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On propagation of anti-plane shear waves in piezoelectric plates with surface effect

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ABSTRACT

Based on the surface piezoelectricity model, the anti-plane or horizontally polarized shear (SH) waves propagating in an infinite piezoelectric plate of nano-thickness are investigated to show the surface effect on wave characteristics. The influence on the overall properties of piezoelectric structures resulting from the surface effect is treated as a spring force exerting on the boundary of the bulk. The frequency equations of anti-symmetric and symmetric waves are presented analytically for the electrically short-circuited case. Numerical results show that the wave properties are size-dependent, and the surface effect becomes very pronounced at a high frequency.

1. Introduction

Piezoelectric materials/structures have found a wide range of applications in smart devices such as sensors, actuators, transducers, harvesters and so on. With the rapid development of nanotechnology, piezoelectric nanostructures have attracted tremendous attention due to their potential application in piezoelectric nano-devices [1–4]. For example, Wang [5] and Wang's group [6–9] have used piezoelectric zinc-oxide nanowire arrays to demonstrate a novel concept of nanogenerator, which can convert efficiently the nanoscale mechanical energy into the electric energy. At the nanoscale, however, piezoelectric structures exhibit obvious size-dependent phenomena, which have been demonstrated in recent experimental and theoretical studies [10–12]. The size-dependent properties of piezoelectric nanostructures, though, can be well explained from the viewpoint of the atomic simulation or the first principle. The analysis of piezoelectric structures at the nanoscale, using a continuum surface model, will allow us to better understand their behavior.

Surface effect is one of the causes for size-dependent behavior of nanostructures. At the nanoscale, a surface plays a substantial role in their overall properties due to the large ratio of surface to bulk. For elastic solids, a rigorous mathematical theory, which is referred to as the GM theory and incorporates surface effects, was originally developed by Gurtin and Murdoch [13,14]. In the GM theory, the elastic solids are divided into two parts: the surface and the bulk. The surface is regarded as a thin elastic membrane (without thickness) perfectly bonded to the bulk, and the analysis model is known as the surface layer model. Based on the GM theory, the static and dynamic size-dependent behaviors of elastic ultra-thin beam- and plate-like structures have been investigated by many researchers [15–18]. Lü et al. [19,20] studied the functionally graded ultra-thin films at the nanoscale and developed a corresponding refined plate theory involving surface effect. Green's functions for isotropic elastic half-space were derived by Chen and Zhang [21], He and Lim [22] and Koguchi [23] for different situations. It is worth mentioning that Gurtin and Murdoch [14,24] studied the effects of surface stress on waves propagating in elastic solids using the GM theory. It is shown that the surface effect results in a remarkable deviation from the classical theory at high frequencies.

For piezoelectric materials, the surface should have significant effect on its overall properties at the nanoscale as well. The surface stresses depend on not only surface strains but also electric fields. In order to capture the surface effect of piezoelectric nanostructures, Huang and Yu [25] firstly developed a surface piezoelectricity model which is an extension of the surface elasticity theory [13]. It is observed that the surface effect has a considerable influence on the electromechanical behavior of piezoelectric nanostructures. More recently, Pan et al. [26] proposed a phenomenological continuum theory of surface piezoelectricity

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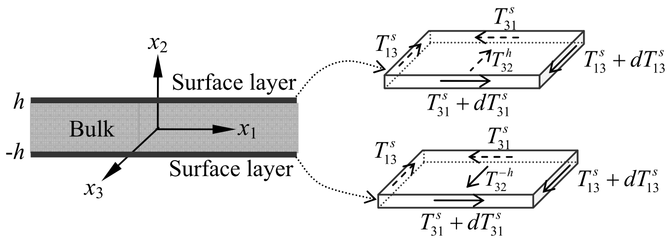

Fig. 1. Schematic sketch of the piezoelectric plate with surface effect.

accounting for the linear superficial interplay between electricity and elasticity. Based on the surface piezoelectricity model proposed by Huang and Yu [25], Yan and Jiang [27,28] investigated the surface effects on the electromechanical coupling and bending behaviors of nanowires and the vibration and buckling behaviors of the piezoelectric nanobeams using Euler–Bernoulli beam theory. The same authors also studied the electromechanical responses of curved piezoelectric nanobeams [29]. Chen [30] established a surface piezoelectricity theory using a different approach in which the surface layer is treated to be of a small thickness, and studied the surface effect on the propagation of Bleustein–Gulyaev wave in a piezoelectric half-space.

The principle of many piezoelectric devices is based on the wave propagation mechanism, so it is crucial to investigate the properties of waves propagating in piezoelectric nanostructures for applications in nanoelectromechanical systems (NEMS). To the authors' knowledge, there is only few relevant research on wave propagation in piezoelectric nanostructures by incorporating surface effects, except the paper of Chen [30], which considers the half-space model only. This motivates us to study the size-dependent behavior of wave propagation in piezoelectric structures at the nanoscale. In this Letter, anti-plane or horizontally polarized shear (SH) waves propagating in an infinite piezoelectric plate of nano-thickness are investigated based on the surface piezoelectricity model [25]. The piezoelectric plate is modeled as two surface layers of zero thickness plus the bulk core layer. The surface effects are thus modeled by the non-classical boundary conditions in which some sort of spring force is acting on the bulk core.

2. Basic equations

In this Letter, we consider an infinite piezoelectric plate of nano-thickness as shown in Fig. 1. The thickness of the plate is $2h$. The ox_1x_3 plane is identical with the mid-plane of the plate. The piezoelectric plate is poled in the x_3 direction. The surface layer model is adopted in our simulation of piezoelectric nanostructures, whose schematic diagram is shown in Fig. 1. The surface layer and the bulk layer have different material properties. For the bulk, the constitutive equations are the same as the classical piezoelectricity [31]. They are

$$T_{ij} = c_{ijkl}S_{kl} - e_{ijk}E_k, \tag{1}$$

$$D_i = e_{ikl}S_{kl} + \varepsilon_{ik}E_k, \tag{2}$$

where T_{ij}, S_{kl}, u_i, D_i and E_k are stresses, strains, displacements, electric displacements and electric fields, and c_{ijkl}, e_{ijk} and ε_{ij} are elastic, piezoelectric and dielectric constants of the bulk, respectively. The convention of summation over repeated indices is employed and a comma followed by an index indicates differentiation with respect to the corresponding coordinate variable in the Cartesian coordinate system. The subscripts i, j, k and l range over 1, 2, 3 and a, b in the following range over the integers 1, 3 throughout this Letter.

The strain-displacement and electric field-potential relations are

$$S_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \tag{3}$$

$$E_i = -\phi_{,i}, \tag{4}$$

where ϕ is the electric potential. Eqs. (3) and (4) are the "geometric" relations of the piezoelectric materials, which are also valid for the surface layer. Under the quasi-electrostatic assumption and in the absence of external body forces and free charges in the bulk, the equations of motion and the charge equation of electrostatics are

$$T_{ij,j} = \rho\ddot{u}_i, \qquad D_{i,i} = 0, \tag{5}$$

where ρ is the mass density of the bulk.

For the surface layer of the piezoelectric plate, the constitutive equations proposed by Huang and Yu [25] are

$$T_{ab}^s = T_{ab}^0 + c_{abcd}^s S_{cd} - e_{abk}^s E_k,$$
$$D_i^s = D_i^0 + e_{iab}^s S_{ab} + \varepsilon_{ij}^s E_j, \tag{6}$$

where T_{ab}^s and D_i^s are the surface stresses and electric displacements, T_{ab}^0 and D_i^0 are the surface residual stresses and electric displacements, and c_{abcd}^s, e_{iab}^s and ε_{ij}^s are the surface elastic, surface piezoelectric and surface dielectric constants, respectively. The surface constitutive model of Eq. (6) is a natural extension of the GM theory. As mentioned earlier, the relations of strain-displacement and electric field-potential of the surface are still given by Eqs. (3) and (4). From the GM theory [14], the equations of motion for the surface layer are written as

$$T_{ia,a}^s + T_{i2} = \rho^s \ddot{u}_i, \tag{7}$$

where ρ^s is the surface mass density. The charge equation $(5)_2$ for the bulk is essentially the differential form of Gauss' law. So, for the surface layer, the charge equation of electrostatics is the same as Eq. $(5)_2$ in form, but we need to replace D_i by D_i^s.

The surface model by Eq. (6), which is under the assumption of linear theory, shows that the surface effect of the surface on the whole structure includes two parts: (1) the residual stress and electric displacement which is strain-independent and (2) the surface piezoelectricity which is strain-dependent. The residual part is caused from the relaxation when a new surface is created. It may in fact reduce a compressive or tensile initial state, which has been explained at length by Zhang et al. [32,33]. Park [34,35] studied the effect of the surface residual stress on the resonant frequencies and the critical buckling strains of silicon nanowires at the basis of finite deformation theory. The residual stress induced by the initial strain has some impact on the properties of structures, but the surface elasticity (strain-dependent) will play a dominant role with decreasing the size. For example, the results in [35] have shown that the strain-dependent part of the surface stress has an increasing important effect on the resonant frequencies with decreasing nanowire size. In addition to the residual part of the surface are constants and hence their space or time derivatives always are zero. The residual stress and electrical displacement of the surface will be neglected for convenience in this Letter. In other words, we mainly study that "how the surface piezoelectricity affects the wave properties propagating in piezoelectric nano-plates" in this Letter.

In the fundamental equations (6) and (7) for the surface layer of the piezoelectric plate, there appear a group of new surface material constants and only few of them are known from the first principle computation. This therefore causes remarkable difficulty in the study of relevant problems of piezoelectric nanostructures. By comparing their equations governing the material surface with