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Review

Hamilton approach to QCD in Coulomb gauge

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ABSTRACT

I will review results obtained recently within the Hamilton approach to QCD in Coulomb gauge.

1. Introduction

In this talk I will give an overview over recent results obtained within the Hamilton approach to QCD in Coulomb gauge. I will briefly describe the basic ingredients of the Hamilton approach to QCD. Then I will present results obtained for pure Yang–Mills theory at zero temperature and compare with the lattice results. After that the approach will be generalized to finite temperature Yang–Mills theory, where I will concentrate on the description of the deconfinement phase transition. Finally, the approach is extended to full QCD by including also the quark fields.

Canonical quantization of Yang–Mills theory is usually performed in Weyl gauge $A_0 = 0$ and results in the following Hamiltonian

$$H = \frac{1}{2}\int \left(\vec{\Pi}^2 + \vec{B}^2\right), \tag{1}$$

where $\vec{\Pi} = \delta/i\delta\vec{A}$ is the canonical momentum operator and B is the non-Abelian magnetic field. This Hamiltonian is invariant under gauge transformations and the Schrödinger equation

$$H\psi(A) = E\psi(A) \tag{2}$$

has to be solved for the gauge invariant vacuum wave functional, i.e., for the lowest energy eigenstate. Once the vacuum wave functional is known all correlation functions can be evaluated, which defines the full quantum field theory. So far, substantial progress in solving the Yang–Mills Schrödinger equation for the gauge invariant vacuum wave functional has been achieved only in $D = 2 + 1$ dimensions [1–3]. To find an (even approximate) solution to the Yang–Mills Schrödinger equation in $D = 3 + 1$ it is more convenient to fix the gauge and for this purpose Coulomb gauge $\vec{\partial}\vec{A} = 0$ is particularly convenient. The price one pays is that the gauge fixed Hamiltonian is more complicated. In Coulomb gauge the Yang–Mills Hamiltonian is given by Christ and Lee [4]

$$H_{YM} = \frac{1}{2}\int \left(J^{-1}\vec{\Pi}^{\perp}J\vec{\Pi}^{\perp} + \vec{B}^2\right) + H_C, \tag{3}$$

where $J = \mathrm{Det}(-D\partial)$ is the Faddeev–Popov determinant and

$$H_C = \frac{g^2}{2}\int J^{-1}\rho(-D\partial)^{-1}(-\partial^2)(-D\partial)^{-1}J\rho \tag{4}$$

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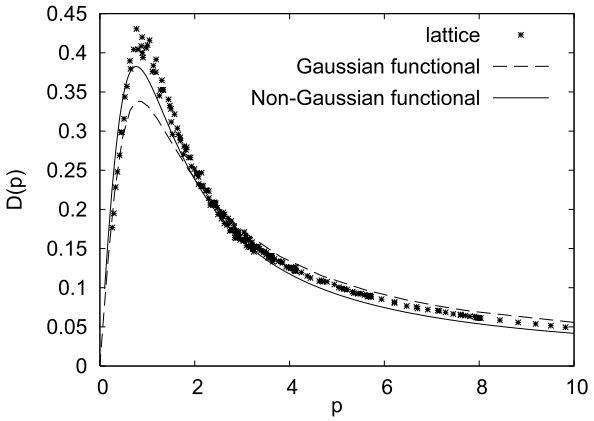

Fig. 1. Comparison of the gluon propagator $1/(2\omega)$ with Gaussian [10] (dashed line) and non-Gaussian [12] (full line) functional to the lattice data [11].

is the Coulomb Hamiltonian, which arises from solving Gauss's law for the longitudinal momentum operator $\vec{\Pi}^{\parallel}$. Furthermore, $\rho = \rho_{gl} + \rho_m$ is the total color charge, which contains beside the charge of the Yang–Mills field $\rho_{gl} = -\hat{\vec{A}} \cdot \vec{\Pi}^{\perp}$ also the charge of the matter fields ρ_m.

In the scalar product of the Yang–Mills wave functionals the transition to Coulomb gauge can be accomplished by using the standard Faddeev–Popov method, which introduces the Faddeev–Popov determinant also in the integration measure

$$\langle \phi | \cdots | \psi \rangle = \int DAJ(A)\, \phi^*(A) \cdots \psi(A). \tag{5}$$

2. Zero-temperature Yang–Mills theory

One can solve the Yang–Mills Schrödinger equation in standard Rayleigh–Schrödinger perturbation theory g [5]. From the Coulomb term (4), which is order g^2, one can extract the running coupling constant and obtains the same result as in ordinary covariant perturbation theory within the functional integral formulation. We are interested here, however, in a non-perturbative solution of the Yang–Mills Schrödinger equation and for this purpose we exploit the variational principle using Gaussian type wave functionals. The first variational calculations in Coulomb gauge were performed in Ref. [6] and later on in Ref. [7]. Our approach [8] differs from previous variational calculations in Coulomb gauge in the ansatz of the vacuum wave functional, in the treatment of the Faddeev–Popov determinant (treated fully in our approach and at least partially neglected in previous approaches) and in the renormalization. The difference between our approach and that of Ref. [7] is discussed in Ref. [9].

The variational approach developed in Tübingen uses the trial ansatz for the vacuum wave functional [8]

$$\psi(A) = \frac{1}{\sqrt{\mathrm{Det}(-\hat{D}\partial)}} \exp\left[-\frac{1}{2}\int A\omega A\right], \tag{6}$$

for which the static gluon propagator is $\langle AA \rangle = 1/(2\omega)$, which shows that ω has the meaning of the gluon energy. Minimizing the vacuum expectation value of the Hamiltonian, $\langle \psi | H | \psi \rangle \to \min$, one finds for the gluon propagator the result shown in Fig. 1 (dashed line) [10] together with the lattice results [11]. At large momenta $\omega(k)$ behaves like the photon energy $\omega(k \to \infty) \sim k$ and is infrared divergent $\omega(k \to 0) \sim 1/k$. This, of course, is the manifestation of the confinement of gluons. As can be seen the gluon propagator obtained from the variational calculation agrees quite well with the lattice data in the infrared and the ultraviolet, while there is some missing strength in the mid-momentum regime around 1 GeV. This can be traced back to the absence of the gluon loop, which escapes the variational calculation with the Gaussian type of ansatz (6). In Ref. [12] the variational approach was extended to non-Gaussian wave functionals including up to quartic terms in the gauge field

$$|\psi[A]|^2 = \exp(-S[A]), \qquad S[A] = \int \omega A^2 + \frac{1}{3!}\int \gamma^{(3)} A^3 + \frac{1}{4!}\int \gamma^{(4)} A^4. \tag{7}$$

To capture the gluon loop in the variational calculation one needs to include at least the three-gluon term $\gamma^{(3)}$ in the exponent of the wave functional. Then one finds the static gluon propagator shown in Fig. 1 (full line), which gives a substantial improvement compared to the propagator obtained with the Gaussian trial wave functional. Fig. 2 shows the result for the three-gluon vertex together with the lattice results [13], for more details see Ref. [12].