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Thin-shell wormhole solutions in Einstein–Hoffmann–Born–Infeld theory

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ABSTRACT

We adopt the Hoffmann–Born–Infeld's (HBI) double Lagrangian approach in general relativity to find black holes and investigate the possibility of viable thin-shell wormholes. By virtue of the non-linear electromagnetic parameter, the matching hypersurfaces of the two regions with two Lagrangians provide a natural, lower-bound radius for the thin-shell wormholes which provides the main motivation to the present study. In particular, the stability of thin-shell wormholes supported by normal matter in higher-dimensional Einstein–HBI–Gauss–Bonnet (EHBIGB) gravity is highlighted.

1. Introduction

It is a well-known fact by now that non-linear electrodynamics (NED) with various formulations has therapeutic effects on the divergent results that arise naturally in linear Maxwell electrodynamics. The theory introduced by Born and Infeld (BI) in 1930s [1] constitutes the most prominent member among such class of viable NED theories. Apart from the healing power of singularities, however, drawbacks were not completely eliminated from the theory. One such serious handicap was pointed out by Born's co-workers shortly after the introduction of the original BI theory. This concerns the double-valued dependence of the displacement vector $\vec{D}(\vec{E})$ as a function of the electric field $\vec{E}$ [2]. That is, for the common value of $\vec{E}$ the displacement $\vec{D}$ undergoes a branching which from physical grounds was totally unacceptable. To overcome this particular problem, Hoffmann and Infeld [2] and Rosen [3], both published successive papers on this issue. Specifically, the model Lagrangian proposed by Hoffmann and Infeld (HI) contained a logarithmic term with remarkable consequences. It removed, for instance, the singularity that used to arise in the Cartesian components of the $\vec{E}$. Being unaware of this contribution by HI, and after almost 70 years, we have rediscovered recently the ubiquitous logarithmic term of Lagrangian while in attempt to construct a model of elementary particle in Einstein–NED theory [4]. In the present study, in addition to the BI and HI's field theoretic contributions we consider the Gauss–Bonnet (GB) extension of general relativity. The reason for adding GB theory and considering EHBIGB theory relies also on the advantages of the GB parameter: for specific choice of such a parameter we eliminate the exotic matter. The double-Lagrangian feature of the problem for two different regions seems to be the pay-off in attaining such a resolution.

In this Letter we wish to make further use of the Hoffmann–Born–Infeld (HBI) Lagrangian in general relativity, in constructing black holes and thin-shell wormholes. This motivation for such a study relies on the fact that the common boundary radius in which each Lagrangian is valid makes a natural shell akin for constructing a thin-shell wormhole. That is, the thin-shell whose energy–momentum maintains the wormhole can be identified as the boundary on which the HBI Lagrangians match. Further, the fact that the shell's radius lies outside the horizon – a motto for traversable wormholes – is satisfied, because the NED parameter can be chosen arbitrary. Our analysis reveals that for the negative GB parameter ($\alpha < 0$) thin-shell wormholes supported by normal matter exists, and they are stable against linear radial perturbations. Due to the intricate structure of the potential function, our stability analysis is carried out numerically and plots are made in $d = 5$.

2. *d*-dimensional stable, normal matter thin-shell wormhole in EHBIGB theory

Our action in d-dimensional EHBIGB theory of gravity is given by (we use $c = \hbar = k_B = 8\pi G = \frac{1}{4\pi\epsilon_\circ} = 1$)

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$$S=\frac{1}{2}\int dx^d\sqrt{-g}\left\{-\frac{(d-2)(d-1)}{3}\Lambda+R+\alpha L_{GB}+\mathcal{L}(\mathcal{F})\right\} \quad (1)$$

with the line element

$$ds^2=-f(r)\,dt^2+\frac{1}{f(r)}\,dr^2+r^2\,d\Omega_{d-2}^2 \tag{2}$$

where $L_{GB}=R_{\mu\nu\gamma\delta}R^{\mu\nu\gamma\delta}-4R_{\mu\nu}R^{\mu\nu}+R^2$, α is the GB parameter and $d\Omega_{d-2}^2$ is the line element of the $(d-2)$-dimensional sphere. In what follows we shall label the spherical coordinates by θ_i for $1\leqslant i\leqslant d-2$. Here we have

$$\mathcal{L}(\mathcal{F})=\begin{cases}\mathcal{L}_-, & r\leqslant\sqrt{qb},\\ \mathcal{L}_+, & r\geqslant\sqrt{qb},\end{cases} \tag{3}$$

in which

$$\mathcal{L}_+=-\frac{2}{b^2}(k+\alpha\epsilon_+-\ln\epsilon_+) \tag{4}$$

and

$$\mathcal{L}_-=-\frac{2}{b^2}\big(k+\alpha\epsilon_--\ln|\epsilon_-|\big) \tag{5}$$

with $q=$ electric charge, $b=$ Born–Infeld (BI) parameter, $\alpha=1$, $k=\ln 2-2$ and $\epsilon_\pm=1\pm\sqrt{1+2b^2\mathcal{F}}$. Our notation is such that, $\mathcal{F}=F_{\mu\nu}F^{\mu\nu}$ and the electric field 2-form is given by

$$\mathbf{F}=E_r\,dt\wedge dr. \tag{6}$$

Having $\mathcal{L}_+$ for $r^4>q^2b^2$ and $\mathcal{L}_-$ for $r^4<q^2b^2$ imposes $(\mathcal{L}_+=\mathcal{L}_-)_{r^4=q^2b^2}$ which is satisfied, as it should. The nonlinear Maxwell equation

$$d\big(\mathcal{L}_F{}^\star\mathbf{F}\big)=0,\qquad \left(\mathcal{L}_F=\frac{\partial\mathcal{L}}{\partial F}\right) \tag{7}$$

in d dimensions leads to the radial electric field

$$E_r=\frac{qr^{(d-2)}}{(q^2b^2+r^{2(d-2)})}. \tag{8}$$

At $r^4=q^2b^2$, one gets $E_r=\frac{1}{2b}$ which is the maximum value that E_r may take. Variation of the action (1) yields the field equations as

$$\mathcal{G}_\mu{}^\nu+\frac{(d-2)(d-1)}{6}\Lambda\delta_\mu{}^\nu=T_\mu{}^\nu, \tag{9}$$

where the energy–momentum tensor is given by

$$T_\mu{}^\nu=\frac{1}{2}\big(\mathcal{L}\delta_\mu{}^\nu-4\mathcal{L}_\mathcal{F}F_{\mu\lambda}F^{\nu\lambda}\big), \tag{10}$$

which clearly gives $T_t{}^t=T_r{}^r=(\frac{1}{2}\mathcal{L}-\mathcal{L}_\mathcal{F}\mathcal{F})$, stating $\mathcal{G}_t{}^t=\mathcal{G}_r{}^r$ and $T_{\theta_i}{}^{\theta_i}=\frac{1}{2}\mathcal{L}$. Our ansatz metric function can be expressed more conveniently by

$$f(r)=1-r^2H(r), \tag{11}$$

in which $H(r)$ is a function to be determined [5]. One should notice that, our choice of $g_{tt}=-(g_{rr})^{-1}$ is a direct result of $\mathcal{G}_t{}^t=\mathcal{G}_r{}^r$ up to a constant coefficient, which is chosen to be one. The Einstein tensor components are found to be

$$\mathcal{G}_t{}^t=\mathcal{G}_r{}^r=-\frac{(d-2)}{2r^{d-2}}\big[r^{d-1}\big(H+\tilde\alpha H^2\big)\big]', \tag{12}$$

$$\mathcal{G}_{\theta_i}{}^{\theta_i}=-\frac{(d-2)}{2r^{d-3}}\big[r^{d-1}\big(H+\tilde\alpha H^2\big)\big]'', \tag{13}$$

where $\tilde\alpha=(d-3)(d-4)\alpha$. Here we use the definition of the energy–momentum tensor (10) and the Lagrangian given by (1) to get

$$T_t^t=T_r^r=-\frac{1}{b^2}\ln\left(1+\frac{b^2q^2}{r^{2(d-2)}}\right), \tag{14}$$

and

$$T_{\theta_i}{}^{\theta_i}=-\frac{1}{b^2}\ln\left(1+\frac{b^2q^2}{r^{2(d-2)}}\right)+\frac{2q^2}{b^2q^2+r^{2(d-2)}}. \tag{15}$$

Having the closed form of energy–momentum tensor enable us to investigate the energy conditions. The weak energy condition (WEC) reads

$$\rho\geqslant 0,\qquad \rho+p_i\geqslant 0, \tag{16}$$

in which $\rho=-T_t^t$ and $p_i=T_i^i$ and $i=1,\dots,d-1$. It is not difficult to see that WEC is satisfied. The strong energy condition (SEC) states that

$$\rho+\sum_{i=1}^{d-1}p_i\geqslant 0,\qquad \rho+p_i\geqslant 0, \tag{17}$$

which after substitution we get

$$\frac{2b^2q^2}{b^2q^2+r^{2(d-2)}}-\ln\left(1+\frac{b^2q^2}{r^{2(d-2)}}\right)\geqslant 0. \tag{18}$$

This condition is not satisfied for arbitrary values of the parameters, rather, for the case $\frac{b^2q^2}{r^{2(d-2)}}\leqslant 0.254$ it is satisfied. Finally the dominant energy condition (DEC) is given by

$$p_{eff}=\frac{1}{d-1}\sum_{i=1}^{d-1}p_i\geqslant 0. \tag{19}$$

This equation, after substitution, reads

$$\frac{d-2}{d-1}\left(\frac{2b^2q^2}{b^2q^2+r^{2(d-2)}}\right)-\ln\left(1+\frac{b^2q^2}{r^{2(d-2)}}\right)\geqslant 0, \tag{20}$$

which is not satisfied in all regions, and depends on dimensions (the range of validity is even smaller than SEC). The tt and rr components of the Einstein equations, upon substitution of (14), read

$$-\frac{(d-2)}{2r^{d-2}}\big[r^{d-1}\big(H+\tilde\alpha H^2\big)\big]'+\frac{(d-2)(d-1)}{6}\Lambda=-\frac{1}{b^2}\ln\left(1+\frac{b^2q^2}{r^{2(d-2)}}\right), \tag{21}$$

whose integral for $H(r)$ yields

$$H(r)+\tilde\alpha H(r)^2=\frac{\Lambda}{3}+\frac{m}{r^{d-1}}+\frac{2}{b^2(d-2)r^{d-1}}\int r^{d-2}\ln\left(1+\frac{b^2q^2}{r^{2(d-2)}}\right)dr, \tag{22}$$

in which m is the ADM mass in the Reissner–Nordstrom (RN) case. As a result, we obtain the metric function to be

$$f_\pm(r)=1+\frac{r^2}{2\tilde\alpha}\left\{1\pm\sqrt{1+4\tilde\alpha\Theta}\right\} \tag{23}$$

where

$$\Theta=\frac{\Lambda}{3}+\frac{m}{r^{d-1}}+\frac{2}{b^2(d-2)r^{d-1}}\int r^{d-2}\ln\left(1+\frac{b^2q^2}{r^{2(d-2)}}\right)dr. \tag{24}$$