M}_i|^2$ and $|\mathcal{M}_{i,\rm LL}|^2$ squared matrix elements, exact and in the LL approximation, respectively, relative to the emission of the ith hard bremsstrahlung photon. The expansion at $\mathcal{O}(\alpha)$ of Eq. (8) coincides now with the exact NLO cross section of Eq. (5) and higher-order LL contributions are the same as in Eq. (1).

3. Numerical results

3.1. Integrated cross sections: Technical tests and radiative corrections

The calculation of QED corrections requires the introduction of the unphysical soft-hard separator ϵ. Therefore, the independence of the predictions for the QED corrected cross section from variation of such a parameter has to be proved, for sufficiently small ϵ values. This is successfully demonstrated, at a precision level of $\sim 0.01\%$, in Figs. 1 and 2, which show the cross section of the photon pair production process, obtained according to the exact $\mathcal{O}(\alpha)$ cross section of Eq. (5) (Fig. 1) and to the matched cross section of Eq. (8) (Fig. 2), as a function of ϵ from 10^{-2} to 10^{-6}. The numerical results shown in Figs. 1 and 2 correspond to the following experimental set up, which models, up to a good accuracy, the selection criteria adopted by KLOE Collaboration at DAΦNE [17]

$$\begin{cases} \sqrt{s} = 1.02\ \text{GeV}, \\ E_\gamma^{\min} = 0.3\ \text{GeV}, \\ \vartheta_\gamma^{\min} = 45^\circ, \\ \vartheta_\gamma^{\max} = 135^\circ, \\ \xi_{\max} = 10^\circ, \end{cases} \tag{10}$$

where $E_\gamma^{\min}$ is the minimum energy threshold for the detection of at least two photons, $\vartheta_\gamma^{\min,\max}$ are the angular acceptance cuts and $\xi_{\max}$ is the maximum acollinearity between the most energetic and next-to-most energetic photon.

As a further test of the approach, we checked that our results for the NLO corrections agree at the 0.1% level with those quoted in Ref. [9] for the exact $\mathcal{O}(\alpha)$ relative corrections to the totally