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Comparison of stochastic resonance in static and dynamical nonlinearities

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ABSTRACT

We compare the stochastic resonance (SR) effects in parallel arrays of static and dynamical nonlinearities via the measure of output signal-to-noise ratio (SNR). For a received noisy periodic signal, parallel arrays of both static and dynamical nonlinearities can enhance the output SNR by optimizing the internal noise level. The static nonlinearity is easily implementable, while the dynamical nonlinearity has more parameters to be tuned, at the risk of not exploiting the beneficial role of internal noise components. It is of interest to note that, for an input signal buried in the external Laplacian noise, we show that the dynamical nonlinearity is superior to the static nonlinearity in obtaining a better output SNR. This characteristic is assumed to be closely associated with the kurtosis of noise distribution.

1. Introduction

Stochastic resonance (SR) is a nonlinear phenomenon where the response of a nonlinear system to input signals can be enhanced by the addition of noise [1–16]. The SR effect was originally found in a bistable system driven by a subthreshold periodic input signal and additive noise [1–4]. Naturally, the coupled [17,18] and parallel arrays [19–24] of nonlinear systems were investigated on exploiting the constructive role of internal noise in further improvement of the system performance. Recently, the SR effects in complex networks [25–27] were also adequately evaluated. It is verified that an interconnected network configuration, as well as the internal noise level, can be optimized to achieve the best network response [25–27].

Initially, the output signal-to-noise ratio (SNR) was adopted as a metric for describing the signature of SR [2,5,28–30], behaving a non-monotonic function of the background noise intensity. Since then, the SNR is frequently employed to show the conventional SR effect in a nonlinear system subjected to a noisy input [2,5,30–36]. Among these researches, it is interesting to note that the output–input SNR gain exceeds unity for a 'soft' double-well potential dynamical model driven by a suprathreshold signal [33,34]. Meanwhile, the SR effect displayed in a class of static nonlinearities that exhibit saturation in their responses to input biased signals [35]. Both models in Refs. [33,35] contain a nonlinear hyperbolic tangent transfer function. Therefore, it deserves to contrastively evaluate the abilities of dynamic and static models to enhance the output SNR within the context of SR.

In this paper, we compare the SR effects in parallel arrays of static and dynamical nonlinearities for processing a periodic signal in additive white noise. It is shown that, in an array of static nonlinearities, the SR phenomenon only occurs as the array size is large enough. While, for a parallel array of dynamical nonlinear elements, the potential bistability might be broken as the system parameters vary, and rich behaviors of the output SNR are shown as a function of the internal noise level and the array size. By comparison, the output maximum SNR of an array of static nonlinearities is predominant in the condition of external Gaussian white noise. However, when the external white noise is a type of non-Gaussian noise, e.g. Laplacian noise, the output SNR of the dynamical nonlinearity is superior to that of the static nonlinearity. We can characterize the non-Gaussian noise distribution with its kurtosis [37,38], and also demonstrate Gaussian noise has the worse for the SNR enhancement of both static and dynamical systems. The present comparison results show that the nonlinear phenomenon of SR can be discovered in both static and dynamical systems for improving the overall performance. While, in view of characteristics of static and dynamical nonlinearities, we need to choose the relative optimal array to process the input signal buried in different noise types.

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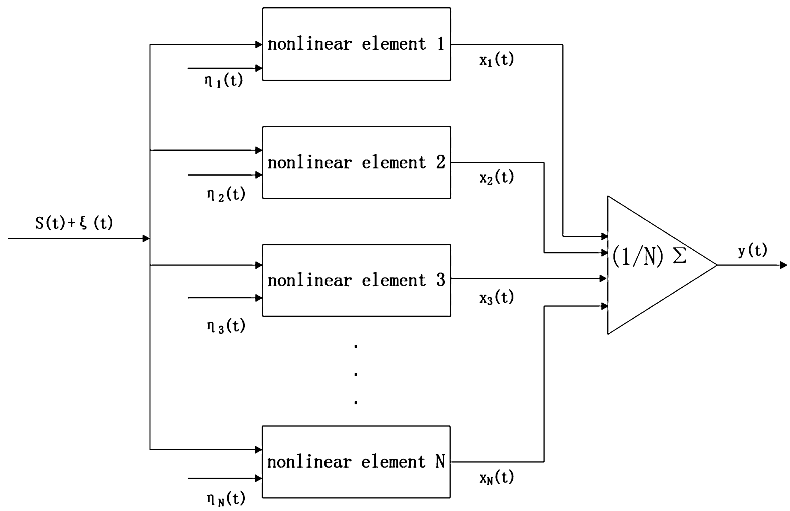

Fig. 1. A parallel array of N nonlinear elements. Each element is subject to the same noisy input $s(t)+\xi(t)$ but independent array noise $\eta_i(t)$. The array output is $y(t)=\sum_{i=1}^{N} x_i(t)/N$.

2. General model

Consider the observation of a process $v(t)=s(t)+\xi(t)$, where the component $s(t)$ is a periodic sinusoidal signal with maximal amplitude A ($|s(t)|\le A$) and period T, and zero-mean additive white noise $\xi(t)$, independent of $s(t)$, has a probability density function (PDF) f_ξ and variance $\sigma_\xi^2=\mathrm{E}_\xi[x^2]=\int_{-\infty}^{\infty}x^2 f_\xi(x)dx$. Next, $v(t)$ is applied to an uncoupled parallel array of N identical static or dynamical nonlinearities [20], as shown in Fig. 1. The terms $\eta_i(t)$, independent of $v(t)$, represent the internal noise components for each element, so as to yield the outputs $x_i(t), i=1,2,\dots,N$. The internal noise components $\eta_i(t)$ are mutually independent and identically distributed (i.i.d.) with the same PDF f_η and variance σ_η^2. Then, as shown in Fig. 1, the array output $y(t)$ is given by

$$y(t)=\frac{1}{N}\sum_{i=1}^{N}x_i(t). \tag{1}$$

Since $s(t)$ is periodic, $y(t)$ is in general a cyclostationary random signal with period T [3]. The nonstationary mean $\mathrm{E}[y(t)]$ is a deterministic periodic function of time t with period T. A classical definition of the SNR, at frequency $1/T$ on the output, follows as the ratio of the power contained in the spectral line alone to the power contained in the noise background in a small frequency band ΔB around $1/T$. The corresponding expression of the output SNR is then given by [3]

$$R_{\mathrm{out}}=\frac{|\langle \mathrm{E}[y(t)]\exp(-\imath 2\pi t/T)\rangle|^2}{\langle \mathrm{var}[y(t)]\rangle H(1/T_s)\Delta B}, \tag{2}$$

where the nonstationary expectation

$$\mathrm{E}\big[y(t)\big]=\frac{1}{N}\sum_{i=1}^{N}\mathrm{E}\big[x_i(t)\big]=\mathrm{E}\big[x_i(t)\big], \tag{3}$$

and nonstationary variance

$$\begin{aligned}\mathrm{var}\big[y(t)\big]&=\mathrm{E}\big[y^2(t)\big]-\mathrm{E}^2\big[y(t)\big]\\&=\frac{1}{N}\mathrm{E}\big[x_i^2(t)\big]+\frac{N-1}{N}\mathrm{E}\big[x_i(t)x_j(t)\big]-\mathrm{E}^2\big[x_i(t)\big],\end{aligned} \tag{4}$$

for $i,j=1,2,\dots,N$ and $i\neq j$. Here, the temporal operator $\langle\cdot\rangle=\int_0^T\cdot\,dt/T$, and $H(\nu)$ is the Fourier transform of the normalized stationary autocovariance

$$h(\tau)=\frac{\langle \mathrm{E}[y(t)y(t+\tau)]-\mathrm{E}[y(t)]\mathrm{E}[y(t+\tau)]\rangle}{\langle \mathrm{var}[y(t)]\rangle}. \tag{5}$$

As the array size $N\to\infty$, we note that Eq. (4) becomes $\lim_{N\to\infty}\mathrm{var}[y(t)]=\mathrm{E}[x_i(t)x_j(t)]-\mathrm{E}^2[x_i(t)]$ and Eq. (5) can be expressed as

$$\lim_{N\to\infty}h(\tau)=\frac{\langle \mathrm{E}[x_i(t)x_j(t+\tau)]-\mathrm{E}[x_i(t)]\mathrm{E}[x_j(t+\tau)]\rangle}{\langle \mathrm{E}[x_i(t)x_j(t)]-\mathrm{E}^2[x_i(t)]\rangle}. \tag{6}$$

In this case, the output SNR of an array of the nonlinearities can be written as

$$R_{\mathrm{out}}^{\infty}=\frac{|\langle \mathrm{E}[x_i(t)]\exp(-\imath 2\pi t/T)\rangle|^2}{\langle \mathrm{E}[x_i(t)x_j(t)]-\mathrm{E}^2[x_i(t)]\rangle H(1/T_s)\Delta B}, \tag{7}$$

for $i\neq j$, which makes the numerical calculation of the output SNR of a parallel array of nonlinearities with infinity size $N=\infty$ possible. Since the indices i and j are different, but arbitrary in Eq. (7), we can adopt two different nonlinear systems, each embedded with independent noise, to evaluate the output SNR of a parallel array with size $N=\infty$ [3,31]. This method is tractable and effective. In the same way, the mixture $v(t)=s(t)+\xi(t)$ has an input SNR defined as

$$R_{\mathrm{in}}=\frac{|\langle s(t)\exp(-\imath 2\pi t/T)\rangle|^2}{\sigma_\xi^2\Delta t\Delta B}, \tag{8}$$

with Δt indicating the time resolution or the sampling period in a discrete-time implementation [3]. For a sinusoidal signal $s(t)=A\sin(2\pi t/T)$ buried in white noise, the input SNR can be simplified as $R_{\mathrm{in}}=A^2/(4\sigma_\xi^2\Delta t\Delta B)$ [3].

3. Comparison results of static and dynamical nonlinearities in array SR

In this subsection, we observe the SR effects in arrays of static and dynamical nonlinearities. First, consider the static nonlinearity with saturation

$$x_i(t)=\tanh\big[\beta\big(s(t)+\xi(t)+\eta_i(t)\big)\big], \tag{9}$$

with slope $\beta>0$, which is linear for small $|u|\ll\beta^{-1}$ and saturates at ±1 for large $|u|\gg\beta^{-1}$ [28,35].

Secondly, a dynamical nonlinearity [33,34]

$$\frac{dx_i(t)}{dt}=-x_i(t)+J\tanh\big[\beta x_i(t)\big]+s(t)+\xi(t)+\eta_i(t), \tag{10}$$