Physics Letters A 379 (2015) 2062–2067

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

# Bucket transport of energetic ions in tokamaks

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ARTICLE INFO

*Article history:*
Received 27 February 2015
Received in revised form 3 June 2015
Accepted 18 June 2015
Available online 23 June 2015
Communicated by F. Porcelli

*Keywords:*
Tokamak
Energetic ions
Transport
Resonance islands
Magnetic perturbations

ABSTRACT

The so-called "bucket transport" of energetic ions — the spatial mixing of these ions due to spatial displacement of resonances — is studied with special attention to quasi-steady-state magnetic perturbations. A Hamiltonian formalism suitable to the case when the resonance displacement results from the collisional slowing down of the particles and the temporal evolution of the safety factor profile is suggested. The energy flux produced due to the bucket transport is shown to be considerable in configurations with low shear. It is shown that the bucket transport flux associated with magnetic islands tends to be localized at some distance from the islands. The bucket transport caused by perturbations with non-zero frequencies is also discussed.



## 1. Introduction

Quasi-steady-state perturbations, i.e., perturbations with negligible rotation in the plasma frame, which violate the symmetry of magnetic configurations, are often observed in tokamak plasmas. In many cases they lead to the formation of magnetic islands around the $q = m/n$ flux surface, where $q$ is the tokamak safety factor (the magnetic winding number), $m$ and $n$ are the poloidal and toroidal numbers of the perturbation, respectively. The nature of these perturbations and concomitant phenomena can be different. Well-known examples are the neoclassical tearing modes [1] and the sawtooth oscillations [2], which are associated with large-scale perturbations resulting in the formation of low-$n$ islands. It seems that low-frequency activity with $n = 1$, which accompanies avalanches of high-frequency instabilities driven by energetic ions (see, e.g., Refs. [3,4]), also has zero frequency in the plasma frame. On the other hand, when the $q$-profile slightly exceeds the resonant magnitude $q = m/n$, quasi-static deformations of flux surfaces occur; in particular, the infernal mode [5] with $(m, n) = (1, 1)$ takes place when the $q$-profile is flat and slightly exceeds unity; a $(1, 1)$-mode may also occur in reversed-shear discharges with $q_{\min}$ slightly exceeding unity [6]. Low-$n$ static magnetic perturbations can also be produced by external coils with the aim to suppress or mitigate ELMs (edge-localized modes) [7].

The purpose of this Letter is to study the so-called "bucket transport" — the plasma mixing due to the displacement of energetic (fast) ions trapped in the resonance when the spatial location of this resonance changes. The analysis is restricted mainly to static perturbations, the case of perturbations with finite frequency ($\omega \neq 0$) is only briefly discussed. Well-passing particles (for which the longitudinal velocity $v_\parallel \approx \text{const}$) are considered.

Note that the bucket transport was studied in Refs. [8–11]. In Refs. [8–10] the transport caused by frequency chirping was considered. In contrast to this, we study the transport caused by collisional slowing down of the particles and/or temporal evolution of the plasma current. The influence of collisions on the transport was studied also in Ref. [11], where the analysis was restricted to studying the bucket motion due to the infernal mode; the motion of non-resonant particles, which, in general, plays an important role in the particle mixing, was not considered. The possibility that the particles trapped in the wave move in radial direction due to collisions was mentioned in an earlier work [12] (where this effect was called the "conveyor belt").

## 2. Hamiltonian formalism

We proceed from the following Lagrangian differential form of the guiding centre motion [13]:

$$\Gamma = \left(\frac{e}{c}\boldsymbol{A} + M v_\parallel \boldsymbol{b}\right) \cdot d\boldsymbol{x} + \frac{Mc}{e}\mu\, d\Theta - \mathcal{E}\, dt, \tag{1}$$

where $\boldsymbol{x}$ is the guiding centre position, $\Theta$ is the gyrophase, $e$, $M$, $\mathcal{E}$, and $\mu$ are the charge, mass, energy, and magnetic moment of

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http://dx.doi.org/10.1016/j.physleta.2015.06.041

the particle, respectively, $\boldsymbol{b} = \boldsymbol{B}/B$, $\boldsymbol{B}$ is the magnetic field, $\boldsymbol{A}$ is its vector potential, and we have assumed that there is no electric field (we consider quasi-steady-state perturbations in this section).

As $\mu$ and $\mathcal{E}$ are both conserved, we can ignore the corresponding terms of the differential form and concentrate on simplifying the remaining term. We use magnetic coordinates $(\psi, \vartheta, \varphi)$ (where $\psi$ is the toroidal flux, $\vartheta$ and $\varphi$ are the poloidal and toroidal angles, respectively). Then we can represent the equilibrium magnetic field in the form (see, e.g., [14])

$$\boldsymbol{B}_0 = \nabla\psi \times \nabla\vartheta + \nabla\varphi \times \nabla\psi_p = \nabla \times (\psi\nabla\vartheta - \psi_p\nabla\varphi) \tag{2}$$

and take the gauge $A_{0\vartheta} = \psi$, $A_{0\varphi} = -\int d\psi\, \iota(\psi)$ and $A_{0\psi} = 0$ for its vector potential $\boldsymbol{A}_0$, where the subscripts denote the covariant components, $\psi_p$ is the poloidal magnetic flux, and $\iota = q^{-1} = d\psi_p/d\psi$. We assume that $\epsilon = r/R_0 \ll 1$ (with $r$ the radial coordinate and $R_0$ the radius of the magnetic axis) and that $\tilde{B}_\parallel/\tilde{B}_\perp \ll 1$, where $\tilde{B}_\parallel$ and $\tilde{B}_\perp$ are the longitudinal and transversal components of the magnetic perturbation, respectively. Then we can choose the gauge

$$\tilde{\boldsymbol{A}} = -\alpha(\psi)B_0R_0^2\cos(m\vartheta - n\varphi)\nabla\varphi \tag{3}$$

for the perturbation, where $\alpha$ characterizes the perturbation amplitude, $B_0 = B(r=0)$; indeed, one can show that this choice provides $\tilde{B}_\parallel/\tilde{B}_\perp \sim \epsilon$. We neglect the contribution of the perturbation to $\boldsymbol{b}$ (on varying the differential form, one can see that the $\boldsymbol{b}$-term is responsible for the particle drift across the field lines) and retain the perturbation only in the $\boldsymbol{A}$-term responsible for the longitudinal motion. Finally, we neglect $b_\vartheta$ and $b_\psi$ (in comparison with $b_\varphi$) and take $B_\varphi \approx B_0R_0$. After normalization by $eB_0/c$, we write equation (1) in the form

$$\Gamma = J\,d\vartheta - H\,d\varphi, \tag{4}$$

where $J = \psi/B_0$ is the flux surface cross-section square divided by $2\pi$ ($J = \kappa r^2/2$ when the cross-section is elliptical with the elongation $\kappa$),

$$H = H_0(J,\theta) + R_0^2\alpha(J)\cos(m\vartheta - n\varphi), \tag{5}$$

$$H_0 = \int_0^J dJ\,\iota(J) - \rho_\parallel R_0, \tag{6}$$

$\rho_\parallel = v_\parallel/\omega_B$, $\omega_B = eB/(Mc)$, $v_\parallel$ is the longitudinal velocity of the particle.

We observe that the differential form $\Gamma$ corresponds to a one-dimensional Hamiltonian system, with $\vartheta$ playing the role of the coordinate, $J$ the momentum, and $H$ the Hamiltonian. The toroidal angle $\varphi$ plays the role of time. If we keep only the first term in $H_0$ and take $\alpha = 0$, Hamilton's equations describe the unperturbed magnetic field lines. On adding the last term of $H$, we obtain equations of the perturbed field lines. The second term of $H_0$ describes the particle drift across the magnetic field. Note that the perturbation in Eq. (5) ($\alpha \neq 0$) does not lead to chaos when $\rho_\parallel = 0$ because in this case the Hamiltonian possesses symmetry in magnetic coordinates.

It is known that the collisional slowing down does not affect the equations for $\dot{\boldsymbol{x}}$ ($\boldsymbol{x}$ is the guiding centre position). Therefore, the Hamiltonian equations

$$\frac{d\vartheta}{d\varphi} = \frac{\partial H}{\partial J}, \qquad \frac{dJ}{d\varphi} = -\frac{\partial H}{\partial\vartheta} \tag{7}$$

remain true in the presence of the slowing down at every point of the particle trajectory if we take the appropriate magnitude of the particle velocity $v$. Now let us consider an ensemble of well passing particles launched at the same time from the same toroidal angle $\varphi$. The quantities $v$ and $\varphi$ are connected with the time $t$ by the relations

$$\dot{\varphi} = v_\parallel/R, \qquad \dot{v} = -\nu_s v, \tag{8}$$

where $R$ is the distance to the axis of symmetry, $\nu_s$ is the slowing-down frequency. Neglecting weak fast variations of $v_\parallel/R$, we can consider $v$ as a function of $\varphi$ for the particles of the ensemble. We conclude that the motion of these particles is described by the Hamiltonian (5) but with $v$ (and thus $\rho_\parallel$) depending on $\varphi$.

Then we perform a canonical coordinate transformation, replacing $(J,\vartheta)$ with action–angle variables of the Hamiltonian $H_0$, which we denote as $(I,\theta)$ (one can show that the difference between $I$ and $J$, as well as between $\theta$ and $\vartheta$ is $\sim \Delta_b/r$, where $\Delta_b \sim q\rho/\kappa$ is the radial deviation of the orbit from the flux surface). In the action–angle variables, the Hamiltonian $H_0$ takes the form

$$H_0 = H_0(I,\varphi) = \int_0^I dI\,\iota_*(I,\varphi). \tag{9}$$

Here $\iota_* = d\theta/d\varphi$, the dependence on $\varphi$ appears only when the collisional slowing down is taken into account.

The quantity $\iota_*$ differs from $\iota$ due to toroidal precession. The calculation of the toroidal precession is rather cumbersome; it requires knowing $\boldsymbol{B}$ with the accuracy of $\epsilon^2$. Following Ref. [15], we write

$$\iota_* = \left\langle\frac{d\vartheta}{d\varphi}\right\rangle = \frac{1}{\bar{q}}\left(1 - \frac{\omega_{\rm pr}}{\omega_\varphi}\right), \tag{10}$$

where the angular brackets denote time averaging, $\bar{q} = \oint d\vartheta\, q/(2\pi)$ with the integral taken along the particle orbit, $\omega_\varphi = \langle\dot{\varphi}\rangle$, $\omega_{\rm pr}$ is the toroidal precession frequency given by

$$\omega_{\rm pr} = \xi v^2/(R_0^2\omega_{B0}) \tag{11}$$

with $|\xi| \lesssim 1$. The coefficient $\xi$ is determined by the $q$- and pressure profiles, the flux surface shape and other factors; it is negative in circular plasmas with low $\beta$ ($\beta = 8\pi p/B^2$, $p$ is the plasma pressure), becoming more negative with the increase of $\beta$, but it may be positive at strong plasma elongation and low $\beta$, see [16].

It should be mentioned that when $H_0$ depends on $\varphi$, the transformation from $(J,\vartheta)$ on $(I,\theta)$ depends on $\varphi$, too, and this dependence results in an additional term depending on $\theta$. However, we neglect this term because it is very small, $\sim qrR_0\nu_s/\omega_B$.

In the action–angle variables the perturbation is no longer single-harmonic because of the toroidal drift of the particle. However, for the sake of simplicity we will disregard the satellite harmonics of the perturbation, as well as the spatial dependence of $\alpha$. Neglecting the difference between $\theta$ and $\vartheta$ in the perturbation term of Eq. (5), we conclude that the resonance condition is

$$\iota_*(I) = n/m. \tag{12}$$

Assuming that the $\iota$-profile in the vicinity of the resonance point can be approximated by a linear function of $I$, we write Eq. (5) in the form

$$H = \iota_*'(I - I_{\rm res})^2/2 + R_0^2\alpha\cos(m\bar{\theta}), \tag{13}$$

where $\bar{\theta} = \theta - (n/m)\varphi$, $I_{\rm res}$ is $I$ at the resonance given by Eq. (12), prime denotes derivative in $I$.

One can see that the Hamiltonian (13) is reduced to that studied in Ref. [8]:

$$H = p^2/2 - \hat{A}\cos[q - \phi(t)], \tag{14}$$