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Type-I intermittency with noise versus eyelet intermittency

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ABSTRACT

In this Letter we compare the characteristics of two types of the intermittent behavior (type-I intermittency in the presence of noise and eyelet intermittency taking place in the vicinity of the chaotic phase synchronization boundary) supposed hitherto to be different phenomena. We show that these effects are the same type of dynamics observed under different conditions. The correctness of our conclusion is confirmed by the consideration of different sample systems, such as quadratic map, Van der Pol oscillator and Rössler system. Consideration of the problem concerning the upper boundary of the intermittent behavior also confirms the validity of the statement on the equivalence of type-I intermittency in the presence of noise and eyelet intermittency observed in the onset of phase synchronization.

0. Introduction

Intermittency is well known to be an ubiquitous phenomenon in nonlinear science. Its arousal and main statistical properties have been studied and characterized already since long time ago, and different types of intermittency have been classified as types I–III intermittencies [1,2], on–off intermittency [3,4], eyelet intermittency [5–7] and ring intermittency [8].

Despite of some similarity (the presence of two different regimes alternating suddenly with each other in the time series), every type of intermittency is governed by its own certain mechanisms and characteristics of the intermittent behavior (such as the dependence of the mean length of the laminar phases on the control parameter, the distribution of the laminar phase lengths, etc.) of different intermittency types are distinct. There are no doubts that different types of intermittent behavior may take place in a wide spectrum of systems, including cases of practical interest for applications in radio engineering, medical, physiological, and other applied sciences.

This Letter is devoted to the comparison of characteristics of type-I intermittency in the presence of noise and eyelet intermittency taking place in the vicinity of the phase synchronization boundary. Although these types of intermittency are known to be characterized by different theoretical laws, we show here for the first time that these two types of the intermittent behavior considered hitherto as different phenomena are, in fact, the same type of the system dynamics.

The structure of the Letter is the following. In Section 1 we give the brief theoretical data concerning both the type-I intermittency with noise and eyelet intermittency observed in the vicinity of the phase synchronization boundary as well as arguments confirming the equivalence of these types of the dynamics. Sections 2–3 aim to verify the statement given in Section 1 by means of numerical simulations of the dynamics of several model systems such as a quadratic map, Rössler oscillators, etc. Eventually, in Section 4 we discuss the problem of the upper boundary of the intermittent behavior. The final conclusions are given in Section 5.

1. Relation between type-I intermittency with noise and eyelet intermittency

First, we consider briefly both eyelet intermittency in the vicinity of the phase synchronization boundary and type-I intermittency in the presence of noise following conceptions accepted generally. The main arguments confirming equivalence of these types of the intermittent behavior are given afterwards.

1.1. Type-I intermittency with noise

The intermittent behavior of type-I is known to be observed below the saddle-node bifurcation point, with the mean length of laminar phases T being inversely proportional to the square root of the criticality parameter $(\varepsilon_c - \varepsilon)$, i.e.

$$T \sim (\varepsilon_c - \varepsilon)^{-1/2}, \tag{1}$$

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where ε is the control parameter and ε_c is its bifurcation value corresponding to the bifurcation point [9]. The influence of noise on the system results in the transformation of characteristics of intermittency [10–12], with the intermittent behavior being observed in this case both below and above the saddle-node bifurcation point ε_c. In the supercritical region [12] of the control parameter values (i.e., above the point of bifurcation, $\varepsilon > \varepsilon_c$) the mean length T of the laminar phases is given by

$$T = \frac{1}{k\sqrt{\varepsilon - \varepsilon_c}} \exp\left(\frac{4(\varepsilon - \varepsilon_c)^{3/2}}{3D}\right), \tag{2}$$

where $k = \text{const}$, D is the intensity of a delta-correlated white noise ξ_n [$\langle \xi_n \rangle = 0$, $\langle \xi_n \xi_m \rangle = D\delta(n-m)$], with Eq. (2) being applicable in the region

$$D^{2/3} \ll |\varepsilon - \varepsilon_c| \ll 1 \tag{3}$$

of the control parameter plane [10,13]. In this region the criticality parameter $(\varepsilon - \varepsilon_c)$ is large enough and, therefore, the approximate equation

$$\ln T = B(\varepsilon - \varepsilon_c)^{3/2} - \ln k \tag{4}$$

(where $B = \text{const}$) is used typically (see [11] for detail) instead of (2). In turn, the distribution $p(\tau)$ of the laminar phase lengths τ is governed by the exponential law [12]

$$p(\tau) = T^{-1} \exp(-\tau/T). \tag{5}$$

1.2. Eyelet intermittency

For the chaotic systems in the vicinity of the phase synchronization boundary (if the natural frequencies of oscillator and external signal are detuned slightly) two types of the intermittent behavior and, correspondingly, two critical values are reported to exist [5,6,14]. Below the boundary of the phase synchronization regime the dynamics of the phase difference $\Delta\varphi(t)$ features time intervals of the phase synchronized motion (laminar phases) persistently and intermittently interrupted by sudden phase slips (turbulent phases) during which the value of $|\Delta\varphi(t)|$ jumps up by 2π. For two coupled chaotic systems there are two values of the coupling strength $\varepsilon_1 < \varepsilon_2$ being the characteristic points which are considered to separate the different types of the dynamics. Below the coupling strength value ε_1 the type-I intermittency is observed, with the power law $T \sim (\varepsilon_1 - \varepsilon)^{-1/2}$ taking place for the mean length of the laminar phases, whereas above the critical point ε_2 the phase synchronization regime is revealed. For the coupling strength $\varepsilon \in (\varepsilon_1; \varepsilon_2)$ the super-long laminar behavior (the so called *"eyelet intermittency"*) should be detected. For eyelet intermittency (see, e.g. [5,6]) the dependence of the mean length T of the laminar phases on the criticality parameter is expected to follow the law

$$T \sim \exp\bigl(\kappa(\varepsilon_2 - \varepsilon)^{-1/2}\bigr) \tag{6}$$

or

$$\ln(1/T) = c_0 - c_1(\varepsilon_2 - \varepsilon)^{-1/2} \tag{7}$$

(c_0, c_1 and κ are the constants) given for the first time in [15] for the transient statistics near the unstable-unstable pair bifurcation point. The analytical form of the distribution of the laminar phase lengths has not been reported anywhere hitherto for eyelet intermittency.

The theoretical explanation of the eyelet intermittency phenomenon is based on the boundary crisis of the synchronous attractor caused by the unstable-unstable bifurcation when the saddle periodic orbit and repeller periodic orbit join and disappear [5, 14]. This type of the intermittent behavior has been observed both in the numerical calculations [5,6] and experimental studies [7] for the different nonlinear systems, including Rössler oscillators.

1.3. Theory of equivalence of the considered types of behavior

Although type-I intermittency with noise and eyelet intermittency taking place in the vicinity of the chaotic phase synchronization onset seem to be different phenomena, they are really the same type of the dynamics observed under different conditions. The difference between these types of the intermittent behavior is only in the character of the external signal. In case of the type-I intermittency with noise the stochastic signal influences on the system, while in the case of eyelet intermittency the signal of chaotic dynamical system is used to drive the response chaotic oscillator. At the same time, the core mechanism governed the system behavior (the motion in the vicinity of the bifurcation point disturbed by the stochastic or deterministic perturbations) is the same in both cases. To emphasize the weak difference in the character of the driving signal we shall further use the terms "type-I intermittency with noise" and "eyelet intermittency" despite of the fact of the equivalence of these types of the intermittent behavior.

Indeed, the phenomena observed near the synchronization boundary for periodic systems whose motion is perturbed by noise (in other words, the behavior in the vicinity of the saddle-node bifurcation perturbed by noise) have been shown recently to be the same as for chaotic oscillators in the vicinity of the phase synchronization boundary [5,12,13,16]. Thus, both for two coupled chaotic Rössler systems and driven Van der Pol oscillator the same scenarios of the synchronous regime destruction have been revealed [16]. Moreover, for two coupled Rössler systems the behavior of the conditional Lyapunov exponent in the vicinity of the onset of the phase synchronization regime is governed by the same laws as in the case of the driven Van der Pol oscillator in the presence of noise [13]. Additionally, when the turbulent phase begins the phase trajectory demonstrates motion being close to periodic both for the eyelet intermittency observed in the vicinity of the phase synchronization boundary (see [14]) and for type-I intermittency with noise. Finally, the repeller and saddle periodic orbits of the same period in the vicinity of the parameter region corresponding to the intermittent behavior tend to coalesce with each other (see, e.g. [5,12]) for both these types of the intermittent behavior. Obviously, if the phenomena observed near the saddle-node bifurcation point for the systems whose motion is perturbed by noise are the same as for chaotic oscillators in the vicinity of the phase synchronization onset, one can expect that the intermittent behavior of two coupled chaotic oscillators near the phase synchronization boundary (eyelet intermittency) is also exactly the same as intermittency of type-I in the presence of noise in the supercritical region.

So, if type-I intermittency with noise and eyelet intermittency taking place in the vicinity of the chaotic phase synchronization onset are the same type of the system dynamics, the theoretical equations (4) and (7) obtained for these types of the intermittent behavior are the approximate expressions being the different forms of Eq. (2) describing the dependence of the mean length of the laminar phases on the criticality parameter. Therefore, Eq. (7) can be deduced from Eq. (4) and vice versa. As a consequence, the coefficients B, k and c_0, c_1 in (4) and (7) are related with each other. Obviously, the mean length of the laminar phases must obey Eqs. (4) and (7) simultaneously, independently whether the system behavior is classified as eyelet intermittency or type-I intermittency with noise. Additionally, the laminar phase length distribution for the considered type of behavior must satisfy the exponential law (5).

The intermittent behavior under study is considered in the coupling strength range $\varepsilon_c < \varepsilon < \varepsilon_2$. In the case of the system driven by external noise (type-I intermittency with noise) the lower boundary value ε_c corresponds to the saddle-node bifurcation point when external noise is switched off. For the dynamical