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Some notes on the big trip

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Abstract

The big trip is a cosmological process thought to occur in the future by which the entire universe would be engulfed inside a gigantic wormhole and might travel through it along space and time. In this Letter we discuss different arguments that have been raised against the viability of that process, reaching the conclusions that the process can actually occur by accretion of phantom energy onto the wormholes and that it is stable and might occur in the global context of a multiverse model. We finally argue that the big trip does not contradict any holographic bounds on entropy and information.

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1. Rather bizarre implications from dark energy models are now being considered that might ultimately make the future of the universe sings a somehow weird melody. It has been in fact recently proposed [1] that if the current value of the equation-of-state parameter w would keep up being less than -1 in the future, then the throat radius of naturally existing wormholes could grow large enough to engulf the entire universe itself, before this reached the so-called big rip singularity [2], at least for an asymptotic observer. This rather astonishing result—which has been dubbed the "big trip"—has proved to be not free from a number of difficulties which has been raised afterwards and that mainly includes: (1) The result is obtained by using a static metric and therefore it has been claimed [3] that the accretion of dark energy cannot significantly change the amount of exotic matter in the wormhole and hence no large increase of the throat radius should be expected; (2) wormhole space–times are all asymptotically flat and thereby a very large increase of the throat size would imply that the insertion of the wormhole cannot be made onto our universe[1]; (3) quantum catastrophic creation of vacuum particles on the chronology horizon would make macroscopic wormholes completely unstable [4,5], and (4) the holographic bound on the entropy [6,7] would prevent any relevant amount of information to flow through the wormholes, so that these wormholes could never be used to circumvent the big rip singularity [8]. The present report aims at discussing these four difficulties. It will be seen that none of the problems (1), (3) and (4) indeed hold for the asymptotic observer, and that problem (2) is a debatable one and might require considering the big trip to take place within the context of a multiverse scenario.

2. Metric staticity may not be a real problem actually. The question that has been posed is that by using a static metric one automatically ensures that there cannot be any energy flow of the exotic stuff making the wormhole and therefore no arbitrarily rapid accretion can take place, let alone the accretion of the

[1] There is no topologically allowed way by which an asymptotically flat wormhole tunneling with the throat larger than a given sphere can be inserted into the interior of that sphere.

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entire universe required by the big trip. It has been in this way claimed that a static metric may well justify small accretion rates by rigorously calculating the integrated stress tensor conservation laws [9] needed to suitably evaluating the accretion of dark energy according to the generalized Michel theory developed by Babichev et al. [10], but it can never describe extreme accretion regimes. However, we shall show in what follows that by using the static four-dimensional Morris–Thorne metric [11] with a zero shift function we obtain exactly the same result on such extreme regimes (that is a big trip) as when we introduce any time dependence in the g_{rr} metric tensor component entering that metric. Let us start with the static Morris–Thorne metric [11] with zero shift function,

$$ds^2 = -dt^2 + \frac{dr^2}{1-\frac{K(r)}{r}} + r^2\, d\Omega_2^2.$$

From just the integration of the conservation laws for the momentum–energy tensor and its projection on four-velocity (energy flux), the following two equations can then be obtained [9]:

$$\frac{ur^2 \exp\left(\int_{\rho_\infty}^{\rho} \frac{d\rho}{(p+\rho)}\right)}{m^2\sqrt{1-\frac{K(r)}{r}}} = A, \tag{1}$$

$$\frac{\sqrt{\frac{u^2+\frac{K(r)}{r}}{1-\frac{K(r)}{r}}}}{\exp\left(\int_{\rho_\infty}^{\rho} \frac{d\rho}{(p+\rho)}\right)}(p+\rho) = B = \hat{A}\big(\rho_\infty + p(\rho_\infty)\big), \tag{2}$$

where m is the exotic mass which can be assumed to be spherically distributed on the wormhole throat, $u = dr/ds$, $K(r)$ is the shape function [11], A, B and $\hat{A}$ are generally positive constants, and the dark-energy pressure, p, and energy density, ρ, bear all time-dependence in these two expressions. Moreover, since Eq. (1) [where the constant A must be dimensionless (note that we are using natural units so that $G = c = \hbar = 1$)] should describe a flow of dark energy onto the wormhole, $u > 0$ and energy conservation ought then to imply that the exotic mass of the wormhole—and hence the radius of its throat—would progressively change with time, as a consequence from the incoming dark energy flow. Thus, even though the starting metric is static, the energy stored in the wormhole must change with time by virtue of dark energy accretion, in the model considered in Refs. [1,9].

Now, in order to implement the effect of a dark energy flow onto the wormhole implied by Eq. (1), we must introduce a general rate of change of the energy stored in the wormhole due to the external accretion of dark energy onto the wormhole. From the momentum density it can be derived that the rate of change of the exotic mass is generally given by $\dot{m} = \int dS\, T_0^r$, where T_ν^μ is the dark momentum–energy tensor of the universe containing a Morris–Thorne wormhole, and $dS = r^2 \sin\theta\, d\theta\, d\phi$. Hence, using a perfect-fluid expression for that momentum–energy tensor, $T_{\mu\nu} = (P+\rho)u_\mu u_\nu + g_{\mu\nu} p$, and Eqs. (1) and (2), we obtain finally for $p = w\rho(t)$,

$$\dot{m} = -4\pi m^2 A\hat{A}\sqrt{1-\frac{K(r)}{r}}(p+\rho).$$

For the relevant asymptotic regime $r \to \infty$, the rate $\dot{m}$ reduces to $\dot{m} = -4\pi m^2 A\hat{A}(1+w)\rho(t)$, whose trivial integration using the general phantom scale factor $a(t) = a_0(1-\beta(t-t_0))^{-2/[3(|w|-1)]}$, with β a positive constant, yields an increasing expression for $m(t)$ leading to the big trip, provided $w < -1$ and $r \to \infty$, that is [1,9]

$$m \propto K_0 = \frac{K_{0i}}{1-\frac{4\pi Q K_{0i}(|w|-1)(t-t_0)}{(1-\beta(t-t_0))}},$$

where K_{0i} is the initial value of the radius of the wormhole throat and Q is a positive constant. Mere inspection of this equation tells us that the big trip ($K_0 \to \infty$) takes place quite before than big rip ($a \to \infty$) does.

Note that for $r < \infty$ there will be no big trip, but just an increase of the size of the wormhole throat that ceases to occur at a given time. However, because the metric is static, the exotic energy-momentum tensor component describing any internal radial energy flow $\Theta_0^r = 0$, so that, even though the dark energy–momentum tensor component $T_0^r \neq 0$, it has been claimed that accretion of phantom energy following this pattern could only be valid for small rates $\dot{m}$, so that at first sight such a mechanism could not describe arbitrary dark-energy accretion rates and even less so a regime in which the entire universe is accreted.

A more careful consideration leads nevertheless to the conclusion that a big trip keeping the same characteristics as those derived from a static metric stands up as a real phenomenon even when we use a non static metric in such a way that $\Theta_0^r \neq 0$ and $T_0^r \neq 0$ simultaneously. This result can be seen to be a consequence from the fact that the big trip can only occur asymptotically, at $r \to \infty$. In fact, if we start with the corresponding, simplest time-dependent wormhole metric with zero shift function,

$$ds^2 = -dt^2 + \frac{dr^2}{1-\frac{K(r,t)}{r}} + r^2\, d\Omega_2^2,$$

(where the shape function $K(r,t)$ is allowed to depend on time both when tidal forces are taken into account and when such forces are disregarded. In the latter case, all time-dependence in the metric is concentrated on the radius of the wormhole throat, that is it is assumed that the wormhole evolves with time by changing its overall size, while preserving its shape, such as it is thought to occur during the big trip) then Eqs. (1) and (2) would be modified to read

$$\frac{ur^2 \exp\left(\int_{\rho_\infty}^{\rho} \frac{d\rho}{(p+\rho)}\right)}{m^2\sqrt{1-\frac{K(r,t)}{r}}} = AD^{1/2},$$

$$\exp\left(-\int_{\rho_\infty}^{\rho} \frac{d\rho}{p+\rho}\right) D^{1/2}\Bigg\{T_0^r + \frac{\dot{K}_0 K_0}{r}\Bigg[\frac{E}{r^2}(p-T_r^r) + \frac{F(T_0^0 - T_r^r)}{r^2} + \frac{D^{1/2}}{r}\int dr\Bigg(rED^{1/2}\Bigg(\frac{2F(p-T_r^r)}{r^5 D^2} + \frac{d(p-T_r^r)}{r^2 D\, dr}\Bigg) + \frac{F d(T_0^0 - T_r^r)}{r D\, dr}\Bigg)\Bigg]\Bigg\} = B,$$