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Strange attractor in the Potts spin glass on hierarchical lattices ☆

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ABSTRACT

The spin-glass q-state Potts model on d-dimensional diamond hierarchical lattices is investigated by an exact real space renormalization group scheme. Above a critical dimension $d_l(q)$ for $q > 2$, the coupling constants probability distribution flows to a low-temperature *strange attractor* or to the high-temperature paramagnetic fixed point, according to the temperature is below or above the critical temperature $T_c(q,d)$. The strange attractor was investigated considering four initial different distributions for $q = 3$ and $d = 5$ presenting strong robustness in shape and temperature interval suggesting a condensed phase with algebraic decay.

1. Introduction

The q-states Potts model, proposed a long time ago by Domb as the subject of Potts doctoral thesis [1], has found a wide range of applicability in many fields of both basic and material sciences. The Potts model was conceived as a generalization of the Ising model [2], when $q = 2$, and the Askin–Teller model ($q = 4$) [3]. It also mimics the problem of percolation ($q = 1$) [4,5] and even the problem of the linear resistor networks ($q = 0$) [6]. All of the above mentioned problems were also encompassed by the random-cluster model introduced by Fortuin and Kasteleyn [7]. Furthermore, the degeneracy of the ground state of the *antiferromagnetic* Potts models was shown to be related with the *q-coloring* problem [7]. It is important to emphasize that the most important feature of the mathematical structure of the Potts model is the equivalence between its partition function and Tutte polynomial [8]. Concerning applications, the Potts model has been applied in many fields, such as biology [9], sociology [10] and material science [11]. In the latter, for instance, the technique of Monte Carlo simulations on the Potts model has been applied to a wide variety of phenomena, such as diffusion in polycrystalline microstructures [12] and the study of viscous instabilities in foam-flow behavior [13].

In this Letter, the properties of the q-state Potts model with random competing interactions are investigated. This model is called Potts glass in allusion to the particular case when $q = 2$, widely known in the literature as the Ising spin-glass model. The absence of spin-inversion symmetry and a different nature of the frustration effects distinguish it from its Ising counterpart, exhibiting rather a richer critical behavior in mean-field theory [14–20], in contrast with the pure and disordered Potts model (without frustration) and related models, which have been widely investigated in the past [6,21]. Site and bond diluted versions of the Ferro and Antiferromagnetic Potts model were also studied by Monte Carlo simulations on two and three-dimensional regular lattices [22–24] showing signatures of first and second order phase transitions.

More recently, the nature of phase transitions in the q-state Potts-glass model has been investigated via Monte Carlo simulation in two and three dimensions for values of the number of states $q = 3$ [25], $q = 4$ [26], $q = 5$ and 6 [27], $q = 7$ [28] and $q = 10$ [25,29].

The present work focuses on the study of the q-state Potts model with random frustrated exchange interactions on a family of diamond-type hierarchical lattices [30] with scale factor $b = 2$. When symmetrical zero-centered random exchange couplings distributions are considered the system undergoes a phase transition from a paramagnetic high-temperature phase to a low-temperature condensed phase above some dimension, $d_l(q)$. The phase diagram of the Potts-glass model had been investigated

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before, by the Migdal–Kadanoff (MK) renormalization group (RG) scheme, indicating the presence of a condensed phase at finite temperatures when $q > 2$ and $d = 4$ [31] as well as in the limit of large q [32]. In Ref. [31], however, it was assumed (as working *hypothesis*) that the initial symmetrical Gaussian probability distribution of coupling constants with variance σ is transformed under renormalization into another symmetrical Gaussian distribution but with variance $\sqrt{b^{d-1}\sigma}$.

The MK real-space RG method for Bravais lattices is known to be equivalent to exactly solving the model on diamond-like hierarchical lattices [33,34]. In a recent paper [35], however, the authors studied the occurrence of phase transitions of the Potts-glass model using this exact approach by numerically following the flow of the renormalized probability distribution in an appropriated parameter space. Such space was previously considered in Ref. [36,37] to study the Ising spin-glass model. The q-state Potts-glass model was considered on lattices with several fractal dimension, determining the critical temperature and the upper and lower bounds for the associated lower critical dimension $d_l(q)$. For instance, for $q = 3$ the lower (upper) bound was found to be 4.46 (4.58), in contrast with the result obtained in Ref. [31], which founds the transition occurring for $d < 4$. Here we further explore the flow of the renormalized probability distribution in the whole parameter space and investigate the nature of the low-temperature stable fixed point, which surprisingly appeared like a *strange attractor*.

2. Renormalization procedure in disordered systems

For pure systems, the renormalization procedure consists in finding the equivalent exchange interaction for a pair of spins after eliminating several spins in the lattice. For a disordered system, however, the renormalization procedure will affect the whole distribution of coupling constants, the renormalized distribution, $P'(J)$, is related with the previous (non-renormalized) distribution, $P(J)$, by,

$$P'(J) = \int \cdots \int \prod_{\langle ij \rangle} P(J_{ij})\, dJ_{ij}\, \delta\big(K - K'(K)\big), \tag{1}$$

where $K = \beta J$ is a reduced coupling constant, $K'(K)$ is the renormalization equation, and the product runs over all the pairs of spins $\langle ij \rangle$.

Eq. (1) should be iterated until the renormalized distribution reaches a fixed point distribution, characteristic of the thermodynamic phase. A zero-centered Dirac-delta distribution, for instance, indicates a paramagnetic phase. The procedure adopted in this work is to produce a sample of random coupling constants from an initial probability density function, feed the renormalization equation to find a sample of the same size of renormalized coupling constants, estimating the physical quantities numerically from the samples. The process is repeated until the fixed point distribution is reached.

3. The Potts Hamiltonian and the renormalization equation

The Potts Hamiltonian is written as

$$\mathcal{H} = -\sum_{\langle ij \rangle} q\, J_{ij} \delta_{\sigma_i \sigma_j}, \tag{2}$$

where the sum is taken over all bonds in the lattice, the Kronecker δ symbol takes the values 1 if $\sigma_i = \sigma_j$ or 0 otherwise, and $\sigma_i = 1, 2, \ldots, q$ are the q-states Potts spins variables, located at each site of a diamond-type hierarchical lattice with p branches and scale factor b. The basic unit of such lattice is illustrated in

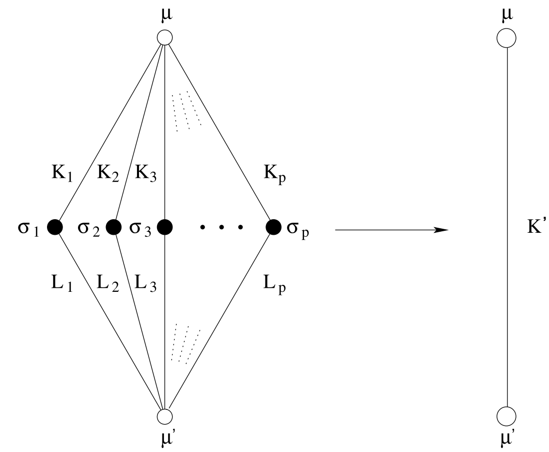

Fig. 1. Renormalization group scheme on the d_f dimension diamond-type hierarchical lattice with p branches and scaling factor $b = 2$.

Fig. 1, where μ and μ' are called external sites and the set $\{\sigma_i\}$ represents the internal sites [30]. K_i and L_i are reduced coupling constants, $K_i \equiv \beta J_{\mu\sigma_i}$ and $L_i \equiv \beta J_{\mu'\sigma_i}$. The lattice generations or hierarchies are successively built by replacing each connection of the basic unit by the basic unit itself, yielding to a graph with fractal dimension,

$$d_f = 1 + \frac{\ln p}{\ln 2}.$$

The exact renormalization process on a n-generation lattice consists in partially tracing the partition function along all the internal sites introduced in the nth generation leading to a $(n-1)$-generation lattice with a set of effective reduced coupling constants $\{K_i'\}$ given by

$$K' = \frac{1}{q} \sum_{i=1}^{p} \left[\frac{(q-1) + \exp(qK_i + qL_i)}{(q-2) + \exp(qK_i) + \exp(qL_i)} \right]. \tag{3}$$

Eq. (3) is the local renormalization equation. The right-hand side of Eq. (3) receives values for the reduced coupling constants calculated from the n-generation of the coupling constant distribution, $P(J)$, resulting in one of the possible values of the renormalized reduced coupling constant of the $(n-1)$-generation. The renormalization procedure starts from the thermodynamic limit (generation $n \to \infty$) where the coupling constants are assumed to have a well-known distribution, actually, Gaussian, delta-bimodal, uniform, or exponential, and well-defined temperature T.

3.1. Probability distribution renormalization flow

For each n-generation lattice we can define a set of *thermal transmissivities* variables $\{t_{ij}\}$, each one associated with the respective bond, i.e.,

$$t_{ij} \equiv \frac{1 - \exp(-q\beta J_{ij})}{1 + (q-1)\exp(-q\beta J_{ij})}. \tag{4}$$

Thermal transmissivity t_{ij} represents the pair correlation function Γ_{ij} between sites ij [21].

A system with a probability distribution of coupling constants $P(J) \equiv P(\{J_{ij}\})$ yields a thermal transmissivity variance,

$$\Delta^2 \equiv \left[\left(t_{ij} - [t_{ij}]_J\right)^2\right]_J, \tag{5}$$

where $[\cdots]_J$ means the average over the probability distribution, $P(J)$.

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