Physics Letters A 378 (2014) 131–138

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Symmetrical positive–negative annular phase object for optical nonlinearity characterization

Zhongquan Nie, Zhongguo Li, Xueru Zhang *, Guanghong Ao, Guang Shi, Yuxiao Wang, Yinglin Song *

Department of Physics, Harbin Institute of Technology, Harbin 150001, PR China

ARTICLE INFO

Article history:
Received 9 May 2013
Received in revised form 14 October 2013
Accepted 16 October 2013
Available online 22 October 2013
Communicated by C.R. Doering

Keywords:
Positive–negative annular phase object
Nonlinear refractive index
Phase delay
Phase contrast signal

ABSTRACT

A novel symmetrical positive–negative annular phase object (PNA-PO) is proposed in the $4f$ coherent imaging system and a third-order nonlinear refraction of the material is measured. By decomposing the normalized incident field passing through the PNA-PO into three top-hat beams with various radii and phase delays, and by using some applicable approximations, analytical solution of the phase contrast signal ΔT is revealed. Furthermore, variations of the analytical solution and numerical simulation of ΔT as a function of the on-axis nonlinear phase shift ϕ_0 are shown. CS_2 as a standard sample is performed using a PNA-PO to illustrate the feasibility and predominance of our system.

1. Introduction

In 2004, a nonlinear imaging technique with a phase object (NIT-PO) was pioneered to investigate the cubic nonlinear index of materials [1]. By numerically fitting the intensity change of the nonlinear image, the third-order nonlinear refractive index n_2 was extracted. This NIT-PO can also be used to study the kinetics of photo-induced effects by shot-by-shot measurements [2]. Subsequently, G. Boudebs et al., using the first-order approximation, obtained an analytical solution of the field in the image plane [3]. Furthermore, they discussed the optimized parameters (L_p/R_a and ϕ_L) to increase the sensitivity of the measurement system. They further showed a limitation of nonlinear phase shift ($|\phi_0| \leqslant \pi$) in this single shot technique, and the variation of ΔT performed oscillation around a saturation value beyond this region.

In 2006, Y. Li et al. optimized the values of L_p/R_a and ϕ_L in the $4f$ coherent imaging system to maximize the sensitivity [4]. Then, they examined the nonlinear refraction and absorption simultaneously by using NIT-PO [5]. For both nonlinear refraction and absorption measurements, ϕ_L was determined to be 0.5π with L_p/R_a equal to 0.3 to obtain the best sensitivity.

In 2008, Y. Li et al. reported a positive–negative circular phase object (PNC-PO), it was sensitive to positive nonlinear phase shift in the positive nonlinear medium yet was more sensitive to negative nonlinear phase shift [6]. As a consequence, this new PO could induce positive and negative phase contrast signals simultaneously, thus the sensitivity of the $4f$ system could be improved. It was worthy to notice that the enhancement of sensitivity was particularly significant at the negative nonlinear phase shift area, and it almost reached an order of magnitude when the nonlinear phase shift $\phi_0 = -\pi$. Moreover, compared with the Z-scan technique [7,8] in $|\phi_0| \leqslant \pi$ conditions, the sensitivity of the $4f$ system with PNC-PO was about 1.7–3.0 times greater. However, since the PNC-PO was asymmetrical, the analytical expression of the field distribution cannot be obtained. Recently, we proposed a positive–negative bar phase object (PNB-PO) [9] to facilitate the fabrication of a PO. Regrettably, the analytical formalism was also out of reach. In other words, it is difficult to obtain an analytical solution when PO is asymmetric. M. Shui et al., in 2010, displayed an analytical expression of the traditional circular symmetric PO to characterize the pure nonlinear refractive index based on first-order approximation [10]. This analytical expression can be used to optimize the sensitivity and determine monotonic interval for optical nonlinearity characterization.

In this paper, we present a novel PNA-PO in the $4f$ system, and develop an analytical expression of ΔT and sensitivity $d\Delta T/d\phi_0$ when the nonlinear phase shift is small. Two kinds of special cases when the radius of the middle ring is close to the radii of the inner circle and the aperture are also given, which correspond to the conventional PO and the PO consisting of a circle with

* Corresponding authors. Tel.: +86 451 86414128; fax: +86 451 86414128.
E-mail addresses: xrzhang@hit.edu.cn (X. Zhang), ylsong@hit.edu.cn (Y. Song).

0375-9601/$ – see front matter
http://dx.doi.org/10.1016/j.physleta.2013.10.022

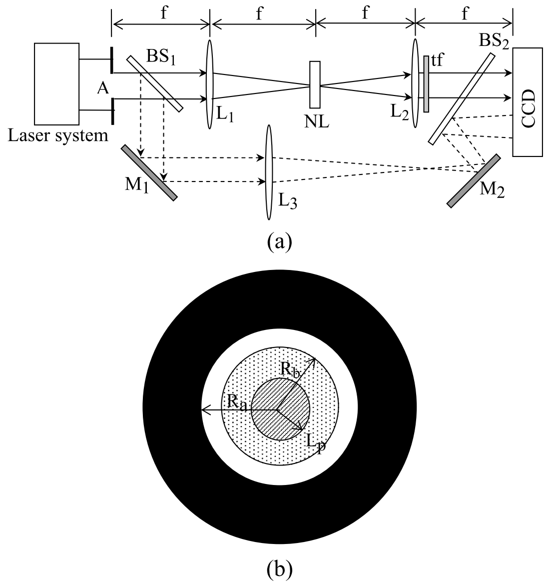

Fig. 1. (a) Schematic of a $4f$ coherent imaging system using a new three-zone PNA-PO. NL is the nonlinear material, L_1, L_2 and L_3 are lenses; a neutral filter (tf) is located after L_2; BS_1 and BS_2 are beam splitters; M1 and M2 are mirrors. (b) Sketch map of the three-zone PNA-PO used at the entry of the set-up.

positive phase shift and a ring with negative phase shift, respectively. Moreover, the results of analytical solution of ΔT compared with numerical simulation as a function of ϕ_0 are shown. The measurement technique with PNA-PO is experimentally validated by measuring the nonlinear refractive index of a standard sample CS_2.

2. Analytical solution

The arrangements of the NIT-PO and PNA-PO are shown in Fig. 1. The novel PO consists of a circular PO of radius L_p, characterized by a uniform phase shift ϕ_L; an annular PO of radius R_b, provides a homogeneous phase retardation $-\phi_L$; and an aperture diaphragm of radius R_a with zero phase shift. This new phase-only pupil filter is placed on the incident plane, while the sample is placed at the confocal plane of lenses L_1 and L_2 and is considered as 'thin' which means the sample's thickness is much less than the depth of focus. A CCD camera at the output plane of the $4f$ system is utilized to collect the image of the laser pulse. It is assumed that the Fourier optics is sufficient to describe the image formation.

2.1. Field distribution in the focal plane

The PNA-PO is illuminated by an expanded Gaussian beam generated from a picosecond pulsed laser. If R_a is much smaller compared to the beam waist of the input Gaussian beam, the incident beam can be regarded as a top-hat beam. For simplicity, we consider an incident unit-amplitude beam at the entry of the object. Consequently, the transmittance function $E_i(r)$, characterizing the phase aperture, can be decomposed into three top-hat beams TH1, TH2 and TH3 in turn as follows

$$\begin{aligned} E_i(r) &= \mathrm{TH1} + \mathrm{TH2} + \mathrm{TH3} \\ &= \mathrm{circ}(r/R_a) + \left[\exp(-i\phi_L) - 1\right]\mathrm{circ}(r/R_b) \\ &\quad + \left[\exp(i\phi_L) - \exp(-i\phi_L)\right]\mathrm{circ}(r/L_p), \end{aligned} \tag{1}$$

where the function $\mathrm{circ}(t)$ is equal to 1 if the radius t is less than 1 and 0 otherwise. The total field amplitude in focal plane is the superposition of the spatial Fourier transform of the incident field TH1, TH2 and TH3

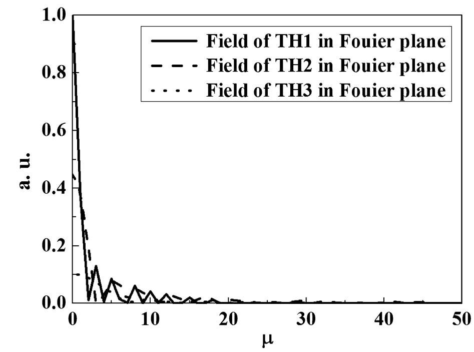

Fig. 2. The field amplitude distribution in the focal plane, $\rho = 0.67$, $\kappa = 0.33$. The both field amplitudes of TH2 and TH3 are amplified properly.

$$\begin{aligned} E_f(\mu) &= \frac{\mathrm{B}\{E_i(r)\}}{\lambda f_1} \\ &= 2E_{f0}\left\{\frac{J_1(\mu)}{\mu} + \left[\exp(-i\phi_L) - 1\right]\rho^2\frac{J_1(\rho\mu)}{\rho\mu}\right. \\ &\quad \left. + \left[\exp(i\phi_L) - \exp(-i\phi_L)\right]\kappa^2\frac{J_1(\kappa\mu)}{\kappa\mu}\right\}, \end{aligned} \tag{2}$$

where B{} represents the Fourier–Bessel transform, $J_1(t)$ denotes a Bessel function of the first kind and first order, $\mu = 2\pi R_a r/\lambda f_1$ is the radial spatial frequency with f_1 denoting the focal length of L_1, $E_{f0} = \pi R_a^2/\lambda f_1$ is the normalized on-axis field distribution in the focal plane, $\rho = R_b/R_a$ and $\kappa = L_p/R_a$ are two dimensionless geometrical factors for characterizing the new phase aperture, respectively.

The intensity $I_f(\mu)$ at the Fourier plane in front of the nonlinear material is expressed as

$$\begin{aligned} I_f(\mu) &= \left|E_f(\mu)\right|^2 \\ &= 4I_{f0}\left\{\left(\frac{J_1(\mu)}{\mu}\right)^2 + 4\sin^2\left(\frac{\phi_L}{2}\right)\left[\left(\rho^2\frac{J_1(\rho\mu)}{\rho\mu}\right)^2\right.\right. \\ &\quad \left. + \rho^2\frac{J_1(\rho\mu)}{\rho\mu}\frac{J_1(\mu)}{\mu}\right] + 4\sin^2(\phi_L)\left(\kappa^2\frac{J_1(\kappa\mu)}{\kappa\mu}\right)^2 \\ &\quad \left. - 4\sin^2(\phi_L)\rho^2\kappa^2\frac{J_1(\rho\mu)}{\rho\mu}\frac{J_1(\kappa\mu)}{\kappa\mu}\right\}, \end{aligned} \tag{3}$$

where $I_{f0} = \pi^2 R_a^4/\lambda^2 f_1^2$ indicates the normalized on-axis irradiance distribution in the focal plane.

On the basis of the property of Fourier transform, the spread of the field in the Fourier plane versus spatial frequency is inversely proportional to the expansion of the input field. According to the relationship $L_p < R_b < R_a$, the field versus spatial frequency of TH3 broadens widest, and that of TH1 broadens narrowest in the focal plane. Thus, one can obtain that the intensity of TH1 is strongest in the Fourier plane, and the corresponding intensity distribution of TH3 is weakest in the low-spatial-frequency region, as shown in Fig. 2. Hereinto, $\rho = 0.67$ and $\kappa = 0.33$ are chosen in the numerical simulation.

In this paper, we focus our attention on the case that the nonlinear material is a lossless Kerr medium, accordingly, the linear and nonlinear absorption are both neglected. When the nonlinear sample is regarded as 'thin', the field at the exit surface of the sample can be written as

$$E_{fL}(\mu) = E_f(\mu)\exp\bigl(j\phi_{NL}(\mu)\bigr), \tag{4}$$

where nonlinear phase shift $\phi_{NL}(\mu)$ is given by

$$\phi_{NL}(\mu) = kn_2 I_f(\mu)L = \phi_0 I_f(\mu)/I_{f0} = \phi_0 F(\mu), \tag{5}$$