Available online at www.sciencedirect.com
ScienceDirect
Physics Letters B 641 (2006) 401–404
PHYSICS LETTERS B
www.elsevier.com/locate/physletb

A tale of two skyrmions: The nucleon's strange quark content in different large N_c limits

Aleksey Cherman, Thomas D. Cohen *

Department of Physics, University of Maryland, College Park, MD 20742, USA

Received 17 July 2006; accepted 16 August 2006

Available online 28 August 2006

Editor: B. Grinstein

Abstract

The nucleon's strange quark content comes from closed quark loops, and hence should vanish at leading order in the traditional large N_c (TLNC) limit. Quark loops are not suppressed in the recently proposed orientifold large N_c (OLNC) limit, and thus the strange quark content should be non-vanishing at leading order. The Skyrme model is supposed to encode the large N_c behavior of baryons, and can be formulated for both of these large N_c limits. There is an apparent paradox associated with the large N_c behavior of strange quark matrix elements in the Skyrme model. The model only distinguishes between the two large N_c limits via the N_c scaling of the couplings and the Witten–Wess–Zumino term, so that a vanishing leading order strange matrix element in the TLNC limit implies that it also vanishes at leading order in the OLNC limit, contrary to the expectations based on the suppression/non-suppression of quark loops. The resolution of this paradox is that the Skyrme model does not include the most general type of meson–meson interaction and, in fact, contains no meson–meson interactions which vanish for the TLNC limit but not the OLNC. The inclusion of such terms in the model yields the expected scaling for strange quark matrix elements.

During the past two decades there has been an extensive experimental program to study strange quark matrix elements of the nucleon [1]. They are of interest in large measure because they are sensitive to physics clearly beyond the naive quark model—they are nonzero only due to closed strange quark loops. Thus they are an ideal way to explore an important theoretical issue: the distinction between two variants of the large N_c limit of QCD. In this paper we focus on strange matrix elements in Skyrme models [2], which are chiral soliton models often justified by appeals to large N_c QCD [3,4]. Attempting to understand the N_c scaling of strange matrix elements in the context of Skyrme models raises an apparent paradox which this Letter resolves.

The traditional method for generalizing QCD to many colors [3,5] treats the quark as being in the fundamental representation of $SU(N)$. We will refer to this approach as the 't Hooft (or "traditional") large N_c (TLNC). Recently, an alternative method—dubbed the "orientifold large N_c" (OLNC) limit [6–10]—for generalizing to large N_c has been proposed, where quarks are taken to be in a two-index representation of color. The principal theoretical motivation for studying this limit was the connection of one flavor QCD in this limit to large N_c supersymmetric Yang–Mills theory; this allows one to exploit powerful mathematical tools in the analysis of one-flavor QCD. However, there is an important connection to phenomenology: for $N_c = 3$ the anti-symmetric representation is isomorphic to the fundamental representation.

The fundamental difference between the two approaches is that the TLNC limit suppresses quark loop effects while the OLNC does not. Quarks are double-color-indexed objects in the OLNC limit and scale in essentially the same was as gluons; all planar diagrams are leading order. Thus, mesons in the OLNC limit scale with N_c in the same way as glueballs [11] which is distinct from the scaling in the TLNC limit:

$$\Gamma_n \sim N_c^{2-n} \quad \text{(OLNC)},$$
$$\Gamma_n \sim N_c^{1-n/2} \quad \text{(TLNC)}, \tag{1}$$

* Corresponding author.
E-mail address: cohen@physics.umd.edu (T.D. Cohen).

0370-2693/$ – see front matter
doi:10.1016/j.physletb.2006.08.027

where Γ_n is a generic n-meson vertex. In effect, there is a rule to convert the generic scaling from the TLNC limit to the OLNC limit, namely, the substitution $N_c^k \to N_c^{2k}$.

An obvious consequence of the scaling of Eq. (1) is on Skyrme models. The N_c scaling in such models is the result of the N_c scaling of the parameters in a model. If one alters the scaling of the parameters of a Skyrmion in the TLNC limit through the generic replacement $N_c^k \to N_c^{2k}$, one finds that the mass of the Skyrmions in the OLNC limit scales as $M \sim N_c^2$. As shown in Refs. [12,13] the N_c scaling of all generic properties of the baryon (mass, couplings, cross-sections, etc.) in the OLNC limit is consistent with the nucleon behaving as a Skyrmion. The consistency of this description is made even stronger due to Bolognesi's observation [12] that the coefficient of the Witten–Wess–Zumino term in the OLNC limit is $N_c(N_c-1)/2 \sim N_c^2$, while in the TLNC limit it is N_c [19].

The consistency of the Skyrme model with large N_c QCD is deeper than merely showing that all of the generic N_c scaling rules apply; spin and flavor play an essential role. The hedgehog structure of the classical solution to the Skyrme model imposes correlations between spatial directions and isospin. These correlations impose relations between certain observables computed at leading order in the collectively quantized Skyrmions which are independent of the details of the Skyrme Lagrangian [14]. In all Skyrme-type models, these relations encode an emergent symmetry of QCD—a contracted $SU(2N_f)$ symmetry where N_f is the number of flavors. These rules follow solely from the fact that the pion–nucleon coupling constant diverges at large N_c, while the pion–nucleon scattering amplitude is finite due to unitarity [15–17]. Since this condition holds for both the TLNC limit ($g_{\pi NN} \sim N_c^{1/2}$) and OLNC limit ($g_{\pi NN} \sim N_c$) the contracted $SU(2N_f)$ spin-flavor symmetry must emerge in both variants of the large N_c limit of QCD.

To begin, let us focus on the Skyrme model, i.e. Skyrme's original model [2], but generalized to three flavors so that the question of strangeness is relevant. The action for the model is

$$S = \int d^4x \left(\frac{f_\pi^2}{4} \operatorname{Tr}\left(L_\mu L^\mu\right) + \frac{\epsilon^2}{4} \operatorname{Tr}\left([L_\mu, L_\nu]^2\right) \right) + S_{\mathrm{WWZ}}, \tag{2}$$

where the left chiral current L_μ is given by $L_\mu \equiv U^\dagger \partial_\mu U$, with $U \in SU(3)_f$ [2,4]; S_{WWZ} is the well-known Witten–Wess–Zumino (WWZ) term, the addition of which is necessary for the Skyrme model to respect the symmetries of QCD [18,19]. The U field can be written as $U = \exp(i\vec{\tau} \cdot \vec{\pi}/f_\pi)$ where $\vec{\pi}$ is the pseudoscalar meson field, and $\vec{\tau}$ is a vector composed of the first three Gell-Mann matrices, $\vec{\tau} \equiv (\lambda_1, \lambda_2, \lambda_3)$. From the scaling rules in Eq. (1), it is apparent that $f_\pi \sim \epsilon \sim N_c^{1/2}$ for the TLNC limit, while for the OLNC limit the scaling is $f_\pi \sim \epsilon \sim N_c$. The only way that N_c enters is through the parameters f_π and ϵ, and through the Witten–Wess–Zumino term [12,13]. To show the N_c dependence of the parameters in an explicit form, we can write

$$\begin{aligned} f_\pi &= \sqrt{N_c}\,\overline{f_\pi}, \qquad \epsilon = \sqrt{N_c}\,\bar{\epsilon} \qquad \text{(TLNC)}, \\ f_\pi &= \sqrt{\tfrac{N_c(N_c-1)}{2}}\,\overline{f_\pi}, \qquad \epsilon = \sqrt{\tfrac{N_c(N_c-1)}{2}}\,\bar{\epsilon} \qquad \text{(OLNC)}, \end{aligned}$$

where the barred quantities do not depend on N_c. This implies that the action can be written as

$$S = N_c \bar{S} \quad \text{(TLNC)}, \qquad S = \frac{N_c(N_c-1)}{2} \bar{S} \quad \text{(OLNC)} \tag{3}$$

with $\bar{S}$ independent of N_c and of the same form for both the TLNC limit and OLNC limit. The choice of the form $\sqrt{N_c(N_c-1)/2}$ rather than N_c for the scaling of the parameters ensures that the Witten–Wess–Zumino term scales in the same way as the rest of the system, and is related to the fact that the baryon consists of $N_c(N_c-1)/2$ quarks in the OLNC limit [12].

This leads to an apparent paradox. In general, when a system is in the semi-classical regime, the size of a prefactor multiplying the action plays two roles: (i) It controls the convergence of the semi-classical expansion, and (ii) specific powers of the prefactor act as multiplicative factors for particular observables. Thus, when N_c is large enough to justify the neglect of subleading effects in both $1/N_c$ expansions, the only effect of going from the TLNC limit to the OLNC limit for the Skyrmion is to make the replacement $N_c \to N_c(N_c-1)/2$ in multiplicative factors for the various observables.

This is a surprising result, because it appears to leave no room for the effects of the different behaviors of quark loops in the two large N_c limits. At leading order, quark loops are suppressed in the TLNC limit, while not being suppressed in the OLNC limit. Thus, one would generically expect that strange quark matrix elements should scale as N_c^2 in the OLNC limit (that is, with leading order scaling), while in the TLNC limit they should be zero at leading order (that is, they should scale as N_c^0, one order below leading). However, given the simple replacement rule above, it appears that the Skyrme model must predict strange quark matrix of the same scale, N_c^0, for both large N_c limits. The paradox is how to reconcile the expectations for the scaling of strange quark matrix elements from a priori quark loop effect considerations with the apparent Skyrme model results.

The resolution would be trivial if the TLNC limit of the Skyrme model had a leading order contribution to strange quark matrix elements. While this is counter to our expectations, calculations of strange quark matrix elements of the nucleon in assorted variants of Skyrme models have larger typical values than for other models on the market [1]. Since the calculations do not include any explicit $1/N_c$ corrections, the very fact that the results are non-zero seems to suggest that the leading order term does survive. However, a careful analysis shows that the strange quark matrix elements of the nucleon in the Skyrme model are zero at leading order in a systematic expansion around the TLNC limit. While there are no explicit $1/N_c$ corrections in the existing calculations based on collective quantization, there are implicit effects which are subleading in $1/N_c$ and which account for the entire result.

To illustrate this, consider the nucleon's strange scalar matrix element at zero momentum transfer for a Skyrmion in the exact $SU(3)$ flavor limit. It is convenient to analyze this matrix element as a fraction, denoted X_s, of the total scalar matrix