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Tunable quantum capacitance and magnetic oscillation in bilayer graphene device

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ABSTRACT

We address the quantum capacitance of a bilayer graphene device in the presence of Rashba spin–orbit interaction (SOI) by applying external magnetic fields and interlayer biases. Quantum capacitance reflects the mixing of the spin-up and spin-down states of Landau levels and can be effectively modulated by the interlayer bias. The interplay between interlayer bias and Rashba SOI strongly affects magnetic oscillations. The typical beating pattern changes tuned by Rashba SOI strength, interlayer bias energy, and temperature are examined as well.

1. Introduction

Bilayer graphene consists of two coupled honeycomb lattices of carbon atoms and provides an interacting two-dimensional electron system [1]. This system can be obtained from graphite crystals through micromechanical cleavage [2]. Unlike monolayer graphene, bilayer graphene has a hyperbolic band structure and a quadratic dispersion relation at the vicinity of the neutrality point (NP, Dirac point) [3]. The tunable gap that can be opened by applying an external perpendicular electric field (interlayer bias) leads to greater expectation in the potential applications of bilayer graphene in electronic devices [4]. Several researchers have exploited bilayer graphene to develop field-effect transistors (FETs) [5–8]. The capacitance characteristic of FETs has elicited extensive attention because of the importance of capacitance in device performance, and some papers have reported experimental results on the capacitance of bilayer graphene [5,6]. In conventional devices, quantum capacitance can be ignored. However, quantum capacitance (C_q) [9] significantly contributes to the total capacitance in such low-dimensional devices [10] given the much thinner oxide layers and higher values of dielectric constant. Meanwhile, quantum capacitance measurements can effectively probe fundamental electronic properties, such as density of states and Landau levels of the material [11–13].

The effects of spin–orbit interaction (SOI) on graphene and its isostructural materials (such as silicene) have also drawn much attention [14–16] because of their potential applications in spintronics and related devices [17,18]. The Rashba SOI [19] induced by breaking the space inversion symmetry of the lattice is considered as the main SOI in monolayer and bilayer graphene. Rashba SOI strength can be as large as or larger than 13 meV [20,21] and can be tuned by varying the electric field. Researchers have recently explored the effects of Rashba SOI on the electronic properties of monolayer graphene [22] and silicene [23] in an external perpendicular magnetic field by calculating the quantum capacitance. Furthermore, Rashba SOI has been theoretically predicted to generate significant changes to the band structure and Landau levels of bilayer graphene [24]. In this Letter, we focus on the magnetic response of bilayer graphene in the presence of Rashba SOI and quantum capacitance in bilayer graphene devices.

2. Model and method

Bilayer graphene, which is considered as two coupled monolayer graphene sheets in a Bernal AB stacking [25], has four different types of sites as follows: **a**, **b** (lower layer) and **A**, **B** (upper layer). Site **A** lies directly on top of **b**, and the two sites are coupled via an interlayer hopping energy of $t \approx 390$ meV. In an external perpendicular magnetic field B, the Landau levels of bilayer graphene without SOI can be expressed as $E_n = \pm\hbar\omega_c\sqrt{n(n-1)}$, where $\omega_c = \frac{2v_F^2 eB}{t}$, $v_F = 10^6$ m/s is the Fermi velocity and $-e$ is the electron charge. Therefore, the two lowest Landau levels of bilayer graphene $E_0 = E_1$ lie at zero energy, unlike those of the monolayer graphene. Given the Rashba SOI, the degeneration of the electron spin is destroyed. Moreover, the effective Hamiltonian

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closest to the K point of the bilayer graphene with an external perpendicular magnetic field and an electric field can be expressed as follows [24,25]:

$$H = \begin{pmatrix} H_+ & H_I \\ H_I^\dagger & H_- \end{pmatrix} + H_R \tag{1}$$

and

$$H_\pm = \begin{pmatrix} \pm U & 0 & v_F\pi^\dagger & 0 \\ 0 & \pm U & 0 & v_F\pi^\dagger \\ v_F\pi & 0 & \pm U & 0 \\ 0 & v_F\pi & 0 & \pm U \end{pmatrix},$$

$$H_I = \begin{pmatrix} 0 & 0 & t & 0 \\ 0 & 0 & 0 & t \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{2}$$

$$H_R = \begin{pmatrix} 0 & i\lambda_R\sigma_- & 0 & 0 \\ -i\lambda_R\sigma_+ & 0 & 0 & 0 \\ 0 & 0 & 0 & i\lambda_R\sigma_- \\ 0 & 0 & -i\lambda_R\sigma_+ & 0 \end{pmatrix} \tag{3}$$

where H is written in terms of the spin basis $(\varphi_{A\uparrow}, \varphi_{A\downarrow}, \varphi_{B\uparrow}, \varphi_{B\downarrow}, \varphi_{a\uparrow}, \varphi_{a\downarrow}, \varphi_{b\uparrow}, \varphi_{b\downarrow})$. $\pi = \pi_x + i\pi_y$ with $(\pi_x, \pi_y) = \vec{P} + e\vec{A}$, where $\vec{P}$ is the momentum operator, and $\vec{A}$ is the magnetic vector potential. The interlayer bias energy U can open a band gap of $2U$, where U and $-U$ are the electric potential on the upper and lower layers respectively. $\sigma_\pm = \frac{1}{2}(\sigma_x \pm i\sigma_y)$ with 2×2 Pauli spin matrices σ_x and σ_y, and λ_R is the Rashba SOI strength. The eigenenergies of this Hamiltonian H can be concisely expressed as follows [24]:

$$E_{n,s}^u = u\sqrt{U^2 + \frac{n}{2}\left(T^2 + 2nW^2 + s\sqrt{4W^4 + 4nW^2T^2 + T^4}\right)} \quad \text{for } n \geqslant 2 \tag{4}$$

$$E_{0,-} = U \quad \text{for } n = 0 \tag{5}$$

$$E_{1,+}^u = u\sqrt{U^2 + T^2 + 2W^2}, \quad E_{1,-} = -U \quad \text{for } n = 1 \tag{6}$$

where $T = \frac{2\sqrt{2}\lambda_R}{t}\hbar\omega$, $W = \frac{2}{t}\hbar^2\omega^2$, $\hbar\omega = \hbar v_F\sqrt{eB/\hbar}$ ($\hbar\omega \approx 25.66$ meV for $B = 1$ T) is the cyclotron energy, $u = \pm$ represents the electron–hole symmetry, and $s = \pm$ denotes the spin-up and spin-down.

Quantum capacitance C_q represents the charge (Q) response in the channel of a device when channel potential U_c ($U_c = E_F/e$) is varied, which can be written as [12,22,23]

$$C_q = e\frac{\partial Q}{\partial E_F} = e^2\int_0^\infty \frac{\partial f(E - E_F)}{\partial E_F} D(E)\,dE \tag{7}$$

where E_F is the Fermi energy. Fermi energy and interlayer bias energy (band gap) can be independently manipulated in dual-gated FETs [7,6]. $f(E - E_F) = [1 + \exp(\frac{E-E_F}{K_BT})]^{-1}$ is the Fermi–Dirac distribution function, and $D(E)$ is the density of states (DOS) per unit area determined by Gaussian-broadened Landau levels.

$$D(E) = \frac{e^2}{2\pi l^2}\sum_{n=0}^{\infty}\sum_{u,s}\frac{1}{\Gamma\sqrt{2\pi}}\exp\left[-\frac{(E - E_{n,s}^u)^2}{2\Gamma^2}\right] \tag{8}$$

Γ is the Gaussian width, and $l = \sqrt{\frac{\hbar}{eB}}$ is the magnetic length.

3. Results and discussions

The numerical values of quantum capacitance as functions of Fermi energy given the limit of zero temperature (T) are shown in Fig. 1. We adopt $B = 2$ T, $\lambda_R = 13$ meV, and $\Gamma = 0.5$ meV with different interlayer bias energies $U = 0$ meV (a) and 25 meV (b). Quantum capacitance can be effectively tuned from minimum to maximum by the Fermi energy via gate voltage. This oscillatory quantum capacitance of bilayer graphene in the magnetic field can be clearly observed in recent experiments [6]. The numerical values of quantum capacitance are symmetrical at about $E_F = 0$ because of the electron–hole symmetry, which is unaffected by interlayer bias energy. We then focus on the characteristics of quantum capacitance at $E_F = 0$, which can reflect the energy levels at NP. The $n = 0$ Landau level and the zero-mode of the $n = 1$ Landau level degenerate at NP without the interlayer bias energy. Therefore, a single peak (a maximum) is observed in C_q at $E_F = 0$ and $U = 0$, which are unaffected by the Rashba SOI. However, C_q equals to zero (a minimum) at $E_F = 0$ for the interlayer bias energy $U \neq 0$, because the interlayer bias splits the degenerate zero-energy level into one electron and one hole level. This split of quantum capacitance at NP can also be generated by the strain energy [26] or the hybridization energy [27] in topological insulators. Clearly, the interlayer bias energy can form a region ($-U < E_F < U$) in which quantum capacitance vanishes. Each Landau level ($n \geqslant 2$) noticeably splits into two sublevels because of the Rashba SOI, which can lead to the splitting of quantum capacitance. When the interlayer bias energy $U = 25$ meV, two peaks are observed at $E_F \approx \pm 26.8$ meV as the result of the mixing between spin-up and spin-down Landau levels.

Fermi energy can sweep across different Landau levels with an increasing magnetic field, which leads to quantum capacitance oscillations. We show the magnetic oscillations of the bilayer graphene device with different Rashba SOI strengths and interlayer bias energies in Fig. 2. We set $\Gamma = 0.5$ meV, $E_F = 30$ meV, and $T = 0$ K to clearly observe the Rashba SOI and the interlayer bias effects. Under a low magnetic field, a beating pattern [22] with well-defined nodes can be observed with Rashba SOI strengths $\lambda_R = 4$ meV (a) and 13 meV (b) because the Landau level separation reveals a minimum (maximum) oscillation at the Fermi energy. The amplitude of the beating pattern and the number of nodes both increase with the increase in Rashba SOI strength. However, a strong Rashba SOI effect destroys the beating patterns when $\lambda_R = 40$ meV, as shown in Fig. 2(c), because of the overlarge spin-splitting of the Landau level. The amplitude of the beating pattern decreases as the interlayer bias energy increases. Moreover, a strong interlayer bias energy [such as $U = 25$ meV in Fig. 2(a)] can virtually suppress the beating patterns given that Rashba SOI strength is sufficiently weak. The interlayer bias energy can also change the node position of the beating pattern, as shown in Fig. 2(b). Unordered oscillations are observed when $U = 25$ meV and $\lambda_R = 40$ meV under a low magnetic field because of the strong competition between interlayer bias energy and Rashba SOI energy, as shown in Fig. 2(c). The cyclotron energy becomes larger, and a Shubnikov–de Haas oscillation with a high magnetic field is clearly revealed [23]. A regular Shubnikov–de Haas oscillation is observed with a lower Rashba SOI strength $\lambda_R = 4$ meV, and the interlayer bias energy can obviously tune the oscillation frequency. As the Rashba SOI strength increases, the oscillations reveal split quantum capacitance peaks because of the stronger spin-splitting.

The thermal effects on magnetic oscillations under different temperatures, namely, $T = 0, 2, 4$, and 8 K with $\lambda_R = 13$ meV, $E_F = 30$ meV, and $U = 0$, are explored and shown in Fig. 3. Oscillation amplitude decreases with an increase in temperature. A similar phenomenon can also be observed in the quantum capacitance measurements of topological insulators [28]. It is found