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Gated-controlled electron pumping in connected quantum rings

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ABSTRACT

We study the electronic transport across connected quantum rings attached to leads and subjected to time-harmonic side-gate voltages. Using the Floquet formalism, we calculate the net pumped current generated and controlled by the side-gate voltage. The control of the current is achieved by varying the phase shift between the two side-gate voltages as well as the Fermi energy. In particular, the maximum current is reached when the side-gate voltages are in quadrature. This new design based on connected quantum rings controlled without magnetic fields can be easily integrated in standard electronic devices.

1. Introduction

Recently developed nanofabrication of quantum rings and dots by self-assembling [1–4], lithographic [5,6] or etching techniques [7] has opened an active area of research, both theoretical and experimental. Due to their high crystalline quality, the coherence length in these nanostructures is in the micron scale, usually larger than their size, and then electron transport is ballistic [8]. As a consequence, quantum effects, such as quantum interference, play a major role in the design of future nanodevices based on quantum dots and rings. A cornerstone of coherent electron transport and the subsequent interference effects in mesoscopic rings is the celebrated Aharonov–Bohm (AB) effect [9]. Electrons passing through the two arms accumulate a phase difference due to the magnetic flux threading the ring. The resulting interference pattern leads to a modulation of the conductance as a function of the magnetic flux. The predicted phase shift was indeed soon confirmed experimentally [10]. Remarkably, even excitons can undergo AB oscillations [11–15], in spite of being neutral entities.

The combination of two different quantum effects, namely persistent currents in a quantum ring threaded by a static magnetic field [16] and electron pumping under two periodically varying potentials with different phases [17], makes it possible to establish an electric current through connected double-ring systems [18,19]. The resulting current remains finite even if the two leads have identical chemical potentials and the system is in equilibrium. A key ingredient in the AB pump based on connected double-rings is the symmetry breaking due to the oscillating potentials with different phases. But pumping becomes impossible without the static flux that yields persistent current [18]. In this context, Ramos et al. have considered a quantum ring with a quantum dot embedded in one of its arms [20]. A time-harmonic gate voltage was applied to the quantum dot. These authors have demonstrated that the inversion symmetry is not essential to pump electrons, provided that the quantum ring is threaded by a static magnetic flux.

Interference effects of coherent electrons open the possibility of controlling quantum transport without relying on potential barriers or magnetic flux [21,22]. In contrast to previous studies mentioned above on magnetically induced interference effects, in this work we consider a new design of quantum pump based on connected quantum rings, in which electron transport is controlled without applying a magnetic field. We demonstrate that pumped charge can be tuned by applying oscillating side-gate voltages across the rings instead. We show that in this case the phase shift between the two side-gate voltages breaks the symmetry and the system behaves as an efficient quantum pump. In other words, a net electric current is obtained even in equilibrium, and the magnitude of the current is controlled by the phase shift. The pumped current vanishes only for certain values of the phase shift or the chemical potential.

2. Coupled quantum rings under AC side-gate voltages

The system under consideration consists of two quantum rings connected in series and attached to two leads (source and drain). Side-gate voltages $V_{\pm}(t)$ break the symmetry of the upper and lower arms of the rings, as shown in Fig. 1, and act as additional parameters for controlling the electric current through the device, as recently suggested for graphene-based [21,22] and semiconductor nanorings [23]. We assume that the side-gate voltages can be

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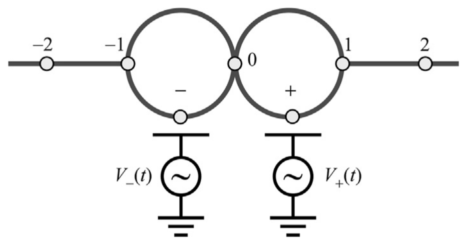

Fig. 1. Schematic diagram of the quantum rings subjected to side-gate voltages $V_\pm(t)$. The equivalent lattice model replaces the side-gate voltages by time-dependent site energies $\varepsilon_\pm(t)$ at sites labeled $\pm$ and two other sites with index ± 1 are attached to semi-infinite chains.

modulated harmonically in time with frequency ω. We neglect capacitance effects as those described in Ref. [24] within the context of quantum rings threaded by an oscillating AB flux.

In order to study electron transport across the connected quantum rings, we mapped the system onto a much simpler yet nontrivial lattice model, depicted in Fig. 1. We replace each quantum ring by three sites of a lattice within the tight-binding approximation. One of the sites, labeled 0, is shared by the two quantum rings. Two sites, labeled $\pm$, have time-dependent energies $\varepsilon_\pm(t)$ and the other two sites, labeled ± 1, are connected to semi-infinite chains. The time-dependent site energies are given by $\varepsilon_\pm(t) = V_0\cos(\omega t \pm \phi)$, where 2ϕ is the phase shift between the two side-gate voltages. To avoid the profusion of free parameters, we assume a uniform transfer integral and vanishing site energies except at sites $\pm$, without loosing generality. The common value of the transfer integral will be set as the unit of energy and we take $\hbar = 1$ throughout the paper.

The time-dependent Schrödinger equation for the amplitude $\psi_j(t)$ at site j reads

$$i\dot{\psi}_j = \varepsilon_\pm(t)\delta_{j,\pm}\psi_j - \sum_{\ell(j)} \psi_{\ell(j)}, \tag{1}$$

where the index $\ell(j)$ runs over the neighboring sites of j and the dot indicates the derivative with respect to time. Using the Floquet formalism, the solution can be expressed in the form

$$\psi_j(t) = \sum_{n=-\infty}^{\infty} A_{n,j} e^{-iE_n t}, \tag{2a}$$

where $E_n = E + n\omega$, E being the quasienergy and n the sideband channel index. Since we are interested in electron transmission across the system, we take the following ansatz for the coefficients $A_{n,j}$ in the expansion (2a)

$$A_{n,j} = \begin{cases} \delta_{n0} e^{ik_n j} + r_n e^{-ik_n j}, & j \le -1, \\ g_n^\pm, & j = \pm, \\ f_n, & j = 0, \\ t_n e^{ik_n j}, & j \ge 1. \end{cases} \tag{2b}$$

This solution corresponds to an electron propagating from the left to the right. Since the phase ϕ breaks the inversion symmetry of the system, the resulting transmission coefficient is not the same as that obtained for an electron coming from the right to the left. This broken symmetry will ultimately lead to the quantum pumping effect.

Inserting the ansatz (2) into the Schrödinger equation (1) yields

$$\begin{aligned} &\delta_{n0} + r_n = f_n + g_n^-, \\ &E_n g_n^- - \alpha_- g_{n+1}^- - \alpha_+ g_{n-1}^- + f_n + \delta_{n0} e^{-ik_n} + r_n e^{ik_n} = 0, \\ &E_n f_n + \delta_{n0} e^{-ik_n} + r_n e^{ik_n} + t_n e^{ikn} + g_n^+ + g_n^- = 0, \\ &E_n g_n^+ - \alpha_+ g_{n+1}^+ - \alpha_- g_{n-1}^+ + f_n + t_n e^{ik_n} = 0, \\ &t_n = f_n + g_n^+, \end{aligned} \tag{3}$$

where for brevity we have defined $\alpha_\pm = (V_0/2)e^{\pm i\phi}$. After straightforward algebra one gets

$$\begin{aligned} &\left[ie^{ik_n}\cot(k_n/2) - e^{-ik_n}\right] g_n^\pm - \alpha_\pm g_{n+1}^\pm - \alpha_\mp g_{n-1}^\pm \\ &\quad + ie^{ik_n}\cot(k_n/2) g_n^\mp = -1 - e^{\mp ik_n}. \end{aligned} \tag{4}$$

Once the amplitudes $g_n^\pm$ have been calculated, the transmission amplitudes t_n are easily obtained with the help of (3). Finally, the transmission probability for the electron coming from the left is

$$T_\rightarrow(E,\omega,\phi) = \sum_n \frac{\sin k_n}{\sin k_0}|t_n|^2, \tag{5}$$

where the sum runs over the propagating channels, namely those channels for which $E_n = E + n\omega$ lies within the band of the leads.

The pumped current also depends on the transmission probability for an electron coming from the right, $T_\leftarrow(E,\omega,\phi)$. Due to the symmetry breaking caused by the side-gate voltages oscillating out of phase, $T_\rightarrow(E,\omega,\phi)$ and $T_\leftarrow(E,\omega,\phi)$ are different. It is a matter of simple algebra to demonstrate that the corresponding equations for an electron incoming from the right are similar to (3) but replacing $g_n^\pm \to g_n^\mp$ and $\alpha_\pm \to \alpha_\mp$. Finally, the pumped current density in equilibrium at zero temperatures is given by [18]

$$J(E_F,\omega,\phi) = \frac{2e}{h}\int_{-2}^{E_F} \left[T_\rightarrow(E,\omega,\phi) - T_\leftarrow(E,\omega,\phi)\right] \mathrm{d}E, \tag{6}$$

where E_F is the Fermi energy and -2 is the minimum energy of the bands at the leads. The time-reversal symmetry is maximally broken when the side-gate voltages are in quadrature ($\phi = \pi/4$) and one would expect maximum pumping efficiency. We will show that this is indeed the case.

3. Results

3.1. Zero side-gate voltages

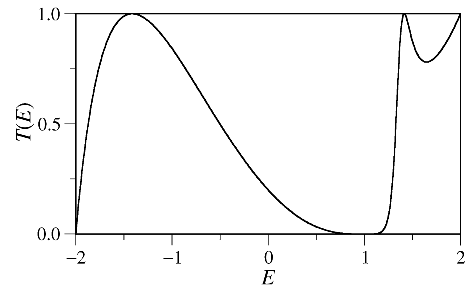

Fig. 2. Transmission probability as a function of energy at zero side-gate voltage. Transmission vanishes at $E = 1$ and displays two local maxima at $E = \pm\sqrt{2}$.

To gain insight into the transmission properties of the connected quantum rings, it is instructive to consider the time-independent case by setting $V_0 = 0$ for the moment. Under these circumstances, only the elastic channel is relevant and the transmission probability $T(E) = |t_0|^2$ can be easily computed with the aid of (3)

$$T(E) = \frac{(E+2)(E-1)^2}{2E^3 - 11E + 10}. \tag{7}$$

Fig. 2 shows the transmission probability as a function of the energy of the incoming electron at zero side-gate voltage. As seen in the figure, transmission vanishes at $E = 1$ and reaches unity at $E = \pm\sqrt{2}$ and at the upper band edge.

The vanishing of the transmission profile at $E = 1$ shown in Fig. 2 can be understood from the coupling of the local modes