Physics Letters A 376 (2012) 2822–2829

Contents lists available at SciVerse ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Spin rotation by Earth's gravitational field in a "frozen-spin" ring

Yuri Orlov [a,*], Eanna Flanagan [b], Yannis Semertzidis [c]

[a] Department of Physics, 310 Physical Sciences Building, Cornell University, Ithaca, NY 14853, USA
[b] Departments of Physics and Astronomy, Cornell University, Ithaca, NY 14853, USA
[c] Brookhaven National Laboratory, Upton, NY 11973-5000, USA

ARTICLE INFO

Article history:
Received 17 April 2012
Received in revised form 2 July 2012
Accepted 7 August 2012
Available online 14 August 2012
Communicated by P.R. Holland

Keywords:
Particle spin
Gravitation
Electric dipole moment
Magic momentum
Frozen-spin storage ring

ABSTRACT

Detailed calculations of spin rotation by the Earth's gravitational field in a frozen-spin ring are presented in three different coordinate systems and used (a) to show that the systematic error caused by gravitation in a proposed electric dipole moment measurement can be unambiguously determined, and (b) to propose measuring the spin-gravity effect in a dedicated frozen-spin ring using electrons.

1. Introduction and basic equations

This Letter was chiefly motivated by proposals to measure the electric dipole moments (EDMs) of various charged particles in "frozen-spin" storage rings [1–3]. EDM here means a vector, $\vec{d}$, whose rest-frame energy in an electric field equals $U = -\vec{d} \cdot \vec{E}$ (as it does for any classical electric dipole), but whose direction is defined by a particle's rest-frame spin, $\vec{s}$, $\vec{d} = (e\hbar/2mc)\eta\vec{s}$. Such a $\vec{d}$ violates T- and P-symmetries and has never been directly observed. The Standard Model predicts EDMs at a level much below 10^{-29} e cm, the current hoped-for sensitivity of proposed proton and deuteron EDM measurements. However, SUSY and some other physical models predict much larger ones. Discovery of an EDM at the level of $d > 10^{-29}$ e cm, which for protons corresponds to $\eta_p > 2 \times 10^{-15}$, would obviously open a window to new physics. At that level of accuracy, unavoidable perturbations like spin rotation by the Earth's gravitational field and by the Earth's rotation must be seriously investigated as sources of systematic error in EDM measurement. Our Letter focuses on the leading gravitation spin effect because, in a ring, the Earth's rotation is not nearly as significant a source of error as the Earth's gravitational field. The effect of the Coriolis force, for example, averages to zero due to particle revolutions. (See Section 5 for further discussion of this point.)

A frozen-spin ring is designed so that the initial polarization of a particle relative to the equilibrium orbit will remain constant ("frozen") in the absence of EDM. Then, ideally, only the EDM will rotate the spin — in the plane perpendicular and tangential to that orbit. This requires a combination of particle momentum p, $p/mc = \beta\gamma$, $\gamma = \varepsilon/mc^2$, and lab-frame vertical magnetic B-field and radial electric E-field such that,

$$\vec{\omega}_a = -\frac{e}{mc}\left[a\vec{B} + \left(a - \frac{1}{\gamma^2\beta^2}\right)\frac{\vec{E}\times\vec{v}}{c^2}\right] = 0. \tag{1.1}$$

ω_a is the rotation frequency of the *planar* spin projection relative to the rotation frequency of the momentum [1]; $a = (g-2)/2$ where g is the gyromagnetic factor. When $\omega_a = 0$, the planar angle between $\vec{p}$ and $\vec{s}$ remains constant in time. Some feedback system is needed to hold this condition experimentally. We assume the existence of such a system, as well as a system canceling spin rotations in the vertical plane by perturbations other than EDM. (We refer the reader to [4] for discussion of these non-EDM problems, including betatron and synchrotron stability issues.)

* Corresponding author. Tel.: +1 607 255 3502; fax: +1 607 254 4552.
E-mail address: yfo1@cornell.edu (Y. Orlov).

0375-9601/$ – see front matter
http://dx.doi.org/10.1016/j.physleta.2012.08.011

In a frozen-spin EDM ring, the EDM detector(s) measure the vertical spin component as a function of time,

$$\frac{d\vec{s}}{dt} = \frac{\eta e}{2mc}(\vec{E} + \vec{\beta} \times \vec{B}) \times \vec{s}. \tag{1.2}$$

The initial spin polarization should be along the orbit. If (1.1) is satisfied, the longitudinal spin component, s_l, on the right side of (1.2) as well as E_r, β_l, and B_z remain constant, so s_z is growing.

Gravitation is involved here because, among other things, it rotates the spin in the same vertical plane tangential to the orbit. Rough preliminary estimates show that for the case of protons or deuterons, the scale of the EDM spin rotation in the vertical plane at $d = 10^{-29}$ e cm can be ~ 10 nrad/s, depending on the accessible electric field. The scale of the spin rotation by the Earth's gravitational field in the same plane is $g/c = 30$ nrad/s, which corresponds to a large systematic error in the EDM measurement.

As follows from (1.1), it is possible to use a purely electric frozen-spin ring for particles with a positive anomalous magnetic moment, $a = (g-2)/2 > 0$, which includes protons, muons and electrons but not deuterons, whose $a < 0$. In this case, $B = B_z = 0$ in Eq. (1.1) and, obviously, the particle momentum must equal $p = p_{mag} = mc/\sqrt{a}$ to freeze the spin. This is the so-called magic momentum introduced in 1970 for the muon $g-2$ experiment [5]. For protons, $p_{mag} = 0.7007$ GeV/c; for muons, $p_{mag} = 3.09$ GeV/c; for electrons, $p_{mag} = 15.005$ MeV/c. Such a regime has the following big advantage. Two beams of particles can be injected and rotated in opposite directions, meeting the same perturbations simultaneously, and the corresponding systematic errors can be reduced by combining the clockwise and counterclockwise data [2]. This idea is based on the fact that most perturbations (including the gravitational field) do not violate T-symmetry, whereas the EDM does. In the deuteron case, the magnetic field cannot be zero and the momentum cannot be magic. One can nevertheless try to extract the true EDM value by using clockwise and counterclockwise beams *in turn*, changing the particles' initial conditions and the sign of the magnetic field accordingly [3].

In Sections 2–4 we calculate gravitational spin rotations for the general case of a frozen-spin ring, including the purely electric version, investigating the same spin-gravity effect in different coordinate systems. This approach addresses a problem apparently unremarked in the spin-gravitation literature concerned with rings (see [6], for example): that the measured magnitude of spin rotation in the vertical plane is very sensitive to one's definition of "vertical" *direction*, which itself is determined by the experimental setup. Thus, computations of the spin-gravity effect in different coordinate systems can (as they do here) yield different results, because the vertical directions of the different coordinate systems do not coincide. We need to establish the scale of this unwelcome sensitivity, its sources, and ways to avoid it. We also need to address a parallel question about sensitivity to the choice of "vertical" *position* when the "vertical" direction is given.

Our calculation technique is based on manifestly covariant relativistic spin equations in the framework of Riemannian geometry, with the Schwartzschild metric linearized with respect to g, $g = 9.80665$ m/s^2, and small deviations from the circular equilibrium orbit. (Similar covariant equations were used much earlier in [7] with $g = 0$, but in connection with spin.) These equations are the general relativity extension of the quasi-classical Thomas–BMT equation [8] into which we introduce the EDM term:

$$\frac{DS^i}{cD\tau} \equiv \frac{dS^i}{cd\tau} + \Gamma^i_{kl} S^k u^l = \frac{e(1+a)}{2mc^2}\left(F^i_k S^k + u^i F^{kl} S_k u_l\right) - u^i S_k \frac{Du^k}{cD\tau} + \frac{e\eta}{2mc^2}\varepsilon^{iklm} F_{kj} S_l u^j u_m, \tag{1.3}$$

together with the Lorentz equation,

$$\frac{Du^i}{Dc\tau} \equiv \frac{du^i}{dc\tau} + \Gamma^i_{kl} u^k u^l = \frac{e}{mc^2} F^i_k u^k, \tag{1.4}$$

where

$$\Gamma^i_{kl} = \frac{1}{2} g^{im}\left(\frac{\partial g_{mk}}{\partial x^l} + \frac{\partial g_{ml}}{\partial x^k} - \frac{\partial g_{kl}}{\partial x^m}\right), \tag{1.5}$$

$\varepsilon^{iklm} \equiv e^{iklm}/\sqrt{-|g|}$; $|g|$ is the determinant of g_{ik}. We adopt the convention that Roman indices run over 0, 1, 2, 3. The spacetime interval is $ds \equiv cd\tau$, $ds^2 = g_{ik}\, dx^i\, dx^k$. For the purpose of this Letter, we need to know only Γ^2_{00}, Γ^2_{11}, Γ^3_{00}, Γ^3_{11} in order to calculate the spin rotation by gravitation in the tangential vertical planes of a frozen-spin ring. The components Γ^k_{i2}, Γ^k_{i3} are not needed because, in the equations of motion we use, they are multiplied by u^2, S^2, u^3, S^3, which by design equal either zero or almost zero in the equilibrium. $\Gamma^k_{01} = 0$ because our g_{ik} do not depend on x^0, x^1 and $g_{01} = 0$. $\Gamma^0_{00} = \Gamma^0_{11} = \Gamma^1_{00} = \Gamma^1_{11} = 0$, in our case.

The last term of (1.3) represents the EDM. Although the main actor in the EDM phenomenon is the dual EM field tensor $e^{iklm} F_{lm}/2$, additional factors u^j and u_m must be included in the last term to satisfy conditions $u_i S^i = 0$ and $S_i S^i = constant$. The next-to-last term of (1.3) describes the Thomas precession [9]. Combining this term with the second term in parentheses gives us the generalized BMT lab-frame spin equation,

$$\frac{dS^i}{dc\tau} + \Gamma^i_{kl} S^k u^l = \frac{e}{mc^2}\left[(1+a) F^i_k S^k + a u^i \left(F_{lm} S^l u^m\right)\right] + \text{EDM}; \tag{1.6}$$

$a = (g-2)/2$, where g is the particle gyromagnetic factor. (Below, we will use notation g only for gravity acceleration and $|g|$ for $\det g_{ik}$.)

For the metric g_{ik} we use the Schwarzschild solution in Schwarzschild coordinates [10]:

$$ds^2 = (1 - R_g/R)c^2\, dt^2 - R^2\left(\sin^2\theta\, d\varphi^2 + d\theta^2\right) - dR^2/(1 - R_g/R), \tag{1.7}$$

where $R_g = 2kM/c^2$ is the Earth's gravitational radius; $-kM/R = \Phi = \Phi_0 + g\bar{x}^3$, $\bar{x}^3 = R - R_0$, see Fig. 1; Φ is the gravitational potential, $2\Phi/c^2 \ll 1$; and $\Phi = \Phi_0$ is the potential at the Earth's radius, R_0:

$$\Phi_0/c^2 = -gR_0/c^2, \qquad R_g/R_0 = 2gR_0/c^2. \tag{1.8}$$

R_0 is defined by the Earth's circumference, $L = 2\pi R_0$. The equations for our electro-magnetic fields in a curvilinear coordinate system with gravitation are: