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Effect of a nonuniform magnetic field on the Landau states in a biased AA-stacked graphene bilayer

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ABSTRACT

We study the Landau states in the biased AA-stacked graphene bilayer under an exponentially decaying magnetic field along one spatial dimension. The results show that the energy eigenvalues of the system are strongly dependent on the inhomogeneity of the magnetic field and the bias voltage between the graphene layers, and in particular the reordering and mixing of finite Landau states could occur. Moreover, we also demonstrate that the current carrying states induced by the decaying magnetic field propagate vertically to the magnetic-field gradient within the graphene sample and can be further modulated by the bias voltage between the layers.

Graphene, with a two-dimensional (2D) hexagonal honeycomb structure, has become a star nanomaterial in the last few years, because of its unique physical properties and potential applications [1]. Afterwards, graphene bilayer (GBL) has surged as another attractive 2D carbon material and exhibited interesting physical properties [2–13]. Generally, the natural form for graphene bilayer is AB or staggered stacking. However, recent experiments have shown that the AA-stacked GBL is another stable structure [14–17]. It is known that different from the AB-stacked GBL, the energy bands of the AA-stacked GBL are just the double copies of that of graphene monolayer (GML) shifted up and down by the interlayer coupling [17,18].

Clearly, the main feature of the zero-field subbands of the GBL should be reflected in the Landau states when a magnetic field is applied perpendicularly to the graphene plane. Recent studies have shown that under constant and perpendicular magnetic fields, the Landau states in the AB-stacked GBL exhibit many peculiar properties as compared with the GML [3–5,12,13], especially in the presence of a vertical electric field [6,19–23]. But, the stationary Landau states in uniform magnetic fields do not propagate and give no contribution to the conductance. In contrast, the Landau states in spatially inhomogeneous time-independent magnetic fields are peculiar. Actually, the electron trajectories of these states are open orbits drifting in a direction perpendicular to the magnetic-field gradient. Also, we should mention that the Schrödinger equation in the 2D or quasi 2D systems [24–26] and the Dirac–Weyl equation in the graphenes [27–31] have been analytically solved for a single electron (hole) in the presence of inhomogeneous distributions of magnetic fields, including magnetic barriers, magnetic steps and decaying magnetic fields. However, little work has payed attention to the influence of nonuniform magnetic fields on the Landau states in the AA-stacked GBL with a bias voltage between the graphene layers.

In this Letter, we study the Landau states in a biased AA-stacked GBL subjected to the decaying magnetic field, and reveal the reordering and mixing of the finite Landau states in this system. We consider the graphene sample to be in the xy plane with a magnetic field applied in the z direction, but exponentially damped in the x direction [25,26,31], i.e.,

$$\boldsymbol{B} = B_0 e^{-x/\lambda}\hat{\boldsymbol{z}}, \tag{1}$$

where λ is the penetration depth of the magnetic field, and x ranges from 0 to ∞. Such a decaying magnetic field can be produced as follows: When a homogeneous magnetic field is applied parallel to one planar surface of a type-I superconductor, then the magnetic field with exponentially decaying variation occurs inside the superconductor within the London penetration depth, and this design can be used to raise an inhomogeneous magnetic field in our system [25].

In the continuum limit and in the effective-mass approximation, the Hamiltonian near the K valley in the AA-stacked GBL with the interlayer biased potential U can be described by [32,33]

$$\mathcal{H} = \mathcal{H}_0 + (U/2)\tau_z, \tag{2}$$

where

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$$\mathcal{H}_0 = \begin{pmatrix} v_F \boldsymbol{\sigma} \cdot (\boldsymbol{p} + e\boldsymbol{A}) & \gamma_\perp \mathbf{I} \\ \gamma_\perp \mathbf{I} & v_F \boldsymbol{\sigma} \cdot (\boldsymbol{p} + e\boldsymbol{A}) \end{pmatrix}, \tag{3}$$

and

$$\tau_z = \begin{pmatrix} \mathbf{I} & 0 \\ 0 & -\mathbf{I} \end{pmatrix}. \tag{4}$$

Here $U = (U_1 - U_2)$ with U_1 and U_2 being the electrostatic potential at the two layers, which can be induced by an external electric field vertical to the sample plane. $\boldsymbol{\sigma} = (\sigma_x, \sigma_y)$ and $\mathbf{I}$ are, respectively, the 2×2 Pauli matrices in the isospin space and the identity matrix. $\boldsymbol{p} = (p_x, p_y)$ is the two-dimensional momentum operators. The magnetic vector potential is chosen as $\boldsymbol{A} = (0, -B_0\lambda(e^{-x/\lambda} - 1), 0)$ within the Landau gauge and it can directly recover the uniform magnetic field case in the limit of $\lambda \to \infty$. $v_\mathrm{F} \approx 10^6$ m/sec is the Fermi velocity, e is the absolute value of electron charge, and $\gamma_\perp$ is the interlayer coupling parameter.

The eigenstates of the Hamiltonian $\mathcal{H}$ in Eq. (2) are the four-component spinors, i.e., $\Psi = [\psi_{\mathrm{A1}}, \psi_{\mathrm{B1}}, \psi_{\mathrm{A2}}, \psi_{\mathrm{B2}}]^\mathrm{T}$, where $\psi_{\mathrm{A1(A2)}}$ and $\psi_{\mathrm{B1(B2)}}$ are the envelope functions associated with the probability amplitudes of the wave functions on the A1 (A2) and B1 (B2) sublattices of the upper (lower) layer. Since $[\mathcal{H}, p_y] = 0$, the components of the spinor Ψ can be rewritten as $\psi_{\mathrm{A1}} = \phi_{\mathrm{A1}}(x)e^{ik_y y}$, $\psi_{\mathrm{B1}} = \phi_{\mathrm{B1}}(x)e^{ik_y y}$, $\psi_{\mathrm{A2}} = \phi_{\mathrm{A2}}(x)e^{ik_y y}$ and $\psi_{\mathrm{B2}} = \phi_{\mathrm{B2}}(x)e^{ik_y y}$. Accordingly, the x dependences of the spinor components are then given by

$$i\left(\xi\frac{\partial}{\partial\xi} - \xi + \xi_0\right)\phi_{\mathrm{A1}} = \left(\varepsilon - \frac{u}{2}\right)\phi_{\mathrm{B1}} - \gamma'_\perp\phi_{\mathrm{B2}}, \tag{5a}$$

$$i\left(\xi\frac{\partial}{\partial\xi} + \xi - \xi_0\right)\phi_{\mathrm{B1}} = \left(\varepsilon - \frac{u}{2}\right)\phi_{\mathrm{A1}} - \gamma'_\perp\phi_{\mathrm{A2}}, \tag{5b}$$

$$i\left(\xi\frac{\partial}{\partial\xi} - \xi + \xi_0\right)\phi_{\mathrm{A2}} = \left(\varepsilon + \frac{u}{2}\right)\phi_{\mathrm{B2}} - \gamma'_\perp\phi_{\mathrm{B1}}, \tag{5c}$$

$$i\left(\xi\frac{\partial}{\partial\xi} + \xi - \xi_0\right)\phi_{\mathrm{B2}} = \left(\varepsilon + \frac{u}{2}\right)\phi_{\mathrm{A2}} - \gamma'_\perp\phi_{\mathrm{A1}}, \tag{5d}$$

where

$$\xi = \frac{eB\lambda^2}{\hbar}e^{-x/\lambda}, \tag{6}$$

and

$$\xi_0 = k_y\lambda + \frac{eB\lambda^2}{\hbar}. \tag{7}$$

Here, we employ the dimensionless quantities for simplicity, in which all the energy terms, i.e., ε, u and $\gamma'_\perp$, are rescaled by $\hbar v_F/\lambda$. The unitless parameter ξ_0 is used to characterize the inhomogeneity of the magnetic field, which is identical to the parameter s in Ref. [26].

After some straightforward algebra, the energy eigenvalues of the AA-stacked GBL in the decaying magnetic field can be expressed as

$$E_n = \pm\big(L_n(\lambda, \xi_0) \pm \Gamma(U)\big), \tag{8}$$

where

$$L_n(\lambda, \xi_0) = \frac{\hbar v_F}{\lambda}\sqrt{(n+1)(2\xi_0 - n - 1)}, \tag{9a}$$

$$\Gamma(U) = \sqrt{\gamma_\perp^2 + \frac{U^2}{4}}. \tag{9b}$$

The above results indicate that the interlayer biased potential U can modify the energy eigenvalues of the system, and further the

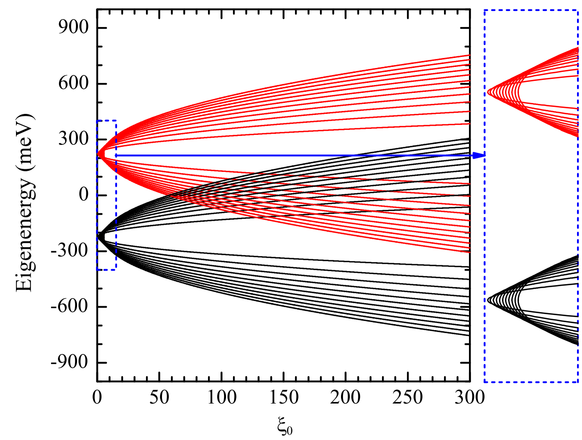

Fig. 1. (Color online.) Energy eigenvalues of the AA-stacked GBL in the decaying magnetic field as a function of ξ_0 for $\lambda = 100$ nm at $U = 200$ meV. For clarity, the blue rectangular area in the figure is enlarged in the right inset.

accidental degeneracy of the Landau states with the same index n can take place at zero energy when $U = 2\sqrt{L_n^2 - \gamma_\perp^2}$. Also, the number of the Landau states in this system is dependent on ξ_0, and the maximum number of states is limited to

$$n_{\max} = \lfloor 2\xi_0 - 1 \rfloor, \tag{10}$$

where $\lfloor x \rfloor$ means the integer part of x. In particular, if the following relationship is satisfied for each positive integer value of $2(\xi_0 - 1)$,

$$n_1 + n_2 = 2(\xi_0 - 1), \tag{11}$$

the Landau states with different indices n_1 and n_2 merge each other, which is here referred to as the mixing of Landau states.

Furthermore, the Landau states in the AA-stacked GBL subjected to a decaying magnetic field can drift vertically to the magnetic-field gradient in the graphene sample, which is somewhat similar to the snake states discussed before [24,28]. But, there is obvious difference between the two cases. In our case the current carrying states induced by the decaying magnetic field are strongly dependent on the penetration depth of the magnetic field and can be further modulated by the bias voltage between the graphene layers. From Eq. (8), the drift velocity of the Landau states of the system is

$$v_y = \hbar^{-1}\frac{\partial E_n}{\partial k_y} = \mathrm{sgn}(E_F - \Gamma)\sqrt{\frac{n+1}{2\xi_0 - n - 1}}v_F. \tag{12}$$

Here, sgn(s) is a sign function such that it is positive if $E_F > \Gamma(U)$, and negative if $E_F < \Gamma(U)$, in which E_F is the energy of the Fermi level. Thus, the sign of the drift velocity in the system can be affected by the interlayer biased potential, which is starkly different from the behavior in the conventional 2D electron systems or the GML with spatially inhomogeneous magnetic fields [25,26,28].

Based on the above formulation, we calculate the energy eigenvalues of the biased AA-stacked GBL in the decaying magnetic field, and analyze the effects of the inhomogeneity of the magnetic field and the bias voltage between the layers on the Landau states of the system. In the following calculation, we assume, without loss of generality, that the interlayer coupling parameter $\gamma_\perp = 200$ meV.

Fig. 1 shows the energy eigenvalues of the AA-stacked GBL in a decaying magnetic field as a function of ξ_0 for $\lambda = 100$ nm at $U = 200$ meV. As can be seen from Fig. 1, the interlayer biased potential U can vary the energy eigenvalues of the system, and then the Dirac points are located at $\pm\sqrt{\gamma_\perp^2 + U^2/4}$ instead of $\pm\gamma_\perp$.