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On the confinement of a Dirac particle to a two-dimensional ring

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ABSTRACT

In this contribution, we propose a new model for studying the confinement of a spin-half particle to a two-dimensional quantum ring for systems described by the Dirac equation by introducing a new coupling into the Dirac equation. We show that the introduction of this new coupling into the Dirac equation yields a generalization of the two-dimensional quantum ring model proposed by Tan and Inkson [W.-C. Tan, J.C. Inkson, Semicond. Sci. Technol. 11 (1996) 1635] for relativistic spin-half quantum particles.

1. Introduction

Recent studies of the interaction between a relativistic spin-half particle with the harmonic oscillator potential has shown, in the nonrelativistic limit of the Dirac equation, the impossibility of recovering the harmonic oscillator Hamiltonian due to the presence of a quadratic potential [1–4]. Therefore, the relativistic harmonic oscillator has been investigated by introducing a coupling into the Dirac equation in such a way that the Dirac equation remains linear both in momenta and coordinates, and such that one can recover the Schrödinger equation for a harmonic oscillator in the nonrelativistic limit of the Dirac equation. This new coupling introduced into the Dirac equation is called the Dirac oscillator [1]. Another way of obtaining a relativistic oscillator was investigated in [5] by introducing a scalar and vector potentials which are quadratics in coordinates. In recent years, the Dirac oscillator has been used in studies of the Ramsey-interferometry effect [4], in the quantum Hall effect [6], and has also been investigated in the presence of an external magnetic field [7].

It is worth mentioning that the introduction of the Dirac oscillator does not allow us to make a complete study of the confinement of a relativistic quantum particle to a two-dimensional quantum ring. In the nonrelativistic context of the quantum mechanics, the confinement of quantum particles into a two-dimensional quantum ring has presented interesting results, such as, a nonparabolic spectrum of energy [8], the arising of persistent currents due to the dependency of the energy levels on the Berry's phase [9], a parabolic spectrum of energy [10–13], and the arising of persistent currents due to the dependency of the energy levels on the Aharonov–Casher quantum flux [13,14].

In this Letter, based on the coupling corresponding to the Dirac oscillator [1], we propose the introduction of a new coupling into the Dirac equation to study the confinement of a spin-half particle to a two-dimensional quantum ring in condensed matter systems described by the Dirac equation. This is the case of an ultra-relativistic quasiparticle system, where the system is characterized by a linear dispersion relation with respect to the velocity obeyed by the quasiparticles. For instance, near the Fermi points in graphene, the dispersion relation is linear with respect to momenta, where the Fermi velocity plays the role of velocity of the light [15,16]. We show that the introduction of this new coupling into the Dirac equation yields a generalization of the two-dimensional quantum ring model proposed by Tan and Inkson [8] for relativistic quantum particles. By using this model, we can also obtain a nonparabolic and discrete spectrum of energy for a spin-half particle confined to two-dimensional quantum ring analogous to the Tan–Inkson model [8] in the nonrelativistic limit. Furthermore, we show that this relativistic model allows us to discuss both the confinement of a spin-half particle to a quantum dot, and the interaction with a quantum antidot in systems described by the Dirac equation.

The structure of this Letter is as follows: in Section 2, we begin by introducing a new coupling into the Dirac equation to describe the confinement of a relativistic spin-half particle to a two-dimensional quantum ring; in Section 3, we consider the

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presence of a magnetic flux in the center of the two-dimensional quantum ring, and solve the Dirac equation exactly. We show that the relativistic energy levels has a dependence of the magnetic quantum flux which gives rise to the arising of persistent currents in the two-dimensional quantum ring; in Section 4, we discuss the nonrelativistic limit of the Dirac equation; in Section 5, we discuss the limit where this model describes the confinement of a spin-half particle to a quantum dot, and the interaction with a quantum antidot in systems described by the Dirac equation. In Section 6, we present our conclusions.

2. Two-dimensional quantum ring model

Let us begin by introducing a new coupling into the Dirac equation. As we have discussed in the introduction, the interaction between a relativistic spin-half particle with the harmonic oscillator potential has shown problems in recovering the harmonic oscillator Hamiltonian in the nonrelativistic limit of the Dirac equation due to the presence of a quadratic potential [1–4]. Therefore, a new coupling has been introduced into the Dirac equation in such a way that this equation remains a linear equation both in momenta and coordinates in order to study the relativistic harmonic oscillator. This coupling, which is known in the literature as the Dirac oscillator [1], is given by

$$\vec{p} \to \vec{p} - im\omega\rho\gamma^0\hat{\rho}, \tag{1}$$

where $\hat{\rho}$ is the unit vector in the radial direction. In this way, based on the coupling which gives rise to the Dirac oscillator, we introduce the following coupling into the Dirac equation

$$\vec{p} \to \vec{p} + i\left[\frac{\sqrt{2ma_1}}{\rho} + \sqrt{2ma_2}\rho\right]\gamma^0\hat{\rho}, \tag{2}$$

where a_1 and a_2 are control parameters. In the following, we show that the introduction of the coupling (2) gives rise to a model for studies of the confinement of a relativistic spin-half particle to a two-dimensional quantum ring yielding a generalization of the Tan–Inkson model for a two-dimensional quantum ring [8]. We also show that, by taking the parameter $a_1 = 0$, the relativistic energy levels correspond to both the energy levels of the Dirac oscillator, and the confinement of a relativistic spin-half particle to a quantum dot. Moreover, by taking the parameter $a_2 = 0$, we have the case where the relativistic spin-half particle interacts with a quantum antidot, and no bound states can be achieved.

3. Solutions of the Dirac equation in a two-dimensional ring

In this section, we obtain the solutions of the Dirac equation in the presence of the confining potential described by Eq. (2). Since the coupling (2) takes into account the cylindrical symmetry, we work the Dirac equation with curvilinear coordinates. In this case, the line element of the Minkowski spacetime is written in cylindrical coordinates as follows: $ds^2 = -dt^2 + d\rho^2 + \rho^2 d\varphi^2 + dz^2$. Thus, by applying a coordinate transformation $\frac{\partial}{\partial x^\mu} = \frac{\partial \bar{x}^\nu}{\partial x^\mu}\frac{\partial}{\partial \bar{x}^\nu}$, and a unitary transformation on the wave function $\psi(x) = U\psi'(\bar{x})$, the Dirac equation can be written in any orthogonal system in the following form [17]:

$$i\gamma^\mu D_\mu\psi + \frac{i}{2}\sum_{k=1}^{3}\gamma^k\left[D_k \ln\left(\frac{h_1h_2h_3}{h_k}\right)\right]\psi = m\psi, \tag{3}$$

where $D_\mu = \frac{1}{h_\mu}\partial_\mu$ is the derivative of the corresponding coordinate system, and the parameters h_k are the scale factors of the corresponding coordinate system. For instance, in cylindrical coordinates, the scale factors are given by $h_0 = 1$, $h_1 = 1$, $h_2 = \rho$, and $h_3 = 1$. In this way, the second term in (3) gives rise to a term called the spinorial connection [17–22]. The matrices γ^μ are the Dirac matrices given in the Minkowski spacetime [23,24], that is,

$$\gamma^0 = \hat{\beta} = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}; \qquad \gamma^i = \hat{\beta}\hat{\alpha}^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix};$$
$$\Sigma^i = \begin{pmatrix} \sigma^i & 0 \\ 0 & \sigma^i \end{pmatrix}, \tag{4}$$

with I, $\vec{\Sigma}$ and σ^i being the 2×2 identity matrix, the spin vector and the Pauli matrices, respectively. The Pauli matrices satisfy the relation $(\sigma^i\sigma^j + \sigma^j\sigma^i) = 2\eta^{ij}$, where $\eta^{\mu\nu} = \mathrm{diag}(-+++)$ is the Minkowski tensor.

Let us also consider a field characterized by a potential vector $\vec{A} = \frac{\phi}{2\pi\rho}\hat{e}_\varphi$, with ϕ being the magnetic flux on the z direction, where the magnetic field is given by $\vec{B} = \phi\delta(x)\delta(y)\hat{z}$. The purpose for introducing this magnetic flux or the Aharonov–Bohm flux is to investigate the arising of persistent currents in the nonrelativistic case [8]. In the present case, we have added this flux in order to obtain persistent currents in the quantum ring described by the model (2), and compare the relativistic behavior with the well-known results of the nonrelativistic case [8].

Hence, by introducing the coupling (2) into the Dirac equation in the presence of the magnetic flux, the Dirac equation becomes

$$i\frac{\partial\psi}{\partial t} = m\hat{\beta}\psi - i\hat{\alpha}^1\left[\frac{\partial}{\partial\rho} + \frac{1}{2\rho} - \hat{\beta}\frac{\sqrt{2ma_1}}{\rho} - \hat{\beta}\sqrt{2ma_2}\rho\right]\psi$$
$$- i\frac{\hat{\alpha}^2}{\rho}\left[\frac{\partial}{\partial\varphi} - i\frac{\phi}{\phi_0}\right]\psi - i\hat{\alpha}^3\frac{\partial\psi}{\partial z}, \tag{5}$$

where q corresponds to the electric charge of the particle, and $\phi_0 = 2\pi/|q|$. The solution of the Dirac equation (5) is given in the form:

$$\psi = e^{-i\mathcal{E}t}\begin{pmatrix} \eta \\ \chi \end{pmatrix}, \tag{6}$$

where $\eta = \eta(\rho,\varphi,z)$ and $\chi = \chi(\rho,\varphi,z)$ are two-spinors. Thus, substituting (6) into the Dirac equation (5), we obtain two coupled equations for η and χ, where the first coupled equation is

$$(\mathcal{E} - m)\eta = -i\sigma^1\left[\frac{\partial}{\partial\rho} + \frac{1}{2\rho} + \frac{\sqrt{2ma_1}}{\rho} + \sqrt{2ma_2}\rho\right]\chi$$
$$- i\frac{\sigma^2}{\rho}\left[\frac{\partial}{\partial\varphi} - i\frac{\phi}{\phi_0}\right]\chi - i\sigma^3\frac{\partial\chi}{\partial z}, \tag{7}$$

while the second coupled equation is

$$(\mathcal{E} + m)\chi = -i\sigma^1\left[\frac{\partial}{\partial\rho} + \frac{1}{2\rho} - \frac{\sqrt{2ma_1}}{\rho} - \sqrt{2ma_2}\rho\right]\eta$$
$$- i\frac{\sigma^2}{\rho}\left[\frac{\partial}{\partial\varphi} - i\frac{\phi}{\phi_0}\right]\eta - i\sigma^3\frac{\partial\eta}{\partial z}. \tag{8}$$

By eliminating χ in (8) and substituting it in (7), we obtain the following second-order differential equation:

$$(\mathcal{E}^2 - m^2)\eta = -\frac{\partial^2\eta}{\partial\rho^2} - \frac{1}{\rho}\frac{\partial\eta}{\partial\rho} + \frac{\eta}{4\rho^2} - \frac{\sqrt{2ma_1}}{\rho^2}\eta$$
$$+ \sqrt{2ma_2}\eta + i\frac{\sigma^3}{\rho^2}\frac{\partial\eta}{\partial\varphi} + \frac{2ma_1}{\rho^2}\eta$$
$$+ 4m\sqrt{a_1a_2}\eta - 2i\sigma^3\frac{\sqrt{2ma_1}}{\rho^2}\frac{\partial\eta}{\partial\varphi}$$
$$- 2\sigma^3\frac{\phi}{\phi_0}\frac{\sqrt{2ma_1}}{\rho^2}\eta + 2ma_2\rho^2\eta$$