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Two-stream instabilities in degenerate quantum plasmas

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ABSTRACT

The quantum effects on the plasma two-stream instability are studied by the dielectric function approach. The analysis suggests that the instability condition in a degenerate dense plasma deviates from the classical theory when the electron drift velocity is comparable to the Fermi velocity. Specifically, for a high wave vector comparable to the Fermi wave vector, a degenerate quantum plasma has larger regime of instability than predicted by the classical theory. A regime is identified, where there are unstable plasma waves with frequency 1.5 times of a normal Langmuir wave.

1. Introduction

The physics of dense plasmas becomes increasingly important [1–3]. Especially, an accurate theory for the two-stream instability is critical in order to understand the dense electron beams in the inertial confinement fusion [1,4,5] and astrophysical events such as the gamma ray burst [6–9]. It is noteworthy to realize that many classical theories need to be revised in dense plasmas because the quantum effects cannot be ignored; a few physical processes deviating from the classical prediction have been identified [2,3,10–18]. The main goal of this paper is to study the electron quantum effects on the two-stream instabilities.

There has been a general theoretical attempt to study the quantum effects on the two-stream instabilities by utilizing fluid-type equations [10,11]. In this paper, the author instead utilizes the Lindhard dielectric function [19] with a view of including the kinetic effects. The relative advantage of the current approach over the quantum fluid theory will be discussed. Two cases are studied; the first case when a plasma has two different groups of electrons and the second case when the electrons have a drift velocity different from the ions. The analysis in this paper suggests the following. When the electron drift velocity is comparable to or lower than the Fermi velocity, the quantum effect cannot be ignored; the instability regime is larger than the classical prediction. Also, a regime is identified where the unstable Langmuir wave can have higher frequency than a normal Langmuir wave by 1.5 times.

This paper is organized as follows. In Section 2, the dielectric function approach in the two-stream instability is introduced. In Section 3, the case, when there are two different groups of degenerate electrons with different drift velocities, is considered. In Section 4, the case, when the degenerate electrons have a drift velocity different from the ions, is studied. In Section 5, the summary and discussion are provided.

2. Dielectric function for the two-stream instability analysis

The longitudinal dielectric function of a plasma is given as

$$\epsilon(\mathbf{k},\omega) = 1 + \frac{4\pi e^2}{k^2}\sum \chi_i, \tag{1}$$

where the summation is over the groups of particle species and χ_i is the susceptibility. Given the dielectric function $\epsilon(k,\omega)$, the analysis of the two-stream instability can be performed by finding the roots of $\epsilon(k,\omega)=0$; If the dielectric function has a root $\epsilon(k,\omega)=0$ in the complex upper-half plane, the plasma is unstable to the two-stream instabilities. In classical plasmas, the susceptibility is given as

$$\chi_i^C(k,\omega) = \frac{n_i Z_i^2}{m_i}\int\left[\frac{\mathbf{k}\cdot\nabla_v f_i}{\omega - \mathbf{k}\cdot\mathbf{v}}\right]d^3\mathbf{v} \tag{2}$$

where m_i (Z_i, n_i) is the particle mass (charge, density) and f_i is the distribution with the normalization $\int f_i d^3\mathbf{v} = 1$. For degenerate electrons, the susceptibility χ_e by Lindhard [19] is given as

$$\chi_e^Q(\mathbf{k},\omega) = \frac{3n_e}{m_e v_F^2}h(z,u), \tag{3}$$

where $v_F = \sqrt{2E_F/m_e}$ is the Fermi velocity, $E_F = \hbar^2 k_F^2/2m_e$ ($k_F = (3\pi^2 n_e)^{1/3}$) is the Fermi energy (Fermi wave vector), $z = k/2k_F$, $u = \omega/kv_F$, and $h = h_r + ih_i$. The real part of h is given from Lindhard [19] as

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$$h_r = \frac{1}{2} + \frac{1}{8z}\left(1-(z-u)^2\right)\log\left(\frac{|z-u+1|}{|z-u+1|}\right) + \frac{1}{8z}\left(1-(z+u)^2\right)\log\left(\frac{|z+u+1|}{|z+u+1|}\right). \tag{4}$$

The imaginary part h_i is from [19]

$$\begin{aligned} h_i &= \frac{\pi}{2}u, \quad \text{for } |z+u|<1, \\ &= \frac{\pi}{8z}u, \quad \text{for } |z-u|<1<|z+u|, \\ &= 0, \quad \text{if } |z-u|>1. \end{aligned} \tag{5}$$

In the next two sections, the author considers two cases. The first case is when there are two groups of electrons, where each group is the Maxwellian distribution (the degenerate Fermi distribution) with different drift velocities. The second case is when the Maxwellian (completely degenerate) electrons have a different drift velocity from the ions. In the first case, the dielectric function is given as

$$\begin{aligned} \epsilon &= 1 + \left(4\pi e^2/k^2\right)\left[\chi_e^C(\omega,k) + \chi_e^C(\omega - \mathbf{k}\cdot\mathbf{v}_0, k)\right] \\ &\text{or } 1 + \left(4\pi e^2/k^2\right)\left[\chi_e^Q(\omega,k) + \chi_e^Q(\omega - \mathbf{k}\cdot\mathbf{v}_0, k)\right], \end{aligned} \tag{6}$$

where $\mathbf{v}_0$ is the drift velocity. In the second case, the dielectric function is given as

$$\begin{aligned} \epsilon &= 1 + \left(4\pi e^2/k^2\right)\left[\chi_e^C(\omega,k) + \chi_i^C(\omega - \mathbf{k}\cdot\mathbf{v}_0, k)\right] \\ &\text{or } 1 + \left(4\pi e^2/k^2\right)\left[\chi_e^Q(\omega,k) + \chi_i^C(\omega - \mathbf{k}\cdot\mathbf{v}_0, k)\right]. \end{aligned} \tag{7}$$

In deriving Eqs. (6) and (7), the author uses Eqs. (2) and (3). In the classical plasmas, it is trivial to add two susceptibilities of two different groups of electrons; the first equations in Eq. (6) and Eq. (7), which are the sum of the two susceptibilities, is obvious. On the other hand, it is not clear for the second one in Eq. (6) because the existence of another group of electrons would affect the electron transition due to the degeneracy. In this paper, the author assumes that this interference is negligible or $\hbar k/m_e < v_0$. If $\hbar k/m_e \cong v_0$, the treatment of the degeneracy would be more complicated and beyond the scope of this paper. In the example as illustrated in Fig. 2, the condition, $\hbar k/m_e < v_0$, is satisfied. The validity of the second equation in Eq. (7) is straightforward as the ions does not affect the degeneracy of the electron transition.

3. When there are two different groups of (degenerate) electrons

In this section, the author analyzes Eq. (6). In the conventional classical two-stream instability, the threshold condition for the two-stream instability is that there should be a local maximum between $\omega_{pe} < \omega < kv_0$ and that the local maximum should be less than 0. In Fig. 1, the author plots the classical dielectric function as a function of ω for a particular k. The hump at $\omega \cong 1.3\omega_{pe}$ is the local maximum less than 0; the plasma Langmuir wave is unstable to the two-stream instability.

For a fixed electron density and temperature (or completely degenerate), the unstable regimes are estimated by plotting $\epsilon(k,\omega)$ as a function of ω and using the two criteria given above. The analysis for various v_0 and various wave vector k suggests that the plasma is unstable when $k < k_C(v_0)$ where $k_C(v_0)$ is a threshold value. In Fig. 2, the author plots the instability boundary for a plasma with $n_e = 10^{24}$/cc. In the figure, for the classical plasmas (Classical 21 eV), the instability begins to emerge when $v_0/v_F \geq 3.5$. As the electron temperature of the classical plasma gets lower, the cutoff velocity decreases. For the complete degenerate plasma, the instability begins to emerge when $v_0/v_F \geq 2.75$, which is the minimal drift for the existence of the two-stream instability for any temperature. From Fig. 2, it can be concluded

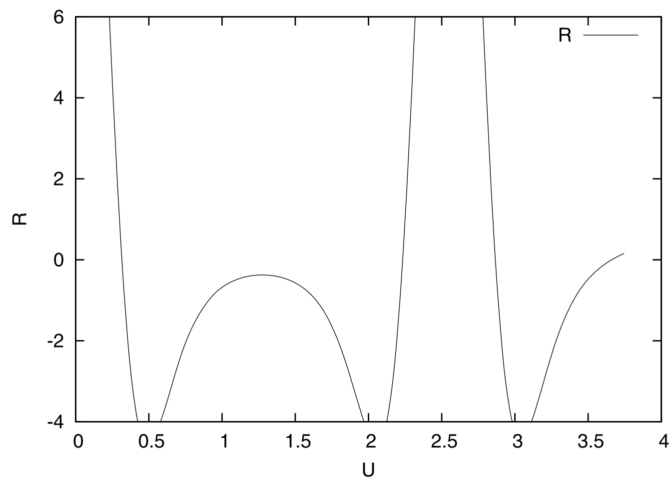

Fig. 1. The real part of the dielectric function $R = \mathrm{Re}[\epsilon(k,\omega)]$ as a function of ω. The x-axis is $U = \omega/\omega_{pe}$ and the y-axis is $R = \mathrm{Re}[\epsilon]$. In this example, $n_e = 10^{24}$/cc, $T_e = 21$ eV, $k\lambda_{de} = 0.26$, $\lambda_{de} = (T_e/4\pi n_e e^2)^{1/2}$ is the Debye length and the drift velocity has the electron kinetic energy of 720 eV so that $kv_0/\omega_{pe} \cong 2.29$. The local maximum of the real part at $\omega = 1.5\omega_{pe}$ is less than 0 so that the plasma is unstable to the two-stream instability.

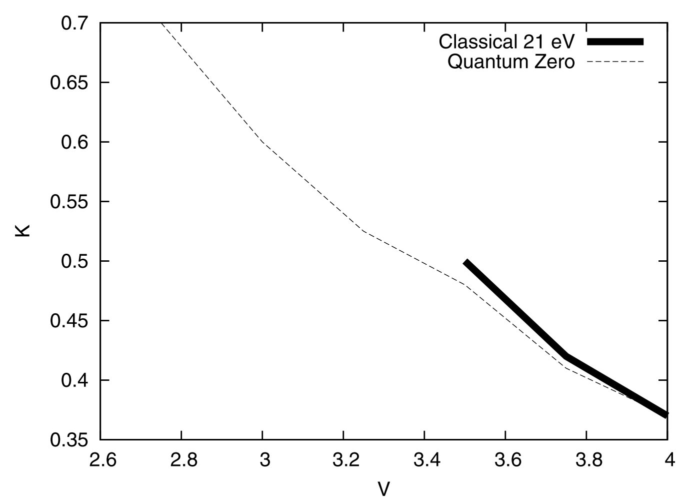

Fig. 2. The boundary wave-vector k_C as a function of the drift velocity. In this example, $n_e = 10^{24}$/cc and $E_F = 36$ eV. The x-axis is $V = v_0/v_F$ and the y-axis is $K = k_C/k_F$. In this figure, the author plots the zero temperature quantum plasma (Quantum Zero). In addition, the author plots the dielectric function of the classical plasmas, where the electron temperature is assumed to be $T_e = 0.6E_F = 21$ eV since the average kinetic energy of the electron in the completely degenerate case is $0.6E_F$.

that the regime of the two-stream instability is larger in the quantum prediction than the classical prediction. As the drift velocity becomes larger than $v_0 > 3.5v_F$, the difference between the quantum plasma and the classical plasma is negligible. For more dense plasma, the regime of the quantum deviation, where the degenerate plasma (classical plasma) is unstable (stable), widens further. In the case considered, the imaginary part of the dielectric function h_r is zero so that the Landau damping is ignored.

The cutoff at $v_0 = 3.5v_F$ ($v_0 = 2.7v_F$), or the end point of the left of each curve in Fig. 2 is due to the finite electron temperature effect; the cutoff is the upper limit where the collective waves can be sustained. The higher the temperature is, the higher the cutoff becomes. There is no such cutoff in the classical plasma (quantum plasma) with the zero electron temperature (no degeneracy), which suggests that the quantum fluid theory cannot predict such cutoff accurately [10,11]. For this reason, the kinetic approach in