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On geometry-dependent vortex stability and topological spin excitations on curved surfaces with cylindrical symmetry

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ABSTRACT

We study the Heisenberg model on cylindrically symmetric curved surfaces. Two kinds of excitations are considered. The first is given by the isotropic regime, yielding the sine-Gordon equation and π solitons are predicted. The second one is given by the XY model, leading to a vortex turning around the surface. Helical states are also considered, however, topological arguments cannot be used to ensure its stability. The energy and the anisotropy parameter which stabilizes the vortex state are explicitly calculated for two surfaces: catenoid and hyperboloid. The results show that the anisotropy and the vortex energy depends on the underlying geometry.

1. Introduction

Geometrical and topological concepts and tools are important in many branches of natural sciences, particularly, in physics. For instance, the idea of symmetry, which is intimately associated with geometry, is a keystone for studying a number of fundamental properties of several physical systems, e.g., the Noether theorem asserts that there is a conserved quantity to each continuous symmetry of the associated action. Topology, in turn, is crucial for classifying and for giving stability to certain excitations, such as solitons, extending objects having finite energy, and vortices, presenting a non-vanishing vorticity around a given singular point or a topological obstruction. In addition, the observed vortex-pair dissociation is the mechanism behind the topological phase transition [1]. Vortices and solitons have been observed in a number of systems, such as superconductors, superfluids, and magnetic materials [2,3].

Curvature effects play an important role in the characteristics of these topological structures. For instance, Vitelli et al. have shown that in-plane vortices interact not only with each other, but also with the curvature of the substrate [4]. Curvature is also an important factor in the magnetic systems behaviour, in which the interaction of the out-of-plane component of magnetic vortices with curved defects must cause a chiral symmetry breaking in its gyrotropic motion due the thin-film roughness [5,6]. Furthermore, the easy-surface Heisenberg model in magnetic spherical shells predicts a coupling between the localized out-of-surface component of the vortex with its non-localized in-surface structure, associated with the curvature of the underlying geometry [7] and still, the smooth and variable curvature of ferromagnetic nanotorus ensures the stability of the vortex for smaller radius than their nanoring counterparts [8].

In the case of two-dimensional systems, vortices and solitons can appear like solutions of the continuous Heisenberg model, which has been used to analyse the dynamic and static properties of vortices, showing that the energy of these excitations is closely linked to the geometrical properties of the surface [9–14]. It has also been shown that, for simply connected surfaces, the vortex energy presents a divergence, which can be controlled by the insertion of a cutoff in the region where the continuous limit of Heisenberg Hamiltonian has not validity. In the case of magnetic systems, this divergence must be controlled by the development of an out-of-plane component in the vortex core region. Soliton-like solution has also been considered in the above cited works and it has been shown that its characteristic length depends on the length scale of the surface. For finite surfaces, fractional/half-soliton solutions have been found [11,15]. Furthermore, the interaction of an external magnetic field with Heisenberg spins on a cylindrical surface yields a 2π soliton-like solution, inducing a deformation at the sector where the spins are pointing in the opposite direction to the

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magnetic field [16]. A 2π soliton has also been predicted to appear on curved surfaces with cylindrical symmetry, provided the magnetic field is coupled with the curvature of the substrate [17,18].

In this Letter, we study the anisotropic Heisenberg model on curved surfaces with cylindrical symmetry. We are interested in studying a class of topological spin textures on these manifolds in such way that both, soliton and vortex-like solutions are considered. Solitons are predicted to appear on an infinite cylindrically symmetric surface, if the isotropic case is taken into account. In our assumptions, the soliton characteristic length is rescheduled to one and does not depend on the characteristic length scale of the surface. For finite surfaces, fractional solitons, which have not topological stability, are found.

Our analysis includes the study of the XY model and vortex-like solutions are considered. It is shown that, for non-simply connected manifolds, the obstruction of the surface ensures the vortex topological stability due the removal of spurious divergences appearing in the core region. The XY model can also yield a helical-like state, however topological arguments must not be used to ensure the stability of this spin configuration. The energy of these spin textures is calculated and we get that it depends on the surface curvature. Furthermore, we calculate the critical anisotropy parameter for which the vortex appears as the ground state and we show that it is also associated to the geometrical properties of the surface. The vortex energy and the critical anisotropy parameter for two different geometries: the catenoid and hyperboloid, which are negatively curved and non-simply connected manifolds, are explicitly calculated. The choice for studying these surfaces is associated with the fact that both can be realized in fluid interfaces provided with an orientational ordered phase as a consequence of the interplay between surface tension and orientational elasticity [19]. Furthermore, catenoid is the shape minimizing the curvature elastic energy, appearing in phospholipid vesicles of high topology [20] and the symmetry of these geometries allows us to compare our results to these found for the cylindrical case, largely explored in nanomagnetism researches.

To proceed with our analysis, this work is organized as follows: in Section 2 we present the continuous anisotropic Heisenberg model on rotationally symmetric surfaces. The results and discussions for the isotropic Heisenberg Hamiltonian and the XY model are also considered in this section. Section 3 brings the discussions about the model on the catenoid and hyperboloid surfaces and compare our results with that obtained for the surface of a cylinder. Finally, in Section 4, we present our conclusions and prospects for future works.

2. Continuum Heisenberg model on curved surfaces

The anisotropic exchange Heisenberg model, for nearest neighbour interacting spins on a two-dimensional lattice, is given by the Hamiltonian below:

$$H_{\text{latt}} = -J' \sum_{\langle i,j\rangle} \left[m_i^x m_j^x + m_i^y m_j^y + (1+\lambda) m_i^z m_j^z \right], \tag{1}$$

where J' denotes the coupling between neighbouring spins, and according to $J' < 0$ or $J' > 0$, the Hamiltonian describes a ferro or antiferromagnetic system, respectively. $\vec{m}_i = (m_i^x, m_i^y, m_i^z)$ is the spin operator at site i and the parameter λ accounts for the anisotropy interaction amongst spins: for $\lambda > 0$, spins tend to align along the internal Z axis (easy-axis regime); for $\lambda = 0$, one gets the isotropic case; for $-1 < \lambda < 0$, we have the easy-plane regime, while the $\lambda = -1$ case yields to the so-called XY model, which has been considered on curved surfaces [21]. If we focus on a two-component spin, imposing $m_z \equiv 0$, so that $\vec{m}_{PRM} = (m_x, m_y)$, we get the planar rotator model (PRM).

In the continuum approach of spatial and spin variables, valid at sufficiently large wavelength and low temperature, the model given by Eq. (1) may be written as follows ($J \equiv J'/2$):

$$\begin{aligned} H = {} & -\frac{4J}{a^2} \iint \frac{1}{\sqrt{|g|}} \left(1 + \lambda m_z^2\right) d\eta_1\, d\eta_2 \\ & + J \iint \sum_{i,j=1}^{2} \sum_{a,b=1}^{3} g^{ij} h_{ab} (1 + \delta_{a3}\lambda) \\ & \times \left(\frac{\partial m^a}{\partial \eta_i}\right)\left(\frac{\partial m^b}{\partial \eta_j}\right) \sqrt{|g|}\, d\eta_1\, d\eta_2, \end{aligned} \tag{2}$$

where a is the network spacing, the surface has curvilinear coordinates η_1 and η_2, $\sqrt{|g|} = \sqrt{|det[g_{ij}]|}$, g^{ij} and h_{ab} are the surface and spin space metrics, respectively (as usual, $g^{ij} g_{jk} = \delta_k^i$). Now, $\vec{m} = (m_x, m_y, m_z) \equiv (\sin\Theta\cos\Phi, \sin\Theta\sin\Phi, \cos\Theta)$ is the classical spin vector field valued on a unity sphere (internal space), so that $\Theta = \Theta(\eta_1, \eta_2)$ and $\Phi = \Phi(\eta_1, \eta_2)$. With this, the Cartesian parametrization for $\vec{m}$ yields to $h_{ab} = \delta_{ab}$. Note that the first term in the first integral in the Hamiltonian (2) is the ground state energy and we will renormalize it to be zero. It can also be noted that, if λ decreases from 0 to -1, the term depending on m_z^2 increases the energy if $m_z \neq 0$, thus, the smallest energy associated to the anisotropic Heisenberg Hamiltonian will occur for $m_z = 0$. The Hamiltonian (2) may be also viewed as the anisotropic non-linear σ model (NLσM), which lies on an arbitrary two-dimensional geometry. Thus, besides ordinary spins, the above model can be used to describe another condensed matter systems, e.g., a superfluid helium film, thin superconducting films [4,22], a nematic liquid crystal confined on curved surfaces [23] or a spin ladder, which consists in two or more coupled spin chains [24].

Our interest is to study the above model on curved surfaces with cylindrical symmetry, which, in cylindrical coordinate system, can be parametrized by $\mathbf{r} = (\rho(z), \phi, z)$, where $\rho(z) \equiv \rho$ is the radius of the surface at height z, and ϕ accounts for the azimuthal angle. In this case, we have that the covariant metric elements are given by

$$g_{\phi\phi} = \frac{1}{g^{\phi\phi}} = \rho^2 \quad \text{and} \quad g_{zz} = \frac{1}{g^{zz}} = 1 + \rho'^2, \tag{3}$$

where $\rho' = d\rho/dz$. In this case, the Hamiltonian (2) can be rewritten as

$$\begin{aligned} H = J \iint \Bigg\{ & \sqrt{\frac{g_{\phi\phi}}{g_{zz}}} \left[f(\Theta)(\partial_z\Theta)^2 + \sin^2\Theta (\partial_z\Phi)^2 \right] \\ & + \sqrt{\frac{g_{zz}}{g_{\phi\phi}}} \left[f(\Theta)(\partial_\phi\Theta)^2 + \sin^2\Theta(\partial_\phi\Phi)^2 \right] \\ & - \sqrt{\frac{g^{zz}}{g_{\phi\phi}}} \frac{4\lambda\cos^2\Theta}{a^2} \Bigg\}\, dz\, d\phi, \end{aligned} \tag{4}$$

where $f(\Theta) = 1 + \lambda\sin^2\Theta$.

The Euler–Lagrange equations derived from the Hamiltonian (4) are evaluated to give

$$\begin{aligned} & 2f(\Theta)\left(\partial_\zeta^2\Theta + \partial_\phi^2\Theta\right) \\ & \quad = \sin 2\Theta \Bigg\{ (\partial_\zeta\Phi)^2 + (\partial_\phi\Phi)^2 - \lambda\left[(\partial_\zeta\Theta)^2 + (\partial_\phi\Theta)^2\right] \\ & \qquad + \frac{4\lambda}{a^2}\sqrt{\frac{g^{zz}}{g_{\phi\phi}}} \Bigg\} \end{aligned} \tag{5}$$