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Fast long-range connections in transportation networks

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ABSTRACT

Multidimensional scaling is applied in order to visualize an analogue of the small-world effect implied by edges having different displacement velocities in transportation networks. Our findings are illustrated for two real-world systems, namely the London urban network (streets and underground) and the US highway network enhanced by some of the main US airlines routes. We also show that the travel time in these two networks is drastically changed by attacks targeting the edges with large displacement velocities.

1. Introduction

Long-range connections and their effects in complex networks have been extensively studied in the last years. The most important effect became known as *small-world effect*, as it provides an elegant explanation for Milgram's experiment of the six degrees of separation [1]. The first model capable of explaining the small-world effect was reported by Duncan J. Watts and Steven Strogatz [2] in 1998. Several applications to real-world problems have been reported. The small-world model of Watts and Strogatz reveals how the inclusion of just a few long-range connections into regular networks can drastically decrease the network's diameter (i.e. the number of edges between two nodes). Provided that the displacements are done along the shortest paths of the network, it is well known (e.g. [3]) that in a d-dimensional regular network, where the nodes establish connections constrained by adjacency rules, the average traveling time is of order $\tau \propto N^{1/d}/v$. Here, v corresponds to the number of edges crossed per unit of time. By adding a small number of long-range connections, it has been observed that the average travel time decreases as $\tau \propto \log(N)/v$. Although this approach is correct for many purely topological systems, such as protein–protein [4], *WWW* [5], citations [6] and collaborations [2] networks, it cannot be directly extended to several real-world systems, where the Euclidean space and the displacement velocity play a crucial rule in determining the transportation properties of the network [7–11]. In such systems, the velocity is defined as the Euclidean length traveled per unit of time.

The above properties have already been explored in several previous works. For example, Hayashi and Matsukubo [11] showed how the addition of long-range connections can improve the robustness of some embedded network models against intentional attacks. Recently, G. Li et al. [12] also studied the effects of long-range connections on regular lattices. The authors considered the addition of long-range links between nodes i and j with probability $p_{i,j} \propto r_{i,j}^{-\gamma}$, where $r_{i,j}$ is the Manhattan distance between the nodes and $\gamma > 0$. Their results indicate that optimal transportation occurs when $\gamma = d + 1$ for a d-dimensional system, independently of the navigation strategy adopted.

In order to illustrate the importance of spatial constraints on the transport properties, we show in Fig. 1 a simple embedded network with three short-range connections and one long-range connection. If we want to reach node 4 after departing from 1, we can choose two different shortest paths (i.e. paths with the minimum total Euclidean length): $\{1, 2, 3, 4\}$ or $\{1, 4\}$. Observe that the first path only uses short-range connections, while the second option uses a long-range link. It is clear that if the displacement velocity is fixed for both types of connections, there is no advantage in using any of the options. However, as we will see, a substantially different result can be obtained when we consider different displacement velocities.

Typically, real transportation networks are generalizations of the simple situations discussed above. Real networks are embedded in three-dimensional spaces and display regular properties, in which each node is connected to a small number of neighbors through short-range connections. This strategy reduces the building cost of real-world structures, whose costs are proportional to the total length of the system [13]. In addition, the network is enhanced by a small number of long-range links that spans the space.

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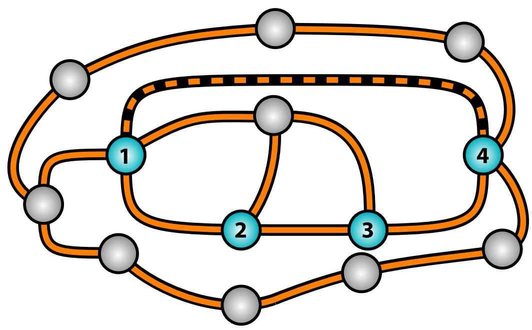

Fig. 1. Sample network with three short-range connections and one long-range connection (dashed line). If we want to reach the node 4 departing from 1, we can choose two shortest paths: {1, 2, 3, 4}, which uses only short-range connections and {1, 4}, which uses the long-range connection. From the point of view of the time spent during travel, there is no difference between the two paths if the displacement velocities are the same along the two types of edges. On the other hand, if we can move faster in the long-range connection, we should choose the second path. Note that the connection lengths correspond to actual Euclidean distances in physical space.

The key point here is to consider that the displacement velocities through the short-range and long-range connections are different. While the displacement along the short-range connections occurs with velocity v_s, in long-range connection this velocity is $v_l = \alpha v_s$, with $\alpha > 1$.

In the current Letter we study two important real-world systems characterized by the features described above, i.e. they have two types of connections with different displacement velocities. The systems that will be considered are: (a) the network of the streets of London plus the respective underground system and commuter trains, and (b) the US highway plus some of the main US airlines connections. We will use multidimensional scaling in order to visualize the effect of the long-range edges and will then quantify the importance of these connections on the transportation properties as well.

This Letter is organized as follow: we start by presenting and discussing the networks used here. Next, multidimensional scaling is used to visualize the effects of the network topology. We then simulate attacks in order to quantify the importance of the long-range connections as compared to the short-range counterparts. Finally, we present the main conclusions and prospects for further investigation.

2. Description of the data

In this section we show how both networks used in this Letter were constructed.

2.1. London Streets Network (LSN)

The central region of London, corresponding to about 13 km × 13 km, was mapped into a network where each node corresponds to the confluence of two or more streets. Also, the underground system and commuter trains of London respective to that same region was then appended into this network. Each underground station was replaced by its closest node from the respective street network. For this network (henceforth called LSN), we have $\alpha \approx 3$ and 3% of the total number of edges correspond to long-range connections. The final network contains 5963 nodes and 8378 edges (8126 from streets and 252 from underground) corresponding to an average degree of 2.81. Fig. 2(a) shows the final version of this network, where the long-range connections are depicted in blue. Observe that, for this network, the long-range connections are 5 times longer than short-range connections, in the average.

2.2. US Highways Network (USHN)

The second network (USHN) considered in this Letter was built using the American highway system enhanced by twenty of the most important airlines connections. The importance of each airline connection was quantified according to the number of passengers that it transports. In this network, the confluences of two or more highways were mapped into nodes. Two nodes are linked if a highway connects them. Moreover, the extremities of the airline connections were replaced by the closest nodes from the highway system. For this network, we have $\alpha \approx 6$ and the fraction of edges corresponding to long-range connections is about 3%, again. The final version of this network is shown in Fig. 2(c). It contains 428 nodes and 674 edges (655 from highways and 19 from airlines), corresponding to an average degree of 3.15. It is interesting to observe that almost all long-range connection tend to link the west coast to the east coast. On average, we found that the length of the long-range connections are 40 times longer than those of the short-range connections.

3. Visualizing the effect of the long-range connections

We applied the classical multidimensional scaling method [14,15] on the networks in order to visualize the effect of the long-range connections. This technique provides a powerful way for obtaining the nodes positions from a dissimilarity matrix, or the so-called *distance cartograms* [16,17]. We denote by $\tau_{i,j}$ the dissimilarity between the nodes i and j. In these networks, this dissimilarity corresponds to the minimum traveling time to reach the destination node j after departing from i. In order to evaluate the value of $\tau_{i,j}$ each edge (i, j) of the network received a weight corresponding to $\tau_{i,j} = \ell_{i,j}/v_{i,j}$, where $\ell_{i,j}$ is the Euclidean distance between i and j and $v_{i,j} = v_s$ if (i, j) is a short-range connection or $v_{i,j} = \alpha v_s$ if (i, j) is a long-range connection. The dissimilarity matrix is defined as the $N \times N$ symmetric matrix $\mathbf{T}$, which has elements $\tau_{i,j}$. Now, the following matrix is obtained from $\mathbf{T}$:

$$\mathbf{B} = -\frac{1}{2}\left(\mathbf{I} - \frac{1}{N}\mathbf{U}\mathbf{U}^T\right)\mathbf{T}'\left(\mathbf{I} - \frac{1}{N}\mathbf{U}\mathbf{U}^T\right), \quad (1)$$

where $\mathbf{U}$ is a vector $N \times 1$ whose elements are all equal to one, $\mathbf{I}$ is the $N \times N$ identity matrix and $\mathbf{T}'$ is a matrix whose elements are equal to the square of the elements of $\mathbf{T}$, i.e. $\tau'_{i,j} = \tau^2_{i,j}$. The eigenvalues of $\mathbf{B}$ are then identified, and only those which are larger than zero are considered in order to build the next matrix, $\mathbf{E}$. Then, these $n \leqslant N$ eigenvalues are sorted in decreasing order, yielding the sequence $\lambda_1 > \lambda_2 > \cdots > \lambda_n > 0$. The respective eigenvectors are stacked as columns of a matrix $\mathbf{E}$ with dimension $N \times n$. The coordinates of the N nodes can now be obtained, up to a rigid body transformation, as:

$$\mathbf{X} = \mathbf{E}\begin{pmatrix} \sqrt{\lambda_1} & 0 & \cdots & 0 \\ 0 & \sqrt{\lambda_2} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \sqrt{\lambda_n} \end{pmatrix}. \quad (2)$$

The dimension of the final coordinates is approximately given by the number of positive eigenvalues, n. The results are shown in Figs. 2(b) and 2(d) for LSN and USHN, respectively. It is clear from these figures that the peripheral regions of London were brought close to the central region. For the USHN case – Fig. 2(d), one can observe a folding effect bringing together the west and the east coast over the US map. In both figures, the edge lengths are approximately proportional to the traveling time required to cross that respective edge. Observe that an element $\tau_{i,j}$ can represent anything that provides some joint information about the nodes i and j. In the special case when this information is given by the 2D