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Neutrino oscillations in the field of a rotating deformed mass

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ABSTRACT

The neutrino oscillations in the field of a rotating deformed mass is investigated. The phase shift is evaluated in the case of weak field limit, slow rotation and small deformation. To this aim the Hartle–Thorne metric is used, which is an approximate solution of the vacuum Einstein equations accurate to second order in the rotation parameter a/M and to first order in the mass quadrupole moment q. Implications on atmospheric, solar and astrophysical neutrinos are discussed.

1. Introduction

In the Standard Model with minimal particle content neutrinos are massless left-handed fermions. The question whether neutrinos have a non-vanishing rest mass influences research areas from particle physics up to cosmology, but it remains an open issue [1]. At present all hints for neutrino masses are connected with neutrino oscillation effects, namely the solar neutrino deficit, the atmospheric neutrino anomaly and the evidence from the LSND experiment [2]. Possible extensions of the Standard Model to generate neutrino masses are reviewed, e.g., in Ref. [3].

Mass neutrino mixing and oscillation in flat spacetime were proposed by Pontecorvo [4]. Later on Mikheyev, Smirnov and Wolfenstein [5] investigated the effect of transformation of one neutrino flavor into another in a medium with varying density. There have been many experimental studies exploring the evidence for oscillations of both atmospheric and solar neutrinos as well as imposing limits on their masses and mixing angle (see, e.g., Ref. [6] and references therein).

The possibility to detect CP violation effects in neutrino oscillations by future experiments has also been considered in recent years [7–10]. Neutrino oscillation experiments are expected to provide stringent bounds on many quantum gravity models entailing violation of Lorentz invariance, so allowing to test quantum gravity theories [11,12]. Planck scale-induced deviations from the standard oscillation length may be observable for ultra-high-energy neutrinos emitted by galactic and extragalactic sources by means of the next generation neutrino detectors such as IceCube and ANITA [13]. Furthermore, since neutrinos can propagate freely over large distances and can therefore pile up minimal length effects beyond detectable thresholds, there is the possibility to explore the presence of a quantum-gravity-induced minimal length using neutrino oscillation probabilities [14].

The effect of gravitation on the neutrino oscillations has been extensively investigated in the recent literature, starting from the pioneering work of Stodolsky [15]. The correction to the phase difference of neutrino mass eigenstates due to the spherically symmetric gravitational field described by the Schwarzschild metric was calculated in various papers within the WKB approximation [16–20]. The results obtained in these papers differ from each other due to different methods used to perform the calculation. For instance, calculating the phase along the timelike geodesic line will produce a factor of 2 in the high energy limit, compared with the value along the null line [21]. A different method was proposed by Linet and Teyssandier [22], based on the world function developed by Synge [23] and defined as half the square of the spacetime distance between two generic points connected by a geodesic path. Unfortunately, the calculation of the world function is not a trivial task. In general, it is performed perturbatively unless the solution of the geodesic equations is explicitly known, as in the very special cases of Minkowski, Gödel, de Sitter spacetimes and the metric of a homogeneous gravitational field [24]. The effect of spacetime rotation on neutrino oscillations has been investigated in Ref. [25], where the Kerr solution was considered. A mechanism to generate pulsar kicks based on the spin flavor conversion of neutrinos propagating in a slowly rotating Kerr spacetime described by the Lense–Thirring metric has been recently proposed [26].

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Furthermore, the neutrino geometrical optics in a gravitational field and in particular in a Lense–Thirring background has been investigated [27]. Finally, in Ref. [28] the generalization to the case of a Kerr–Newman spacetime has been discussed.

In the present Letter we calculate the phase shift in the gravitational field produced by a massive, slowly rotating and quasi-spherical object, described by the Hartle–Thorne metric. This is an approximate solution of the vacuum Einstein equations accurate to second order in the rotation parameter a/M and to first order in the mass quadrupole moment q, generalizing the Lense–Thirring metric. We then discuss possible implications on atmospheric, solar and astrophysical neutrinos. The units $G = c = \hbar = 1$ are used throughout the Letter.

2. Stationary axisymmetric spacetimes and neutrino oscillation

The line element corresponding to a general stationary axisymmetric solution of the vacuum Einstein equations can be written in the Weyl–Lewis–Papapetrou [29–31] form as

$$\begin{aligned} ds^2 = &-f(dt-\omega\, d\phi)^2 \\ &+\frac{\sigma^2}{f}\left\{e^{2\gamma}\left(x^2-y^2\right)\left(\frac{dx^2}{x^2-1}+\frac{dy^2}{1-y^2}\right)\right. \\ &\left.+\left(x^2-1\right)\left(1-y^2\right)d\phi^2\right\} \end{aligned} \tag{1}$$

by using prolate spheroidal coordinates (t, x, y, ϕ) with $x \geqslant 1$, $-1 \leqslant y \leqslant 1$; the quantities f, ω and γ are functions of x and y only and σ is a constant. The relation to Boyer–Lindquist coordinates (t, r, θ, ϕ) is given by

$$t = t, \qquad x = \frac{r-M}{\sigma}, \qquad y = \cos\theta, \qquad \phi = \phi. \tag{2}$$

2.1. Geodesics

The geodesic motion of test particles is governed by the following equations [32]

$$\begin{aligned} \dot t &= \frac{E}{f}+\frac{\omega f}{\sigma^2X^2Y^2}(L-\omega E), \qquad \dot\phi = \frac{f}{\sigma^2X^2Y^2}(L-\omega E), \\ \ddot y &= -\frac{1}{2}\frac{Y^2}{X^2}\left[\frac{f_y}{f}-2\gamma_y+\frac{2y}{X^2+Y^2}\right]\dot x^2 \\ &\quad+\left[\frac{f_x}{f}-2\gamma_x-\frac{2x}{X^2+Y^2}\right]\dot x\dot y \\ &\quad+\frac{1}{2}\left[\frac{f_y}{f}-2\gamma_y-\frac{2y}{X^2+Y^2}\frac{X^2}{Y^2}\right]\dot y^2 \\ &\quad-\frac{1}{2}\frac{e^{-2\gamma}}{f\sigma^4X^2Y^2(X^2+Y^2)}\left\{Y^2\left[f^2(L-\omega E)^2+E^2\sigma^2X^2Y^2\right]f_y\right. \\ &\quad\left.+2(L-\omega E)f^3\left[y(L-\omega E)-EY^2\omega_y\right]\right\}, \\ \dot x^2 &= -\frac{X^2}{Y^2}\dot y^2+\frac{e^{-2\gamma}X^2}{\sigma^2(X^2+Y^2)} \\ &\quad\times\left[E^2-\mu^2 f-\frac{f^2}{\sigma^2X^2Y^2}(L-\omega E)^2\right] \end{aligned} \tag{3}$$

where Killing symmetries and the normalization condition $g_{\alpha\beta}\dot x^\alpha \times \dot x^\beta = -\mu^2$ have been used. Here E and L are the conserved energy (associated with the Killing vector ∂_t) and angular momentum (associated with the Killing vector ∂_ϕ) of the test particle respectively, μ is the particle mass and a dot denotes differentiation with respect to the affine parameter λ along the curve; furthermore, the notation

$$X = \sqrt{x^2-1}, \qquad Y = \sqrt{1-y^2} \tag{4}$$

has been introduced. For timelike geodesics, λ can be identified with the proper time by setting $\mu = 1$. Let U be the associated 4-velocity vector ($U \cdot U = -1$). Null geodesics are characterized instead by $\mu = 0$. Let K be the associated tangent vector ($K \cdot K = 0$).

Let us consider the motion on the symmetry plane $y = 0$. If $y = 0$ and $\dot y = 0$ initially, the third equation of Eqs. (3) ensures that the motion will be confined on the symmetry plane, since the derivatives of the metric functions with respect to y, i.e., f_y, ω_y and γ_y, all vanish at $y = 0$, so that $\ddot y = 0$ too. Eqs. (3) thus reduce to

$$\begin{aligned} \dot t &= \frac{E}{f}+\frac{\omega f}{\sigma^2X^2}(L-\omega E), \qquad \dot\phi = \frac{f}{\sigma^2X^2}(L-\omega E), \\ \dot x &= \pm\frac{e^{-\gamma}X}{\sigma\sqrt{1+X^2}}\left[E^2-\mu^2 f-\frac{f^2}{\sigma^2X^2}(L-\omega E)^2\right]^{1/2}, \end{aligned} \tag{5}$$

where metric functions are meant to be evaluated at $y = 0$.

2.2. Neutrino oscillations

The phase associated with neutrinos of different mass eigenstate is given by [15]

$$\Phi_k = \int_A^B P_{\mu(k)}\, dx^\mu, \tag{6}$$

if the neutrino with 4-momentum $P = m_k U$ is produced at a spacetime point A and detected at B.

The standard assumptions usually applied to evaluate the phase are the following (see, e.g., Ref. [33]): a massless trajectory is assumed, which means that the neutrino travels along a null geodesic path; the mass eigenstates are taken to be the energy eigenstates, with a common energy E; the ultrarelativistic approximation $m_k \ll E$ is performed throughout, so that all quantities are evaluated up to first order in the ratio m_k/E.

The integral is carried out over a null path, so that Eq. (6) can be also written as

$$\Phi_k = \int_{\lambda_A}^{\lambda_B} P_{\mu(k)} K^\mu\, d\lambda, \tag{7}$$

where K is a null vector tangent to the photon path. The components of P and K are thus obtained from Eq. (5) by setting $\mu = m_k$ and $\mu = 0$ respectively. In the case of equatorial motion the argument of the integral (7) depends on the coordinate x only, so that the integration over the affine parameter λ can be switched over x by

$$\Phi_k = \int_{x_A}^{x_B} P_{\mu(k)}\frac{K^\mu}{K^x}\, dx, \tag{8}$$

where $K^x = dx/d\lambda$. By applying the relativistic condition $m_k \ll E$ we find

$$\Phi_k \simeq \mp\frac{1}{2}\sigma^2\frac{m_k^2}{E}\int_{x_A}^{x_B}\frac{xe^\gamma}{\sqrt{\sigma^2(x^2-1)-f^2(b-\omega)^2}}\, dx, \tag{9}$$

to first order in the expansion parameter $m_k/E \ll 1$, where E is the energy for a massless neutrino and $b = L/E$ the impact parameter.