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Review

# Symmetry-adapted no-core shell model applications for light nuclei with QCD-inspired interactions

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ABSTRACT

We use powerful computational and group-theoretical algorithms to perform *ab initio* CI (configuration-interaction) calculations in a model space spanned by SU(3) symmetry-adapted many-body configurations with the JISP16 nucleon–nucleon interaction. We demonstrate that the results for the ground states of light nuclei up through $A = 16$ exhibit a strong dominance of low-spin and high-deformation configurations together with an evident symplectic structure. We also find states among the lowest-lying $0^+$ eigenstates of $^{12}$C and $^{16}$O that are clearly dominated by $\alpha$-clustering correlations. Our findings imply that only a small fraction of the full model space is needed to model nuclear collective dynamics, including deformations and $\alpha$-particle clustering, even if one uses modern realistic interactions that do not preserve SU(3) symmetry. This, in turn, points to the importance of using a symmetry-adapted CI framework, one based on an LS coupling scheme with the associated spatial configurations organized according to deformation.



## 1. Introduction

The predictive power of *ab initio* approaches to nuclear structure depends critically on the choice of a realistic nuclear interaction (such as, [1,2]), and on the ability of a theoretical model to describe multifaceted properties of atomic nuclei from single-particle effects to collective phenomena. Modern *ab initio* approaches to light atomic nuclei [3–5] represent forefront research in nuclear structure and reaction theory [6–10]. These models are built on fundamental principles and therefore hold promise to provide accurate predictions essential for a description of unstable and exotic nuclei, many of which are of significant interest, e.g. in nucleosynthesis, but remain inaccessible even to experiments.

The no-core shell model (NCSM) [3] is an *ab initio* configuration–interaction (CI) method that has achieved a good description of low-lying states and nuclear reactions for nuclei up through the *p* shell [6–9]. This method uses the Lanczos algorithm to compute the lowest few eigenvalues and associated eigenstates of a realistic Hamiltonian matrix whose elements are calculated in an *m*-scheme basis, i.e. a basis of Slater determinants constructed from single-particle wavefunctions of the harmonic oscillator. The main limitation of this approach, and its predictive power, is inherently coupled with the combinatorial growth of the *m*-scheme basis with increasing nucleon number and maximal number of total harmonic oscillator quanta $N_{\max}$ as illustrated in Fig. 1.

We propose a novel model, the *ab initio* symmetry-adapted no-core shell model (SA-NCSM), that adopts the first-principle concept, and utilizes a many-particle basis that is reduced with respect to the physically relevant SU(3) $\supset$ SO(3) subgroup chain. The significance of the SU(3) group for a microscopic description of the nuclear collective dynamics can be seen from

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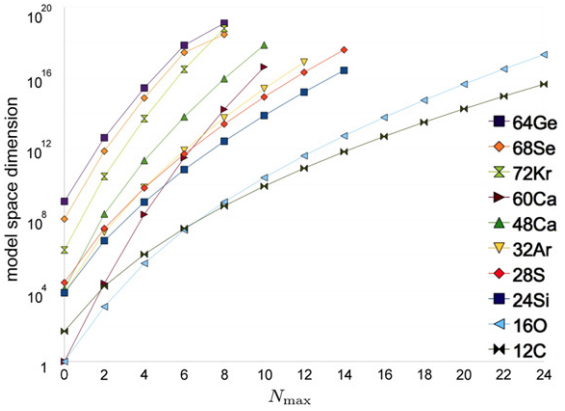


**Fig. 1.** Number of NCSM many-body states as a function of the $N_{\max}$ many-body cutoff for several representative nuclei in the *p*, *sd*, and *pf* shells.

the fact that it is the symmetry group of the Elliott model [11], and a subgroup of the $Sp(3,\mathbb{R})$ symplectic model [12]. Hence, the SA-NCSM holds promise to expand the reach as well as the impact of current *ab initio* approaches toward describing heavier mass nuclei together with collective, deformed, and cluster substructures. This is achieved by recognizing that the choice of coordinates, especially when deformed nuclear shapes dominate, is crucial, and that the SA-NCSM affords a solution in terms of coordinates that reflect symmetries inherent to the nuclear system. While the SA-NCSM states can be obtained through a unitary transformation from the *m*-scheme basis used in the NCSM, and hence span the entire space, the growth of the model space within the SA-NCSM framework can be managed by winnowing to only physically relevant states as determined through symmetry considerations.

## 2. SU(3) symmetry-adapted theoretical framework

The basis states of the *ab initio* SA-NCSM are constructed in the proton–neutron formalism and are labeled by the physical SU(3) $\supset$ SO(3) subgroup chain quantum numbers $(\lambda\,\mu)\kappa L$, and by proton, neutron, and total intrinsic spins $S_\pi$, $S_\nu$, and $S$. The orbital angular momentum $L$ is coupled with $S$ to the total orbital momentum $J$ and its projection $M_J$. Each basis state is thus labeled in the SU(3)-scheme as

$$|\vec{\alpha}\, S_\pi S_\nu S(\lambda\,\mu)\kappa LJM_J\rangle, \tag{1}$$

where the deformation-related $(\lambda\,\mu)$ set of quantum numbers labels SU(3) irreducible representations, irreps, and bring forward important information about nuclear shapes and deformation. For example, (00), $(\lambda\,0)$ and $(0\,\mu)$ describe spherical, prolate and oblate shapes, respectively. The label $\kappa$ distinguishes multiple occurrences of the same $L$ value in the parent irrep $(\lambda\,\mu)$. Symbol $\vec{\alpha}$ schematically denotes the additional quantum numbers needed to unambiguously distinguish between irreps carrying the same $S_\pi S_\nu S(\lambda\,\mu)$ quantum numbers. These irreps compose a well-defined subspace with the unique ability to separate intrinsic and center-of-mass degrees of freedom. The size of these subspaces is typically several orders of magnitude smaller than the full $N\hbar\Omega$ space, where the *m*-scheme basis also allows for an exact factorization of center-of-mass degrees of freedom.

The SA-NCSM implements a set of powerful algorithms [13,14], which facilitate calculations of matrix elements of arbitrary (currently up to two-body, but expandable to higher-rank) operators in the SU(3)-scheme basis. The underlying principle behind the SA-NCSM computational kernel is a SU(3)-type Wigner–Eckhart theorem, which factorizes the problem into reduced matrix elements (*rmes*) and SU(3) coupling/recoupling coefficients. While the latter can be computed using the publicly available library [15], the former is calculated from a set of single-shell *rmes* by the repetitive application of a SU(3)-type reduction formula for *rmes* of operators acting on two independent subsystems. The algorithm is very general. It allows for the evaluation of the Hamiltonian matrix elements and the use of the resulting eigenvectors to determine other experimental observables.

## 3. Results and discussions

We calculated low-lying wavefunctions and some associated physical observables for $^6$Li, $^7$Li, $^{12}$C, and $^{16}$O, using the bare JISP16 interaction [16] for $N_{\max} = 6$ and $\hbar\Omega = 10$ MeV, and compared our results with those obtained using the *m*-scheme based NCSM approach. To demonstrate the efficacy of the symmetry-adapted selection scheme of the SA-NCSM, we show here *ab initio* SA-NCSM calculations for the ground state of $^6$Li and $^7$Li (Fig. 2). The model space includes all the configurations up through $N_{\max} = 4$ (full space) with the $N_{\max} = 6$ subspace restricted to only a few SU(3) and *S* configurations specified in Table 1. The figures reveal that as the model space is expanded by subsequently adding the specified sets of configurations