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Attractors and chaos of electron dynamics in electromagnetic standing waves ☆

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ABSTRACT

In an electromagnetic standing wave formed by two super-intense colliding laser pulses, radiation reaction totally modifies the electron motion. The quantum corrections to the electron motion and the radiation reaction force can be independently small or large, depending on the laser intensity and wavelength, thus dividing the parameter space into 4 domains. The electron motion evolves to limit cycles and strange attractors when radiation reaction dominates. This creates a new framework for high energy physics experiments on the interaction of energetic charged particle beams and colliding super-intense laser pulses.

1. Introduction

New regimes of light-matter interaction emerge with the increase of laser power beyond petawatt, leading to radiation pressure dominant acceleration of ions, bright coherent x-ray generation and electron–positron pair creation [1]. The next generation of high-power short-pulse lasers will soon reach the electromagnetic radiation intensity of 10^{23} W/cm^2 for a 1 μm wavelength [2], 10^5 times greater than the relativistically strong intensity threshold $I_0 = 1.37 \times 10^{18}$ W/cm^2. Then the electromagnetic emission by electrons will be substantial [1,3,4], making the electron dynamics strongly dissipative [5–8] and causing the laser energy fast conversion to gamma-rays [9,10].

The laser intensity above 10^{23} W/cm^2 brings novel physics [11] (see Refs. [1,3,4,12–17] for details), where the electron (positron) dynamics is principally determined by radiation reaction force and quantum electrodynamics (QED) effects. The latter weaken the electromagnetic emission of relativistic electrons, thus decreasing the radiation reaction force [18,19].

Even in the simple case of a standing wave, the electron dynamics is surprisingly complicated. In the magnetic field node plane, the electron motion is unstable [13], with the instability growth rate being about the electromagnetic field frequency. In a linearly polarized standing wave, the electrons are concentrated in the standing wave spatial periods [20–22]. The standing wave configuration is widely used in the QED theory because the description is greatly simplified in the magnetic field node plane: there the electric field vector merely rotates in a circularly polarized or oscillates in a linearly polarized standing wave, with nonzero Poincaré invariants. In addition, in a standing wave formed by two colliding laser puses, the resulting electric field can be higher than that of one pulse, facilitating QED effects, as in the multi-beam configuration [23].

In this Letter we show that in the electron dynamics in a strong electromagnetic standing wave, the quantum corrections and the radiation reaction force can be independently small or large, thus dividing the parameter space into 4 domains. When radiation reaction is significant, a strongly dissipative electron dynamics has limit cycles and strange attractors. Similar structures can be seen even in a transient standing wave formed by two colliding laser pulses.

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2. Radiation reaction

The electron dynamics in the electromagnetic wave is described by the equation

$$\dot{\mathbf{p}} = e(\mathbf{E} + \boldsymbol{\beta} \times \mathbf{B}) + \mathbf{f}_{\rm rad}, \ \ \mathbf{f}_{\rm rad} = G_e \mathbf{f}_{\rm LL}, \tag{1}$$

where $\mathbf{p} = m_e c \gamma_e \boldsymbol{\beta}$, $\boldsymbol{\beta} = \mathbf{v}/c$, $\gamma_e = (1 - \beta^2)^{-1/2}$; $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic fields; e, m_e, $\mathbf{v}$ and $\mathbf{p}$ are the electron charge, mass, velocity and momentum, respectively; c is the speed of light in vacuum. The radiation reaction force in the Landau–Lifshitz [24] form, $\mathbf{f}_{\rm LL}$, is reduced by a factor G_e representing the radiation reaction force weakening due to QED effects, following the approach of Refs. [11,9,14].

In the ultrarelativistic limit $\gamma_e \gg 1$, the radiation reaction force is

$$\mathbf{f}_{\rm rad} \approx -\varepsilon_{\rm rad} G_e m_e c \omega \boldsymbol{\beta} a_S^2 \chi_e^2. \tag{2}$$

Here $\varepsilon_{\rm rad} = 4\pi r_e/3\lambda \approx 1.18 \times 10^{-8}$ (1 μm/λ); $r_e = e^2/m_e c^2 \approx 2.82 \times 10^{-13}$ cm is the classical electron radius; $a_S = eE_S/m_e\omega c \approx 4.12 \times 10^5(\lambda/1\ \mu\text{m})$ corresponds to the QED critical field $E_S = \alpha e/r_e^2$; $\alpha = e^2/\hbar c$ is the fine-structure constant [25]; ω and λ are the electromagnetic wave frequency and wavelength. The relativistic and gauge invariant parameter

$$\chi_e = (\gamma_e/E_S)[(\mathbf{E} + \boldsymbol{\beta} \times \mathbf{B})^2 - (\boldsymbol{\beta} \cdot \mathbf{E})^2]^{1/2} \tag{3}$$

characterizes the probability of a gamma-photon emission by an electron with momentum $\mathbf{p}$. QED effects are negligible for $\chi_e \ll 1$ and become strong for $\chi_e \simeq 1$.

The stronger QED effects, the less radiation is emitted. According to Ref. [18], the total radiated power is reduced by a factor depending on the parameter χ_e. Introduced in Eq. (2) this factor is written using Refs. [25–27] as follows:

$$G_e(\chi_e) = -\int_0^\infty \frac{3 + 1.25\chi_e \xi^{3/2} + 3\chi_e^2 \xi^3}{(1 + \chi_e \xi^{3/2})^4} \mathrm{Ai}'(\xi)\xi d\xi, \tag{4}$$

where $\mathrm{Ai}(x)$ is the Airy function. The discreet nature of the photon emission is neglected (see Refs. [28,29,14]). For computations we approximate Eq. (4) by $G_e(\chi_e) \approx (1 + 18\chi_e + 69\chi_e^2 + 73\chi_e^3 + 5.806\chi_e^4)^{-1/3}$, accurate within 10^{-3} for $0 < \chi_e < 10$, with the same asymptotics at 0 and ∞ as Eq. (4).

3. Charge in a standing wave

We consider the electric field of the one-dimensional (1D) circularly polarized electromagnetic standing wave in the magnetic field node plane: $\boldsymbol{a} = -a(\mathrm{i}_2 \cos\tau + \mathrm{i}_3 \sin\tau)$, where i_2 and i_3 are orthogonal unit vectors perpendicular to the standing wave axis; $\tau = \omega t$, $\mathbf{q} = \mathbf{p}/m_e c$, and $a = eE/m_e\omega c = (I/I_0)^{1/2}(\lambda/1\ \mu\text{m})$. We change to the rotating coordinate system [30],

$$q_\parallel = q_2 \cos\tau + q_3 \sin\tau, \ \ q_\perp = q_2 \sin\tau - q_3 \cos\tau. \tag{5}$$

Eq. (3) yields $\chi_e = (a/a_S)(1 + q_1^2 + q_\perp^2)^{1/2}$.

Substituting Eq. (5) into Eq. (1) and neglecting the electron momentum along the standing wave axis, $q_1 \ll (q_2^2 + q_3^2)^{1/2}$, we obtain

$$\dot{q}_{||} + q_\perp = a - \varepsilon_{\rm rad}\, G_e(\chi_e) a^2 q_{||} q_\perp^2/\gamma_e, \tag{6}$$

$$\dot{q}_\perp - q_{||} = -\varepsilon_{\rm rad}\, G_e(\chi_e)[\gamma_e a + a^2 q_\perp(1 + q_\perp^2)/\gamma_e], \tag{7}$$

where the dot denotes differentiation with respect to τ. Solutions of this system asymptotically tend to steady state (provided $\varepsilon_{\rm rad} > 0$). Following Ref. [6], from Eqs. (6)–(7) we find the critical electromagnetic amplitude determining the radiation reaction strength,

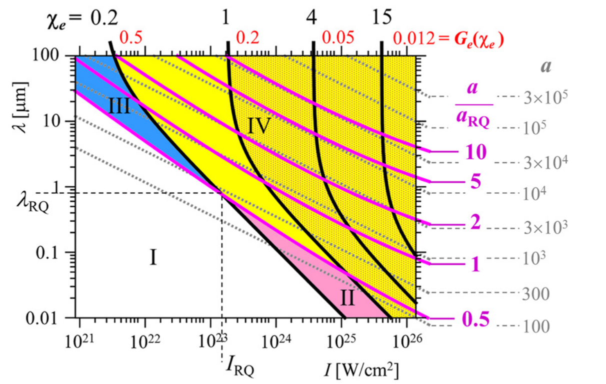

Fig. 1. The characteristics of the electron stationary motion in a rotating electric field. Curves: the field dimensionless amplitude a (gray dashed), ditto normalized to $a_{\rm RQ} = [\varepsilon_{\rm rad} G_e(\chi_e)]^{-1/3}$ (magenta), χ_e parameter (black), factor $G_e(\chi_e)$ (red). Domains I–IV and hatched area: see the text. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

$$a_{\rm RQ} = [\varepsilon_{\rm rad} G_e(\chi_e)]^{-1/3}. \tag{8}$$

Radiation reaction is negligible for $a \ll a_{\rm RQ}$ and becomes substantial for $a \simeq a_{\rm RQ}$. In this limit, for $q_\perp \propto a_{\rm RQ}$ with a factor of the order of unity, we estimate the χ_e parameter as $\chi_m \approx a_{\rm RQ} q_\perp/a_S \approx a_{\rm RQ}^2/a_S$ corresponding to $G_m = G_e(\chi_m)$. This gives the critical wavelength at which radiation reaction becomes substantially quantum,

$$\lambda_{\rm RQ} = 9\pi r_e/2\alpha^3 \chi_m^3 G_m^2. \tag{9}$$

For $\chi_m = 1$ ($G_m \approx 0.18$), we obtain $I_{\rm RQ} = a_{\rm RQ}^2 I_0 = 1.75 \times 10^{24}$ W/cm^2 and $\lambda_{\rm RQ} = 3.1$ μm.

Taking $d/d\tau = 0$ in Eqs. (6)–(7), corresponding to the steady state solution [30], we obtain algebraic dependences of $q_\parallel, \gamma_e$ on $q_\perp$,

$$q_\parallel^2 = (a - q_\perp)(1 + aq_\perp)(1 + q_\perp^2)/[q_\perp(1 + aq_\perp) - a], \tag{10}$$

$$\gamma_e^2 = a^2 q_\perp(1 + q_\perp^2)/[q_\perp(1 + aq_\perp) - a], \tag{11}$$

and the expression

$$a - q_\perp = [\varepsilon_{\rm rad} G_e(\chi_e)]^2 a^2 q_\perp^3 (1 + aq_\perp), \tag{12}$$

implicitly defining $q_\perp$ as a function of a, $\varepsilon_{\rm rad}$, a_S. Thus all the dependent variables are functions of the electromagnetic wave intensity, $I = E^2 c/4\pi$, and wavelength, λ. In this way Fig. 1 shows the χ_e parameter, corresponding factor $G_e(\chi_e)$, and the electromagnetic standing wave amplitude normalized to $a_{\rm RQ}$.

The radiation reaction force becomes substantial when $a \gtrsim 0.5 a_{\rm RQ}$ while QED effects come into play at $\chi_e \gtrsim 0.2$, which corresponds to the radiation reaction force weakening with the factor $G_e \lesssim 0.5$. The intersection of the curves $a/a_{\rm RQ} = 0.5$ and $\chi_e = 0.2$ gives the characteristic intensity $I_{\rm RQ}^* \approx 1.5 \times 10^{23}$ W/cm^2, and wavelength $\lambda_{\rm RQ}^* \approx 0.76$ μm, within an order of magnitude of the estimate presented above. At this point, four different domains meet, Fig. 1: I. radiation reaction is negligible; II. QED effects dominate while the radiation reaction force is small; III. radiation reaction is mostly classical; IV. the radiation reaction force and QED effects are both strong. Beyond $\chi_e \gtrsim 1$, the model is not applicable because of the discrete nature of electron emission.

4. Phase space

In order to generalize the picture given by Eqs. (6)–(7) and investigate the electron dynamics with radiation reaction and QED effects, we numerically solve Eqs. (1), (3), (4). In our setting the electric field oscillates at antinodes, $x = \pm n\lambda/2$, and vanishes at