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Electrostatic double layers in a warm negative ion plasma with nonextensive electrons

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ABSTRACT

The electrostatic double layer (DL) structures are studied in negative ion plasma with nonextensive electrons q-distribution. The extended Korteweg–de Vries (EKdV) equation is derived using a reductive perturbation method. It is found that both fast (compressive) and slow (rarefactive) ion acoustic (IA) DLs can propagate in such type of plasmas. The effects of various plasma physical parameters; such as nonextensivity of electrons, presence of negative ions, temperature of both positive and negative ions and different mass ratios of positive to negative ions on the formation of DL structures are discussed in detail with numerical illustrations.

1. Introduction

Ion acoustic (IA) solitary waves and double layer (DL) structures are found in a wide variety of plasmas, e.g., laboratory plasmas, Earth's magnetosphere, dusty plasmas and quantum plasmas. In the last few years, the formation of DLs have been of great interest [1,2] due to their wide role in astrophysical and laboratory plasmas. The considerable information has been collected on electrostatic DLs with the help of laboratory studies [3,4] as well as the investigations of space plasmas [5,6]. The fundamental properties and applications of DLs have been extensively studied theoretically [7], experimentally [8] and through extensive computer simulations [9]. The linear study of numerous plasma waves are relevant when the amplitude of the wave is small. However, when the amplitude of the wave is sufficiently large, one cannot neglect the nonlinearity in the system. The nonlinearity contributes to the localization of waves and leads to different type of structures, such as solitons, shocks, DLs and vortices [10–15], etc. Several theoretical models have been developed to explain the formation of DLs and related phenomena in plasmas [16–18]. The Korteweg–de Vries (KdV) equation with additional term of cubic nonlinearity is widely employed to describe the propagation characteristics of DLs in different homogeneous plasma systems [19–21].

Negative ion plasma contains positive ions, electrons and negative ions as well. Negative ion plasma occurs naturally in space and astrophysical environments and can be produced in the laboratory. Negative ion plasma have attracted a great deal of interest [22–28] because of their wide technical applications [neutral beam sources [29], plasma processing reactors [30], synthesis of nanomaterials [31], semiconductor material processing [32], etc.] and also their vital roles in astrophysical plasmas. The existence of negative ions in the Earth ionosphere, [33] and cometary comae [34] is also verified. It has been investigated experimentally as well as theoretically that the presence of these negative ions remarkably modifies many characteristic plasma phenomena. For instance, negative ions change the plasma potential and electrons behavior.

Now it is established clearly that the Maxwellian distribution is not the realistic distribution under all circumstances [35]. Several observation of fast ions and electrons in space environments clearly suggests that these particles have a deviated velocity distributions far from Maxwellian behavior [36]. In some space regions the distribution functions having non-Maxwellian energy tails or flat top with pronounced shoulders are observed. During last two decades it has been proven that Boltzmann–Gibbs (BG) statistics is not applicable to describe correctly the systems with long range interaction, long time memory, fractality of the corresponding space–time/phase-space, etc. The main reason of this failure is because BG statistics is extensive or additive in its formalism. In order to analyze the statistical properties of the

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system with long range correlations, Tsallis [37] extended BG thermodynamics by generalizing concept of entropy to nonextensive regime. The nonextensivity means that entropy of the composition $(A+B)$ of two independent systems A and B is equal to $S_q(A+B)=S_q(A)+S_q(B)-(1-q)S_q(A)S_q(B)$ where parameter q defines the nonextensivity and refers as entropic index of nonextensive system under consideration. Lima et al. [38] have found that the q-nonextensive pattern is very important for systems having long-range interactions (i.e., studying interactions comparable with the size of the system under consideration) such as those occurs in astrophysics and plasma physics. Liu et al. [39] and Lima et al. [38] have showed that plasma experimental results lead to non-Maxwellian velocity distribution. Very recently, the Tsallis q-entropy and the ensuing generalized statics have been employed with noticeable success for various plasma physics models [40–44].

The DLs, which are associated with adjacent positive and negative charge regions, are more difficult to generate in laboratory experiments and require a fine tuning of the plasma parameters and hence a more complicated plasma composition with enough tolerance to obey the necessary constraints [45–47]. For example, Jain et al. [45] found that a small concentration of a second negative ion species significantly modifies the characteristics of large-amplitude IA DL. Sagdeev potential analysis has been used by Sahu [47] to address the effect of nonextensive electrons and thermal positrons on the IA solitary waves and DLs. On the other hand, the propagation of IA DLs in a plasma containing a negative ion species are investigated theoretically [48]. It was found that the type of DL exists when the electric potential is positive and the negative ion concentration is greater than certain value. The compressive and rarefactive DLs in a warm multicomponent plasma with negative ions have been analyzed [49]. Recently, the properties of IA solitons and DLs in a plasma consisting of warm positive and negative ions with different concentration of masses, charged states and nonthermal electrons studied under small amplitude approximation limit [50].

In the present Letter, the electrostatic DLs are investigated in a homogeneous plasma comprising of positive and negative ions with nonextensive electrons. The basic set of equations governing the dynamics of nonlinear IA waves in unmagnetized positive, negative ions and nonextensive electrons plasma is presented in Section 2. The extended Korteweg–de Vries (EKdV) equation is derived in Section 3 using reductive perturbation method [51] for electrostatic waves in negative ion plasmas with nonextensive q-distributed electrons. The numerical results and conclusion are presented in Section 4.

2. Basic set of equations

We are studying the nonlinear electrostatic waves in positive, negative ions and nonextensive electron plasmas. The positive and negative ions are assumed to be dynamic and adiabatically heated while electrons are following nonextensive density profile. The normalized nonlinear set of partial differential equations for positive and negative ion fluids is described as follows,

$$\frac{\partial n_i}{\partial t}+\frac{\partial}{\partial x}(n_iu_i)=0, \tag{1}$$

$$\frac{\partial u_i}{\partial t}+u_i\frac{\partial u_i}{\partial x}+\frac{\partial \Phi}{\partial x}+\frac{\theta_i}{n_i}\frac{\partial p_i}{\partial x}=0, \tag{2}$$

$$\frac{\partial p_i}{\partial t}+u_i\frac{\partial p_i}{\partial x}+3p_i\frac{\partial u_i}{\partial x}=0, \tag{3}$$

$$\frac{\partial n_n}{\partial t}+\frac{\partial}{\partial x}(n_nu_n)=0, \tag{4}$$

$$\frac{\partial u_n}{\partial t}+u_n\frac{\partial u_n}{\partial x}-\beta\left(\frac{\partial \Phi}{\partial x}-\frac{\theta_n}{n_n}\frac{\partial p_n}{\partial x}\right)=0, \tag{5}$$

$$\frac{\partial p_n}{\partial t}+u_n\frac{\partial p_n}{\partial x}+3p_n\frac{\partial u_n}{\partial x}=0. \tag{6}$$

The normalized nonextensive electron density profile is given by

$$n_e=\left[1+(q-1)\Phi\right]^{(q+1)/2(q-1)}. \tag{7}$$

The parameter q stands for the strength of electrons nonextensivity. Unlike the description of Maxwellian distribution; that most of the particles center around the thermal speed, the power law distributions characterize the systems containing ample supply of superthermal particles with the constraint $q<1$ or including a large number of low velocity particles by the restriction $q>1$ [44].

Poisson equation is described as

$$\frac{\partial^2\Phi}{\partial x^2}=(\mu n_e+\nu n_n-n_i), \tag{8}$$

where n_i, u_i and n_n, u_n are the normalized densities and velocities of the positive and negative ion fluids respectively. The equilibrium densities of positive, negative ions and electrons are n_{i0}, n_{n0} and n_{e0} respectively. The temperatures of positive ions, negative ions and electrons are T_i, T_n and T_e, respectively. The mass ratio of positive-to-negative ions is $\beta=\frac{m_i}{m_n}$. Also, $\mu=n_{e0}/n_{i0}$, $\nu=n_{n0}/n_{i0}$, $\theta_i=T_i/T_e$ and $\theta_n=T_n/T_e$ are defined. The electric field intensity is defined as $E=-\nabla\varphi$ (where φ is the electrostatic potential). The normalization with $t\to t\omega_{pi}\nabla\to\lambda_D\nabla$, $n_j\to n_j/n_{j0}$, $u_j\to u_j/C_s$, $\Phi\to(e\varphi/k_BT_e)$ are used, where $C_s=\sqrt{k_BT_e/m_i}$, the ion plasma frequency $\omega_{pi}=\sqrt{4\pi n_{i0}e^2/m_i}$ and effective Debye length $\lambda_D=\sqrt{k_BT_e/4\pi n_{i0}e^2}$.

3. Nonlinear evolution EKdV equation

In order to solve the nonlinear set of partial differential equations (1)–(8), we apply the reductive perturbation technique [51] to find the evolution equation: an extended Korteweg–de Vries (EKdV) equation, for DL structures in positive, negative ion and nonextensive electron plasmas. The perturbed quantities can be expanded in the power series of ϵ as follows:

$$\begin{aligned}
n_{i,n,e}&=1+\epsilon n^{(1)}_{i,n,e}+\epsilon^2n^{(2)}_{i,n,e}+\epsilon^3n^{(3)}_{i,n,e}+\cdots,\\
u_{i,n}&=\epsilon u^{(1)}_{i,n}+\epsilon^2u^{(2)}_{i,n}+\epsilon^3u^{(3)}_{i,n}+\cdots,\\
p_{i,n}&=1+\epsilon p^{(1)}_{i,n}+\epsilon^2p^{(2)}_{i,n}+\epsilon^3p^{(3)}_{i,n}+\cdots,\\
\Phi&=\epsilon\Phi^{(1)}+\epsilon^2\Phi^{(2)}+\epsilon^3\Phi^{(3)}+\cdots.
\end{aligned} \tag{9}$$

Here all perturbed quantities are functions of x and t, while ϵ is a small ($\epsilon\ll 1$) expansion parameter characterizing the strength of the nonlinearity.

In order to find the EKdV equation, which leads to a DL solution from nonlinear set of Eqs. (1)–(8), the stretched variables are introduced in the standard fashion as follows [16–18,52]

$$X=\epsilon(x-\lambda t),\qquad \tau=\epsilon^3t. \tag{10}$$

Substituting Eqs. (9) and (10) in Eqs. (1)–(8) and collecting the lowest order ($\sim\epsilon$) terms of continuity and momentum equations of positive and negative ions we have

$$\begin{aligned}
n^{(1)}_i&=\frac{\Phi^{(1)}}{\lambda^2-3\theta_i},\\
u^{(1)}_i&=\frac{\lambda\Phi^{(1)}}{\lambda^2-3\theta_i},\\
p^{(1)}_i&=\frac{3\Phi^{(1)}}{\lambda^2-3\theta_i},\\
n^{(1)}_n&=-\frac{\beta\Phi^{(1)}}{\lambda^2-3\beta\theta_n},
\end{aligned} \tag{11}$$