Physics Letters A 376 (2012) 626–630

Contents lists available at SciVerse ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Propagation of flexural wave in periodic beam on elastic foundations

Dianlong Yu [a,b,*], Jihong Wen [a,b], Huijie Shen [a,b], Yong Xiao [a,b], Xisen Wen [a,b]

[a] Institute of Mechatronical Engineering, National University of Defense Technology, Changsha 410073, China
[b] Laboratory of Science and Technology on Integrated Logistics Support, National University of Defense Technology, Changsha 410073, China

ARTICLE INFO

Article history:
Received 15 October 2011
Received in revised form 18 November 2011
Accepted 28 November 2011
Available online 7 December 2011
Communicated by R. Wu

Keywords:
Phononic crystal
Beam
Band gap
Elastic foundation

ABSTRACT

The propagation properties of flexural wave in the periodic beam on elastic foundations are studied theoretically. The wavenumbers and traveling wave characteristics in the beam on elastic foundations are analyzed. Basing on the equations of motion, the complex band structures and frequency response function are calculated by the transfer matrix method. And the Bragg and locally resonant gaps properties and the effects are researched. A gap with low frequency and wide range can exist in a beam on elastic foundations.

1. Introduction

Extensive effort has been made toward the analysis of the propagation of waves in periodic structures that consist of a number of identical elements joined together in an identical manner [1–3]. Interest in these scenarios has been sparked by the existence of complete elastic band gaps within which all sound and vibration are forbidden.

In the last decade, the propagation of elastic or acoustic waves in periodic composite materials called Phononic Crystals (PCs) has received much attention. Some investigations related with the applications on sound and fundamental physics have been devoted to research deeply, such as sound insulation [4,5], acoustic imaging [6,7], acoustic collimating [8,9], negative dynamic mass [10,11], and so on, that exhibits the common features of the waves coupling with structures. The PCs theory expands the research field and enriches the research content of periodic structure.

There are two kinds of gap formation mechanism for PCs, Bragg scattering mechanism [4] and locally resonant (LR) mechanism [5]. For the PCs with gaps induced by the Bragg scattering mechanism, the spatial modulation of the elasticity must be of the same order as the wavelength in the gap. And the LR gaps can exist in a frequency range of two orders of magnitude lower than the one resulting from the Bragg scattering.

The problem of beams resting on elastic foundation is often encountered in the analysis of the foundations of buildings, highway and railroad structures, and of geotechnical structures in general [12]. The research on the beams on elastic foundation has been become a pop topic [13–16]. Different models of the foundation can be used, such as one-parameter (Winkle) and two-parameter model. More interests are focusing on the calculation method or static analysis for the beams on elastic foundation. To our knowledge, no work appears in open literature studying on the flexural wave in periodic beam on elastic foundation.

In this Letter, the propagation properties of flexural wave in periodic beam on elastic foundations with one-parameter model are studied. We focus on the effects of the elastic foundation on the band gap.

2. Equations of motion and the transfer matrix method

Fig. 1(a) shows a beam on elastic foundation, the stiffness coefficient of the foundation is k_f.

For an Euler-type beam on elastic foundation, the equation governing flexural vibration is given as follows [16]

$$EI\frac{\partial^4 w}{\partial x^4} + \rho S\frac{\partial^2 w}{\partial t^2} + R = 0, \tag{1}$$

where w is the flexural displacement, EI is the flexural rigidity of the pipe, the cross-sectional beam area is S, ρ is density, respectively. And R is anti-force of the elastic foundation. For the one-parameter model,

$$R = k_f w. \tag{2}$$

For a harmonic traveling wave $w(x,t) = We^{kx}e^{i\omega t}$, one can find the dispersion relations corresponding to one-parameter model:

$$EIk^4 + k_f - \rho S\omega^2 = 0. \tag{3}$$

* Corresponding author at: Institute of Mechatronical Engineering, National University of Defense Technology, Changsha 410073, China. Tel./fax: +86 731 8457 4369.
E-mail address: dianlongyu@yahoo.com.cn (D. Yu).

0375-9601/$ – see front matter
doi:10.1016/j.physleta.2011.11.056

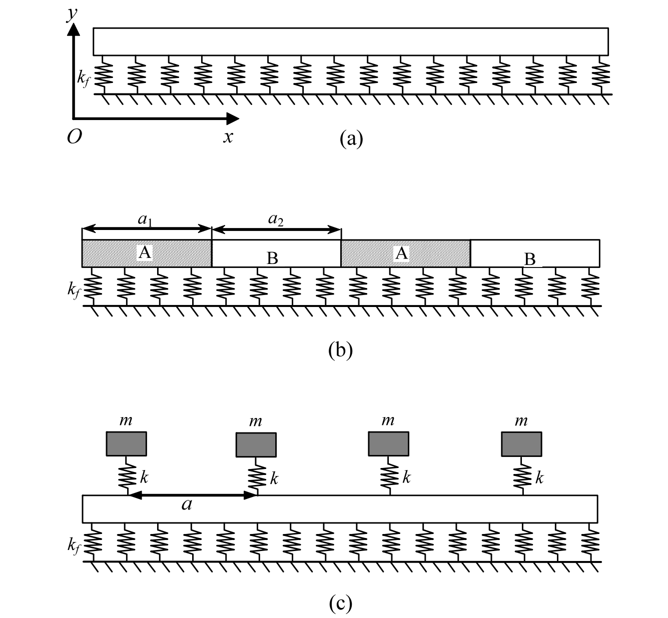

Fig. 1. (a) A sketch of a beam on elastic foundation, (b) a one-dimensional periodic binary beam on elastic foundation, (c) a beam on elastic foundation with periodical LR structures.

For a given ω, the wavenumber roots of Eq. (3) include four different roots, designated k_1, k_2, k_3, k_4.

The harmonic solution of Eq. (1) is

$$w(x,t)=e^{i\omega t}\left(W_1e^{k_1x}+W_2e^{k_2x}+W_3e^{k_3x}+W_4e^{k_4x}\right). \tag{4}$$

Fig. 1(b) shows a periodic binary composite beam on elastic foundation with Bragg band gap mechanism. The beam consists of an infinite repetition of alternating material A with length a_1 and material B with length a_2. And the lattice constant is thus $a=a_1+a_2$.

Fig. 1(c) shows a simple model of a beam on elastic foundation with locally resonant band gap mechanism. The beam is attached periodically with harmonic oscillators. The LR oscillator consists of the spring K and mass M. The lattice constant is a.

For a periodic beam, the continuities of displacement w, slope w', bending moment EIw'' and shear force EIw''' at the interfaces. And following the transfer matrix methodology and Bloch theorem [17], one can derive in the form of a standard eigenvalue problem at hand [18]:

$$\left|\mathbf{T}-e^{iqa}\mathbf{I}\right|=0, \tag{5}$$

where $\mathbf{T}$ is the transfer matrix, and $\mathbf{I}$ is the 4×4 unit matrix. For a given value of ω, Eq. (5) gives corresponding Bloch wave vector of q. Depending on whether q is real or imaginary, the corresponding wave propagates through the beam (pass band) or is damped (band gap).

For a finite periodic structure, the frequency response curve can be calculated to describe its band gap properties. For a periodic beam with m periodic cells, we get

$$W_n=\mathbf{T}^mW_0. \tag{6}$$

3. Wavenumbers and traveling wave characteristics

The material of the beam is chosen as aluminum, whose elastic parameters are $\rho_{\mathrm{Al}}=2730\ \mathrm{kg/m^3}$, $E_{\mathrm{Al}}=7.756\times 10^{10}$ Pa, $\mu_{\mathrm{Al}}=2.887\times 10^{10}$ Pa. The geometrical parameters of the beam with rectangular cross sections are width $b=10$ mm, and thickness $h=10$ mm. And the parameters for foundation are $k_f=1\times 10^5$ Pa.

For the one-parameter model, one can get the wavenumber roots from Eq. (3),

$$k_{1,2}=\pm i\left(\frac{\rho S\omega^2-k_f}{EI}\right)^{\frac14},\qquad k_{3,4}=\pm\left(\frac{\rho S\omega^2-k_f}{EI}\right)^{\frac14}. \tag{7}$$

For the formulation in the following text, we definite a pivotal frequency value

$$f_0=\frac{1}{2\pi}\sqrt{\frac{k_f}{\rho S}}. \tag{8}$$

Fig. 2 describes the relation between the wavenumber roots of flexural wave in aluminum beam on elastic foundation and the frequency, Figs. 2(a) and 2(b) are the real part and imaginary part of k_1, respectively. And Figs. 2(c) and 2(d) are the real part and imaginary part of k_3, respectively.

For k_1, it has both real part and imaginary part while $f<f_0$, and it has only imaginary part in the other frequency range. This indicates flexural wave e^{k_1x} will be attenuate while $f<f_0$, and it is traveling wave $f\geqslant f_0$. So e^{k_1x} is traveling wave, and e^{k_2x} is the negative wave of e^{k_1x}.

And for k_3, it has real part during all frequency range. This indicates flexural wave e^{k_3x} will be attenuate during all frequency range. So e^{k_3x} is near-field wave, and e^{k_4x} is the negative wave of e^{k_3x}.

4. The band gap properties and the effects

4.1. The Bragg gap and effects

The periodic beam with different material parameters is illustrated in Fig. 1. As an example, the material A is chosen as epoxy and material B being aluminum. The elastic parameters of epoxy are $\rho_{\mathrm{ep}}=1180\ \mathrm{kg/m^3}$, $E_{\mathrm{ep}}=4.35\times 10^9$ Pa, $\mu_{\mathrm{ep}}=1.59\times 10^9$ Pa. The lattice constant is chosen to be $a=0.15$ m, and $a_1=a_2=0.075$ m. The materials A and B are chosen to have the same rectangular cross sections with width $b=0.01$ m, and thickness $g=0.01$ m, respectively.

We first calculate the band gaps for the periodic beam without elastic foundation, i.e. $k_f=0$ Pa. The complex band structure for infinite periodic cells can be used to illustrate the band gap and attenuation [19]. The real wave vector is illustrated in Fig. 3(a), and the absolute value of the imaginary part of complex wave vector is illustrated in Fig. 3(b). Two complete band gaps frequency ranges are 387–578 Hz, 1741–3015 Hz. And the imaginary of the complex band structure in Fig. 3(b) can be used to describe the attenuation capability of one periodic cell during the frequency range of band gaps. The maximum attenuation in the two gaps is 0.090 and 0.269, respectively. And the dashed-line denotes the near-field wave, which departs from the traveling wave for all frequencies.

The complex band structure of the periodic material beam on elastic foundation is illustrated in Fig. 4. In this case, the parameter for foundation is $k_f=1\times 10^5$ Pa. The first two band gap frequency ranges are 409–587 Hz and 1746–3020 Hz, respectively. The gap frequencies are enhanced by the elastic foundation. The maximum attenuation in the two gaps is 0.086 and 0.269, respectively. So the attenuation in the gaps is not affected by the elastic foundation.

During 0–114 Hz, no dispersive curve appear in the band structure and there is a stronger attenuation drop in Fig. 4(b). This is because flexural wave is attenuated while $f<f_0$ as mentioned as above. So this attenuation is not the causation of the periodic beam. And for the periodic composite beam, the pivotal frequency value can be given as